A Switched-Capacitor Successive-Approximation A/D Converter

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Abstract—A switched-capacitor successive-approximation analog-to-digital (A/D) converter is developed which incorporates a serial digital-to-analog (D/A) subconverter for generating the threshold voltage sequence. The conversion process is insensitive to parasitic capacitances and offset voltages of a comparator and op-amps. Error analyses and Spice simulations show that a resolution higher than 11 b, a sampling rate up to 440 kps with 10-b resolution, and a power consumption less than 60 mW are attainable with the monolithic implementation using present CMOS technologies. The required chip area is small because of a low device count. The architecture described herein is therefore best suited for high-accuracy, medium-speed A/D converters in ASIC's. A prototype converter breadboarded using discrete components has confirmed the principles of operation.

I. INTRODUCTION

ANALOG-TO-DIGITAL (A/D) conversion is an important topic in digital processing of analog signals. Recent work concentrates on the oversampling $\Delta\Sigma$ converter for audio signal processing and the flash converters for video signal processing. In between these two extremes, a successive-approximation method is still widely used for moderate-speed, high-accuracy A/D conversion. In this method, an input analog voltage is converted to the binary number by being compared successively with the quantized threshold voltage sequence which, starting with a half-reference voltage, increases or decreases depending on the bit value determined in the previous cycle to approach the input analog voltage. Therefore, a crucial component in the architecture is a digital-to-analog (D/A) subconverter to produce the threshold voltage sequence.

In bipolar technologies, such a D/A converter is realized by binary-weighted current sources combined with an R-2R ladder. In CMOS technologies, however, it is difficult to implement accurately weighted current sources because of the relatively large variation in threshold voltage of MOS transistors. Thus the binary-weighted capacitor array or its combination with a tapped resistor string is commonly used instead [1], [2]. This solution is straightforward, but requires a large number of unit capacitors. Therefore, a large chip area and a long settling time are inevitable.

These problems stem from the use of a parallel D/A converter for generating the threshold voltage sequence. A successive-approximation A/D conversion is a serial process. Therefore, replacing the extravagant parallel D/A converter by a simple serial one will resolve the above-mentioned difficulties. Based on this idea, a serial D/A converter is developed which starts a conversion with the most significant bit (MSB) [3], [4]. Following these introductory remarks, the conversion algorithm and the architecture of the successive-approximation A/D converter using this serial D/A converter are described. A resolution, a sampling rate, and a power consumption when the A/D converter is fabricated in the monolithic form using present CMOS technologies are estimated by error analyses and Spice simulations. Experimental results obtained by a prototype converter built using discrete components are also given to confirm the principles of operation.

II. CONVERSION ALGORITHM AND ARCHITECTURES

A. Unipolar Conversion

Figure 1(a) shows a block diagram of the successive-approximation A/D converter developed for unipolar conversion [5], [6]. Its operation is controlled by the non-overlapping three-phase clocks ϕ_1 , ϕ_2 , and ϕ_3 and the state signal S which distinguishes between the sample ($S = "1"$) and conversion ($H = \bar{S} = "1"$) states. Their timing is shown in Fig. 1(b). In the $\phi_1 = "1"$ phase of the sample state, an input analog voltage is sampled into C_1 . In the subsequent conversion state, the analog voltage thus stored in C_1 is compared with the threshold voltage sequence generated by the D/A converter, to be converted into its digital equivalent.

Now assume that the D/A converter produces the voltage sequence $V_i(i)$ in the i -th cycle of the conversion state. The voltage $V_c(i)$ at the inverting input terminal of the comparator is then given by

$$V_c(i) = \frac{-C_1 V_a + C_2 V_i(i)}{C_1 + C_2 + C_p} + V_{os}, \quad (i = 1, 2, \dots, n) \quad (1)$$

where C_p is the parasitic capacitance between the inverting input terminal of the comparator and ground, and V_{os} is the offset voltage of the comparator. It is clear from (1)

Manuscript received July 9, 1992; revised January 25, 1993.

