930 A GENERAL SOLUTION FOR A LOADED BOLT-HOLE IN PIEZO-COMPOSITE PLATES

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This paper deals with a two-dimensional problem of a loaded circular hole in an infinite piezoelectric composite plate by use of the complex variable method. First, we give the Green function for a generalized point load acted at an arbitrary point at the rim of the hole, and then derive a general solution for the case of arbitrarily distributed mechanical and electric loading on the hole surface based on the superposition principle. In general, the solution is in form of series, but its novel feature is that the coefficients involved in the solution can be easily written out once the distributed loading on the hole surface is given.

Key Words : Piezoelectric Plate, Circular Hole, Stress Concentration, General Solution

1. Introduction

Piezoelectric composite plates are being increasingly used in aeronautical and aerospace structures. Holes are usually unavoidable in these structural designs. Existence of holes causes geometric discontinuities and induces high stress concentrations, which may finally leads to fracture/failure of structures. Thus, it is important to study local stress concentrations around holes in piezoelectric materials for the reliability design of smart structures. However, the previous works were mainly restricted in the cases of free hole under remote-field loadings^(1,2). In the present work, we develop a general solution for stress analysis of piezoelectric composite plate with a loaded bolt-hole based on complex variable method. In general, the solution can be expressed in series form, but its novel feature is that the coefficients involved in the solution can be easily written out provided that the loading distribution on the hole surface is specified.

2. Basic equations

Consider a symmetrical cross-ply piezoelectric composite plate that comprises piezo-ceramic fiber composite plies and polymeric matrix material piles. It is assumed that the plate is in the state of a two-dimensional deformation under the mechanical and electric loading in the x_1 - x_2 plane, in which the field variables can be expressed as

$$\langle \sigma_{22}, -\sigma_{12}, \sigma_{11} \rangle = 2 \operatorname{Re} \sum_{k=1}^3 \langle 1, \mu_k, \mu_k^2 \rangle \Phi_k(z_k), \quad z_k = x_1 + \mu_k x_2 \quad (1)$$

$$\langle u_1, u_2 \rangle = 2 \operatorname{Re} \sum_{k=1}^3 \langle p_k, q_k \rangle \varphi_k(z_k) \quad (2)$$

$$\langle E_1, E_2 \rangle = 2 \operatorname{Re} \sum_{k=1}^3 \langle 1, \mu_k \rangle \Phi_k(z_k) \quad (3)$$

$$\langle D_1, -D_2 \rangle = 2 \operatorname{Re} \sum_{k=1}^3 \langle \mu_k, 1 \rangle \Phi_k(z_k) \quad (4)$$

where σ_{ij} , u_i , E_i and D_i are the components of

stress, displacement, electric field and electric displacements, respectively. μ_k , p_k , q_k , κ_k and λ_k are known reduced elastic coefficients⁽¹⁾. Re denotes taking the real part, $\varphi_k(z_k)$ are unknown complex potentials, and $\Phi_k(z_k) = \varphi_k(z_k)/dz_k$.

The complex potentials $\varphi_k(z_k)$ can be determined from the following boundary equations:

$$2 \operatorname{Re} \sum_{k=1}^3 \varphi_k(z_k) = -\int_0^s t_2 ds \quad (5)$$

$$2 \operatorname{Re} \sum_{k=1}^3 \mu_k \varphi_k(z_k) = \int_0^s t_1 ds \quad (6)$$

$$2 \operatorname{Re} \sum_{k=1}^3 \lambda_k \varphi_k(z_k) = -\int_0^s D_n ds \quad (7)$$

where t_1 and t_2 represent the force components along the x_1 and x_2 axes, respectively; s is the arc-length on the boundary, and D_n is the normal component of electric displacement.

3. General solution

Consider a piezoelectric plate with a circle hole Γ , on which a point force (t_{10}, t_{20}) and a point charge Q_0 is exerted at an arbitrary point $z_0 = x_{10} + ix_{20}$, as shown in Fig.1.

