International Journal of Modern Physics A
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Fiber Bundle and Matrix Models*

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Received Day Month Year

Revised Day Month Year

We extend the large N reduction and the compactification in matrix models to those on manifolds represented as fiber bundles. We also discuss application of our results to the AdS/CFT correspondence and topological field theories.

Keywords: fiber bundle; matrix model; N=4 super Yang Mills.

PACS numbers: 11.25.Tq, 11.10.Nx

Emergence of space-time is one of the key concepts in matrix models as non-perturbative definition of superstring^{1,2,3}. This phenomena was first observed in the large N reduction⁴. It states that a large N planar gauge theory is equivalent to the matrix model that is its dimensional reduction to zero dimensions unless the $U(1)^D$ symmetry is broken, where D denotes the dimensionality of the original gauge theory. However, the $U(1)^D$ symmetry is in general spontaneously broken for $D > 2$. There are two improved versions of the large N reduced model that preserve the $U(1)^D$ symmetry. One is the quenched reduced model. The other is the twisted reduced model, which was later rediscovered in the context of the non-commutative field theories. The compactification in matrix models⁵, which we call the matrix T-duality in this talk, shares the same idea with the large N reduced model. The statement of the matrix T-duality is that $U(N)$ Yang-Mills (YM) on $R^p \times S^1$ is equivalent to $U(N \times \infty)$ YM-higgs on R^p which is a dimensional reduction of $U(N \times \infty)$ YM on $R^p \times S^1$ if a periodicity (orbifolding) condition is imposed. Also, deconstruction⁶ and supersymmetric lattice gauge theories inspired by it⁷ are analogs of the matrix T-duality.

The above developments are all concerning gauge theories on flat space-time. It is important to understand how gauge theories on curved space-time is realized in matrix models or gauge theories in lower dimensions, because it would lead us to gain some insights into how curved space-time is realized in the matrix models as nonperturbative definition of superstring. This is the first motivation of our work.

*This talk is based on the collaborations with T. Ishii, G. Ishiki, K. Ohta, S. Shimasaki and Y. Takayama

Note that an interesting approach to description of curved spacetime by matrices was proposed in ⁸. Since the information of topology is relevant for compactification in string theory, it should be included in the matrix models as nonperturbative definition of superstring. Thus it is worthwhile, for instance, to study how topologically nontrivial objects are realized in matrix models. It is also important to investigate realization of the topological field theories in matrix models, because the topological field theories have been developed to efficiently describe the topological aspects of field theories. This is the second motivation of our work. The third motivation of our work is as follows. The AdS/CFT correspondence, a typical example of which is a conjecture of the correspondence between type IIB superstring on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ super Yang Mills (SYM), has been intensively investigated for a decade. However, it has not completely proven yet, partially because it is a strong/weak duality with respect to the coupling constants. It is, therefore, relevant to give a nonperturbative definition of $\mathcal{N} = 4$ SYM which enables us to study its strong coupling regime. The lattice gauge theory is a promising candidate for such a nonperturbative definition. Supersymmetric gauge theories on the lattice is, however, generally difficult to construct, although it should be remarked that there have been some interesting attempts so far. Giving such a nonperturbative definition should provide some insights into the problem of nonperturbative formulation of supersymmetric gauge theories.

In ⁹, we found relationships among the $SU(2|4)$ symmetric theories, which include $\mathcal{N} = 4$ SYM on $R \times S^3/Z_k$, 2+1 SYM on $R \times S^2$ and the plane wave matrix model (PWMM). These theories are obtained by consistently truncating the Kaluza-Klein modes of $\mathcal{N} = 4$ SYM on $R \times S^3$. In particular, the latter two theories can be regarded as dimensional reductions of $\mathcal{N} = 4$ SYM on $R \times S^3$. These theories possess common features: mass gap, discrete spectrum and many discrete vacua. From the gravity duals of those vacua proposed in ¹⁰, the following relations between these theories are suggested: A) the theory around each vacuum of 2+1 SYM on $R \times S^2$ is equivalent to the theory around a certain vacuum of PWMM, and B) the theory around each vacuum of $\mathcal{N} = 4$ SYM on $R \times S^3/Z_k$ is equivalent to the theory around a certain vacuum of 2+1 SYM on $R \times S^2$ with the orbifolding condition imposed. In ⁹, the relations A) and B) were shown directly on the gauge theory side. The results in ⁹ not only serve as a nontrivial check of the gauge/gravity correspondence for the $SU(2|4)$ theories, but they are also interesting from a point of view of the reduced model as follows. In the relation A), it was manifestly shown that the continuum limit of concentric fuzzy spheres correspond to multi monopoles. The relation B) can be regarded as an extension of the matrix T-duality to that on a nontrivial $U(1)$ bundle, S^3/Z_k , whose base space is S^2 . Combining the relations A) and B) leads to the relation C) that the theory around each vacuum of $\mathcal{N} = 4$ SYM on $R \times S^3/Z_k$ is equivalent to the theory around a certain vacuum of PWMM with the orbifolding condition imposed. In particular, for $k = 1$, $\mathcal{N} = 4$ SYM on $R \times S^3$ is realized in PWMM. This suggests an interesting possibility of a nonperturbative formulation of $\mathcal{N} = 4$ SYM on $R \times S^3$ by PWMM, which would lead to a nonperturbative test

of the AdS/CFT correspondence.