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IEEE Log Number 9209220.

Fig. 1. A block diagram (a) and timing diagram of digital signals (b) of a successive-approximation A/D converter for unipolar conversion.

that if $C_1 = C_2$, and

$$V_t(i) = \left(\sum_{j=0}^{i-1} b_j 2^{-j} + 2^{-i} \right) V_r \quad (2)$$

then the value of the i -th bit, b_i , can be determined to be 1 or 0 depending on the output of the comparator. In (2), V_r is the reference voltage and $b_0 = 0$. It should also be noted that the conversion process is insensitive to both the parasitic capacitance C_p and the offset voltage V_{os} . The capacitance mismatch between C_1 and C_2 has no effect upon the conversion linearity, either.

The output b_i of the comparator in each cycle indicates the voltage range to which the analog input voltage V_a belongs. For instance, assume the values of the first three bits to be $b_1 b_2 b_3 = (101)_B$. The most significant bit (MSB) $b_1 = 1$ implies that V_a is in the range from $V_r/2$ to V_r . The next MSB $b_2 = 0$ means that V_a is in the range from $V_r/2$ to $(3/4)V_r$, and $b_3 = 1$ in the range from $(5/8)V_r$ to $(3/4)V_r$. It is obvious from this example that the voltage range is successively halved. Let the lower and upper bounds of the voltage range determined in the i -th cycle be $V_L(i)$ and $V_U(i)$, respectively. Then, $V_L(i+1)$ and $V_U(i+1)$ in the next $(i+1)$ th cycle are given by [4]

$$V_L(i+1) = V_L(i) + b_i 2^{-i} V_r \quad (3)$$

$$V_U(i+1) - V_L(i+1) = \frac{V_U(i) - V_L(i)}{2} = 2^{-i} V_r \quad (4)$$

where $V_L(1) = 0$ and $V_U(1) = V_r$. The threshold voltage $V_t(i+1)$ required for the next comparison is then given by

$$V_t(i+1) = \frac{V_L(i+1) + V_U(i+1)}{2} \quad (5)$$

Rewriting (2) as

$$V_t(i+1) = \frac{V_r}{2} - \sum_{j=1}^i (-1)^{b_j} 2^{-(j+1)} V_r \quad (6)$$

and expressing (3)–(5) in the recursive forms, we can obtain the algorithm for generating the threshold voltage sequence:

$$V_t(i+1) = [V_t(i) + \bar{b}_i V_L(i+1) + b_i V_U(i+1)]/2 \quad (7)$$

$$V_L(i+1) = b_i V_t(i) + \bar{b}_i V_L(i) \quad (8)$$

$$V_U(i+1) = \bar{b}_i V_t(i) + b_i V_U(i) \quad (9)$$

where $V_t(1) = V_r/2$.

The above recursive algorithms indicate that the D/A converter can be realized by the sample/hold and divide-by-2 stages. Such a realization is shown in Fig. 2. In the i -th cycle of the conversion state, three capacitors C_L , C_U , and C_T store the lower bound $V_L(i)$, upper bound $V_U(i)$, and threshold voltage $V_t(i)$ sequences, respectively. C_T is a unit capacitor matched to C_S . In the $\phi_1 = "1"$ phase of the next cycle, either $V_L(i)$ or $V_U(i)$ is updated to $V_t(i)$, depending on the bit value b_i determined by the previous conversion. In the next $\phi_2 = "1"$ phase, C_T is charged to the new $V_L(i+1)$ or $V_U(i+1)$. In the subsequent $\phi_3 = "1"$ phase, C_T and C_S are connected in parallel to perform the divide-by-2 operation involved in (7). The threshold voltage sequence $V_t(i+1)$ thus generated is compared with the input analog voltage, to determine the value of the next bit b_{i+1} . The operation of the D/A converter is also insensitive to parasitic capacitances and offset voltages of op-amps.