Fig.1: A point force and point charge at the hole

Introduce the mapping function $z_k(\zeta_k)$:

$$z_k(\zeta_k) = R_k \left(\zeta_k + \frac{m_k}{\zeta_k} \right), R_k = \frac{a(1-i\mu_k)}{2}, m_k = \frac{1+i\mu_k}{1-i\mu_k} \quad (8)$$

which transforms the exterior of the elliptical contour Γ_i and the points z_{i0} in the z_i -plane into the exterior of a unit circle γ in the ζ_i -plane and the same point σ_0 on the unit circle, respectively, where Γ_i and z_{i0} are formed from Γ and z_0 by the affine transformation $z_i = x_i + \mu_i x_2$. Based on (5)-(8) and the use of Muskhelishvili theory⁽³⁾, we can obtain the complex potentials (Green functions), omitting mathematical details, for this case as

$$\varphi_k(\zeta_k) = \beta_k \ln \zeta_k + \frac{1}{2\pi i} \sum_{j=1}^3 \Lambda_j F_j \ln \left(1 - \frac{\sigma_0}{\zeta_k} \right), \quad k = 1, 2, 3 \quad (9)$$

where $\varphi_k(\zeta_k) = \varphi_k[z_k(\zeta_k)]$, and

$$\beta = \frac{1}{2\pi i} \mathbf{A}^T \mathbf{F}, \quad \mathbf{F} = (t_{20}, -t_{10}, -Q_0)^T, \quad \mathbf{A} = \mathbf{B}^{-1}$$

$$\mathbf{A} = \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ \kappa_1 & \kappa_2 & \kappa_3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 1 & 1 \\ \mu_1 & \mu_2 & \mu_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}.$$

When an arbitrarily distributed load t_{1s} , t_{2s} and Q_s are applied at the rim of the hole, using the superposition principle and the identity:

$$\ln \left(1 - \frac{\sigma_0}{\zeta_k} \right) = - \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\sigma_0}{\zeta_k} \right)^n$$

we have from (9) that

$$\varphi_k(\zeta_k) = \beta_k \ln \zeta_k + \sum_{n=1}^{\infty} a_{k,n} \zeta_k^{-n}, \quad k = 1, 2, 3 \quad (10)$$

where the coefficients $a_{k,n}$ can be determined by

$$a_{k,n} = \frac{1}{2\pi i} \oint_{\gamma} [\Lambda_{k1} t_{2s} - \Lambda_{k2} t_{1s} - \Lambda_{k3} Q_s] \sigma_0^n ds, \quad n = 1, 2, 3, \dots \quad (11)$$

4. Example

Consider a loaded bolt-hole in an infinite plate as shown in Fig.2. It is assumed that the resultant force transferred by the bolt is P , and the normal contact force along the surface of the hole is in the way:

$$p_s(\theta) = p_0 \sin \theta, \quad 0 < \theta < \pi \quad (12)$$

where $p_0 = 2P/\pi a$.

If neglecting the friction effect between the bolt and the hole, one has

$$t_1 ds = p_s ds \cos \theta = ap_0 \sin \theta \cos \theta d\theta$$

$$t_2 ds = p_s ds \sin \theta = ap_0 \sin^2 \theta d\theta$$

$$Q_s ds = 0 \quad (13)$$

Inserting (13) and (12) into (11) results in

$$a_{k,n} = \frac{ap_0}{4\pi i} \left(\frac{\Lambda_{k1} - i\Lambda_{k2}}{n+2} - \frac{2\Lambda_{k1}}{n} + \frac{\Lambda_{k1} + i\Lambda_{k2}}{n-2} \right), \quad \text{for } n = 1, 3, 5, \dots$$

$$a_{k,2} = -\frac{ap_0}{16\pi} (\Lambda_{k1} + i\Lambda_{k2}) \ln(-2), \quad \text{for } n = 2$$

$$a_{k,n} = 0, \quad \text{for } n = 4, 6, 8, \dots \quad (14)$$

Furthermore, substituting (14) into (10) we obtain the final complex potentials as

$$\varphi_k(\zeta_k) = \beta_k \ln \zeta_k + a_{k,2} \zeta_k^{-2} + \sum_{n=1}^{\infty} a_{k,2n+1} \zeta_k^{-(2n+1)}, \quad k = 1, 2, 3 \quad (15)$$

Fig.2: A distributed force on the hole surface

Similarly, one can give more examples using (10) and (11). For example, for the case shown in Fig.3:

Fig.3: A pair of self-equilibrium forces on the hole surface

we have the complex potentials as

$$\varphi_k(z_k) = \frac{P}{2\pi i} \Lambda_{k1} \ln \frac{\zeta_k - i}{\zeta_k + i} \quad (16)$$

and the hoop stress at the point h as

$$\sigma_{\theta}|_{\theta=0} = \sigma_{22} = \frac{P}{\pi a} \operatorname{Re} \left(\frac{i\Lambda_{11}}{\mu_1} + \frac{i\Lambda_{21}}{\mu_2} + \frac{i\Lambda_{31}}{\mu_3} \right) \quad (17)$$

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