In ¹¹, we generalized the matrix T-duality to that on an arbitrary $U(1)$ bundle. In ¹³, we further investigated the large N reduction and the matrix T-duality on curved space. First, we develop a dimensional reduction of YM on the total space to YM-higgs on the base space for a general principal bundle. This also enables us to dimensionally reduce YM on a group manifold to a matrix model. Second, as an extension of the work ¹¹, in the case in which the fiber is $SU(2)$, we show that YM on the total space is equivalent to a certain vacuum of YM-higgs on the base space with the periodicity imposed. This enables us to realize YM on an $SU(2)^k \times U(1)^l$ bundle in YM-higgs on its base space. We apply the above results to the case of $SU(n+1)$ as the total space. $SU(n+1)$ is viewed as $SU(n)$ bundle over $SU(n+1)/SU(n) \simeq S^{2n+1}$ or $SU(n) \times U(1)$ bundle over $SU(n+1)/(SU(n) \times U(1)) \simeq CP^n$, and $SU(n+1)/SU(n) \simeq S^{2n+1}$ is viewed as $U(1)$ bundle over CP^n . By the dimensional reduction, we obtain YM-higgs on S^{2n+1} and CP^n and a matrix model. We found that the commutative (continuum) limit of gauge theory on fuzzy CP^n realized in the matrix model coincides with YM-higgs on CP^n . Namely, we show that the theory around each monopole vacuum of YM-higgs on CP^n is equivalent to the theory around a certain vacuum of the matrix model. By combining this with the extended matrix T-duality, we realize YM-higgs on $SU(n+1)/SU(n) \simeq S^{2n+1}$ in the matrix model. We also showed that the extended matrix T-duality of the $U(1)$ case developed in ¹¹ can be interpreted as Buscher's T-duality.

As an application of our results, in ¹², we showed relationships among Chern-Simons theory on a $U(1)$ bundle over a Riemann surface, BF theory with a mass term on the Riemann surface, which is equivalent to two-dimensional Yang-Mills on the Riemann surface, and a matrix model. We also found that in the case where the Riemann surface is S^2 , the former two (topological) field theories associated with topological strings are realized in the matrix model.

The global gauge symmetry of the reduced model is naturally interpreted as the local gauge symmetry of the original gauge theory. Thus the two improved versions of the reduced model give a nonperturbative definition of the planar non-supersymmetric gauge theory which preserves the gauge symmetry manifestly as a lattice gauge theory. It seems, however, that supersymmetry cannot be manifestly preserved in the twisted reduced model at least on flat space while the gauge symmetry and supersymmetry cannot be manifestly preserved simultaneously in the quenched reduced model.

While the matrix T-duality is not restricted to the planar limit, it requires the size of matrices to be infinite from the beginning for the orbifolding condition to be imposed so that it cannot be used for a nonperturbative definition of supersymmetric gauge theories as it stands. It was argued in ⁷ that a lattice theory for a supersymmetric gauge theory preserving part of supersymmetries manifestly can be obtained from its reduced model by imposing an orbifolding condition, such that few parameters are required to be fine-tuned. This construction can be regarded as

an finite-size matrix analog of the matrix T-duality. However, it has a problem of flat directions which is analogous to the problem of the $U(1)^D$ symmetry breaking. To overcome this problem, for instance, one needs to introduce a mass term for the scalar field, which leads to no preservation of supersymmetries.

In ¹⁴, we propose a nonperturbative definition of $\mathcal{N} = 4$ SYM on $R \times S^3$ which is equivalently mapped to $\mathcal{N} = 4$ SYM on R^4 at the conformal point and possesses the superconformal symmetry, the $SU(2, 2|4)$ symmetry. We restrict ourselves to the planar limit. By referring to the relation C) in ⁹, we regularize $\mathcal{N} = 4$ SYM on $R \times S^3$ nonperturbatively by PWMM. Restriction to the planar limit enables us not to impose the orbifolding condition and to consider finite-size matrices such that the size of matrices play a role of the ultraviolet cutoff. Thus we consider an extension of the reduced model to curved space rather than the matrix T-duality as a relation between $\mathcal{N} = 4$ SYM on $R \times S^3$ and $2+1$ SYM on $R \times S^2$. Possible UV/IR mixing which may break the relation between $2+1$ SYM on $R \times S^2$ and PWMM is probably avoided thanks to restriction to the planar limit and sixteen supersymmetries. Because PWMM is a massive theory, there is no flat direction and the quenching prescription is not needed. Our regularization manifestly preserves the $SU(2, 2|4)$ symmetry, a subgroup of the $SU(2, 2|4)$ symmetry, and the gauge symmetry. In particular, sixteen supersymmetries among thirty-two supersymmetries are respected in our regularization. We check that the 1-loop beta function vanishes, which is consistent with the superconformal symmetry, and that the Ward identity is indeed satisfied at the 1-loop level, which is consistent with the gauge symmetry. We need to verify restoration of the $SU(2, 2|4)$ symmetry in the continuum limit to establish the nonperturbative definition of $\mathcal{N} = 4$ SYM by PWMM.

I would like to thank the organizers for inviting me to this wonderful conference. The present work is supported in part by Grant-in-Aid for Scientific Research (No. 19540294) from the Ministry of Education, Culture, Sports, Science and Technology.

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