B. Bipolar Conversion

In the unipolar conversion described above, the analog input voltage V_a with positive polarity is compared with the positive reference voltage V_r by subtracting V_r from V_a . The subtraction is replaced by the addition, in fact, by inverting the polarity of V_a stored in C_1 . Therefore, the architectures in the previous section can be extended to a bipolar conversion if they are modified such that the D/A converter produces the negative threshold voltage sequence when an input analog voltage is negative. Such modifications are shown in Figs. 3 and 4.

An extra state, designated by H_1 , is introduced into the timing diagram shown in Fig. 3(b) to detect the polarity of an input analog voltage V_a . The sign bit b_0 is set to 1 if $V_a \geq 0$, or to 0 otherwise. This value is held during the subsequent conversion state H_2 by the latch 2 shown in Fig. 3(a). It should be pointed out here that because of

Fig. 2. A circuit diagram of the D/A converter for unipolar conversion.

Fig. 4. A circuit diagram of the D/A converter for bipolar conversion.

Fig. 3. A block diagram (a) and timing diagram of digital signals (b) of a successive-approximation A/D converter for bipolar conversion.

the polarity inversion, the magnitude relation between V_a and $V_t(i)$ is reversed when V_a is negative. The Exclusive-OR gate is used in Fig. 3(a) to retrieve the correct relation.

The recursive algorithms for generating the threshold voltage sequence are the same as those in the unipolar conversion, except for V_r being replaced by $-(-1)^{b_0}V_r$. The D/A converter modified is shown in Fig. 4. It produces the bipolar threshold voltage sequence as follows. In the $\phi_1 = "1"$ phase of the sample state, matched ca-

pacitors C_T and C_S are both discharged. Then in the next $\phi_2 = "1"$ phase, C_T and C_S are charged to V_r with the polarity indicated. In the next $\phi_3 = "1"$ phase, the value of b_0 is determined. If $b_0 = 1$, op-amp A_2 holds the positive reference voltage stored in C_T , to charge C_U to V_r , while op-amp A_1 discharges C_L to ground. Therefore, the lower and upper bounds start with $V_L(0) = 0$ and $V_U(0) = V_r$, respectively. The charge stored in C_S is discharged to ground in the $\phi_1 = "1"$ phase of the H_1 state. If $b_0 = 0$, on the other hand, op-amp A_2 holds the negative reference voltage stored in C_T , to charge C_U to $-V_r$. Therefore, the upper bounds start with $V_U(0) = -V_r$. In either case, the D/A converter follows the same process as that of the unipolar conversion in the subsequent conversion state ($H_2 = "1"$), to generate the threshold voltage sequence. The operation is insensitive again to parasitic capacitances and the offset voltages of op-amps.

III. CMOS DESIGN

The op-amp and the latched comparator have been designed in CMOS. Figure 5 shows a circuit diagram of the op-amp. The W/L ratios, assuming a 3- μm process, are also indicated. The circuitry consists of the input differential pair M_1 and M_2 , folded cascade stage M_7 and M_8 , output stage M_3 – M_6 , constant current source M_9 , current mirror M_{10} and M_{11} , and bias chain M_{12} – M_{15} [7]–[9]. The folded cascade stage makes it possible to realize a high gain without causing the instability. Figure 6(a) shows the frequency characteristics of the op-amp. It is obvious that the load capacitance C_L and the output impedance form the dominant pole. The second pole due to the stray capacitances at the low-impedance node is estimated from the phase characteristics to be at about 500 MHz. These pole locations lead to the unity-gain frequency extending beyond 100 MHz. Connecting the source follower to lower the output impedance, one can obtain the wider bandwidth. Figure 6(b) shows the transient response of the op-amp simulated using Spice. The output voltage of the op-amp loaded by a 2-pF capacitor settles within 0.1% of the final value in 80 ns. The 0.1% settling time can be reduced to 40 ns if the load is decreased to 1 pF.

Figure 7 shows a circuit diagram of the latched comparator. The comparator section consists of four cascaded

tion due to clock feedthrough, and capacitance mismatch between C_T and C_S .

Let the open-loop gain of op-amps A_1 and A_2 be A . Then the recursive form (7) of the threshold voltage sequence is modified as follows:

$$V'_i(i+1) = \left[\frac{A}{A+1} \right] \cdot \frac{V'_i(i)}{2} + \left[\frac{A}{A+1} \right]^2 \cdot \frac{b_i V_U(i+1) + \bar{b}_i V_L(i+1)}{2} \quad (10)$$

where $V'_i(1) = [A/(A+1)] \cdot (V_r/2)$, and a prime denotes the erroneous quantity. This erroneous sequence causes the nonlinearity which depends on the bit pattern. Figures 9(a) and (b) show the differential (DNL) and integral nonlinearity (INL) in 11-b A/D conversion when A is 80 dB. These simulations indicate that the finite gain affects the INL more seriously than the DNL, and that INL becomes maximum when all bits assume 0. If the maximum error is less than 1 LSB, then the conversion is accurate down to its LSB. Formulating this condition, we have the criterion:

$$2^n < \frac{2(A^2 + 3A + 2)}{A^2 \left[\frac{A}{2(A+1)} \right]^n + 3A + 2} \quad (11)$$

Now assume that switches M_{10} and M_{15} in Fig. 2 inject the feedthrough charge Q_f into the inverting input terminals of op-amps A_1 and A_2 , respectively, when they turn off. The threshold voltage sequence including Q_f is given by

$$V'_i(i+1) = \frac{V'_i(i) + b_i V_U(i+1) + \bar{b}_i V_L(i+1)}{2} + \frac{Q_f}{C_n} \quad (12)$$

where $V'_i(1) = V_r/2 + Q_f/C_n$ and $C_n = C_S = C_T$. It is found by the simulations based on (12) that the DNL and INL are of almost the same order of magnitude and independent of the bit pattern. The condition to achieve the 1 LSB accuracy is given by

$$2^n < 1 + \frac{C_n V_r}{2 Q_f} \quad (13)$$

The threshold voltage sequence when capacitor mismatch ϵ is present between C_T and C_S is given by

$$V'_i(i+1) = \frac{(1-\epsilon)V'_i(i) + b_i V_U(i+1) + \bar{b}_i V_L(i+1)}{2-\epsilon} \quad (14)$$

where $V'_i(1) = [(1-\epsilon)/(2-\epsilon)] \cdot V_r$. Simulations based on (14) indicate that capacitance mismatch degrades se-

Fig. 9. The differential nonlinearity (DNL) (a) and the integral nonlinearity (INL) (b) in 11-bit A/D conversion when the finite gain of op-amps is 80 dB.

riously the INL. The condition for the maximum INL to be less than 1 LSB in an n -bit conversion is given by

$$\epsilon < \frac{2}{2^{n-2} + 1} \quad (15)$$

From (11), (13), and (15), one can obtain the requirements for, say, 11-b resolution as follows: $A > 80$ dB, $Q_f/C_n < 0.5$ mV when $V_r = 2$ V, and $\epsilon < 0.25\%$. The latter two requirements are easily met by present MOS technologies. Therefore, the op-amp design is the most critical in realizing a high resolution. The gain-compensated scheme proposed by Nagaraj *et al.* is effective to relax the gain requirement [11], [12]. The D/A converter accommodating such a scheme is shown in Fig. 10. Driven by the four-phase clock, the circuit compensates for the gain error based on the following principle. The gain error term $A/(A+1)$ in (10) is approximated, to first order, by $1 - 1/A$. The error voltage due to the finite gain is thus $-V_o/A$, where V_o is $V_U(i)$ or $V_L(i)$ for op-amp A_1 and $V_i(i)$ for op-amp A_2 . Op-amp A_1 detects the error voltage and stores it into C_f in the ϕ_1 and ϕ_2 phases. The error voltage stored in C_f cancels the gain error that

Fig. 10. A circuit diagram of the gain-compensated D/A converter.

would otherwise appear in the ϕ_3 and ϕ_4 phases. Though the clock phases differ, op-amp A_2 operates similarly to cancel the first-order gain error. The residual error is therefore of the second order, which implies that the effective gain is A^2 . Referring to Fig. 6(a), the open-loop gain of the aforementioned op-amp is 42 dB at 1 MHz when the load is 1 pF. A resolution higher than 11-b can be expected from the improved D/A architecture, if the clock is lower than 1 MHz. Since the open-loop gain rolls off at -6 dB/oct, the resolution decreases to 9-b at 5 MHz.

B. Conversion Speed

The duration of each phase necessary for the proper operation can be determined by examining the operation of the D/A converter shown in Fig. 10. In the ϕ_1 phase, C_T and C_S share the charge, and op-amp A_2 outputs the new $V_i(i)$ thus produced, to determine the value of b_i . Assume that all the capacitors are $C = 1$ pF, and the 'on' resistance R_{on} of each switch is 2 k Ω . Then, the ϕ_1 phase should last $t_s + 7CR_{on} + t_r = 84$ ns, where $t_s = 40$ ns is the 0.1% settling time of the op-amp and $t_r = 30$ ns is the response time of the latched comparator. The charging time of C_{pU} or C_{pL} is $14CR_{on} = 28$ ns, which is shorter than t_r . In the ϕ_2 phase, op-amp A_2 charges C_U or C_L to $V_i(i)$, while op-amp A_1 holds the voltage across C_{pU} or C_{pL} . The time required for charging C_U or C_L is 28 ns, which is shorter than the 40 ns required for op-amp A_1 to settle. The time required for the ϕ_3 phase can be calculated by the same procedure to be $t_s + 14CR_{on} = 68$ ns, and that for the ϕ_4 phase to be $t_s + 7CR_{on} = 54$ ns. Some of the above processes are carried out concurrently. Therefore, the time required for a 1-b conversion is estimated to be 230 ns. This results in the maximum sampling rate 440 ksp/s with 10-b resolution.

C. Power Consumption

The op-amp shown in Fig. 5, which is designed to slew at 280 V/ μ s, dissipates 14 mW when $V_{DD} = -V_{SS} = 5$ V and $C_L = 2$ pF. The power dissipation of the latched comparator is 16 mW. The total power consumption, in-

Fig. 11. The differential nonlinearity (DNL) (a) and the integrated nonlinearity (INL) (b) of the prototype 9-bit successive-approximation A/D converter.

cluding the power dissipated in the digital circuitry and the reference source, is estimated to be less than 60 mW.

V. PROTOTYPE CONVERTER

To confirm the principles of operation, a prototype converter based on Figs. 1 and 2 was breadboarded using discrete components. Op-amps and switches used are LF356 and MC14016, respectively. All the capacitors are 5.6 nF. Capacitance mismatch between C_T and C_S is less than 0.5%. The three-phase clock frequency is 20 kHz.

The DNL and INL of the 9-b prototype converter measured by a dc test are shown in Figs. 11(a) and (b), respectively. The maximum DNL is 0.875 LSB, and thus no missing code appears in the transfer characteristics. The maximum INL is 1.425 LSB. The open-loop gain of

the op-amps used is 50 dB at 20 kHz. The accuracy calculated by substituting this value for A in (11) agrees well with the measurement result.

VI. CONCLUSIONS

A switched-capacitor successive-approximation A/D converter has been described which incorporates a serial D/A subconverter for generating the threshold voltage sequence. Circuit diagrams for the CMOS implementation were also described. Error analyses and Spice simulations show that a resolution higher than 11 b, a sampling rate up to 440 ksp/s with 10-b resolution, and a power consumption less than 60 mW can be expected from its monolithic realization using present 3- μ m CMOS technologies. A higher sampling rate is possible with the time-multiplexed architecture [13], or with finer linewidths. It also features a low device count integrable onto a small chip area. These distinctive features make the present architecture most suited for analog interfaces of mixed analog and digital ASIC's.

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