Relativistic impulse approximation analysis of unstable calcium isotopes: $^{60-74}\text{Ca}$

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Recent relativistic mean-field calculations have provided nuclear distributions of Ca isotopes whose mass numbers are 60 through 74. We calculate observables of proton elastic scattering from these unstable isotopes and discuss relations between observables and nuclear distributions of such unstable nuclei. The calculations are based on relativistic impulse approximation (RIA) at incident proton energies from 100 through 500 MeV where predictions of RIA have been shown to provide good agreement with experimental data. To validate the use of optimal factorization and first-order calculations at these energies, contributions from the Fermi motion of the target nuclei and multiple scattering are estimated and compared with results calculated without these effects.

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I. INTRODUCTION

It is very important to get information about the neutron profile of heavy nuclei like ^{208}Pb to provide a strong constraint on nuclear models [1]. The recent relativistic mean-field (RMF) calculations have provided nuclear distributions for many unstable nuclei [2] as well as stable ones, and consequently there are opportunities to compare theoretical model calculations with experimental results in cases where the number of neutrons is much larger than that of protons or vice versa. Proton-elastic scattering is expected to be the most appropriate experiment to determine the target nuclear distributions for unstable nuclei as well as for stable ones. Observables are, however, expected to be quite restricted for unstable target nuclei, especially in the angular distributions, due to their very short lifetimes even using radioactive beams in inverse kinematics. Therefore one must choose sets of observables when the angular region is not large enough to analyze the target nucleus.

One of the observables of proton-elastic scattering, which is well known to be efficacious to study nuclear distributions, is the reaction cross section, and there have been experimental studies [3] and microscopic analyses [4]. In the present work, as in the previous work, the first dip position of the differential cross section is chosen. The reasons and mathematical basis have already been shown [5] and therefore are briefly reviewed here. For incident energies greater than 800 MeV, another parameter has been chosen as a length scale based on different model of nuclei [6]. The present work is interested in the energy region of the incident proton from 100 to 500 MeV, and in analyses within the relativistic framework.

A model analysis has already been proposed to determine nuclear density distributions of unstable nuclear targets, for example, the ^{60}Ca nucleus [7]. In this analysis two parameters of the nuclear distribution function have been determined by the reaction cross section and the position of the first dip of the differential cross section. The earlier model analysis was based on the optimal factorization and the first-order optical potential calculation. To confirm how well such model analysis

works in the case of unstable nuclei, in this work a medium effect and multiple scattering contributions are investigated. Specifically, the Fermi motion of the target nuclei is taken as the medium effect, and the second-order term in the multiple scattering expansion of the optical potential is taken as the multiple-scattering contribution.

In Sec. II, formulas on which the analysis is based are presented, and numerical results are given in Sec. III. Calculated results including contributions from Fermi motion and multiple scattering are compared to experimental data in Sec. III. The summary of this study appears in Sec. IV.

II. FORMULATION

In accordance with the prescription of the RIA [8], the Dirac optical potential is given in momentum space by

$$\hat{U}(\mathbf{p}, \mathbf{p}') = \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha} \bar{\psi}_{\alpha} \left(\mathbf{k} + \frac{1}{2}\mathbf{q} \right) (-\hat{M}) \psi_{\alpha} \left(\mathbf{k} - \frac{1}{2}\mathbf{q} \right), \quad (1)$$

where ψ_{α} is the relativistic nucleon wave function in the target nucleus and $\hat{M}$ denotes the Feynman nucleon-nucleon amplitudes. It is convenient for calculations to distinguish between proton and neutron contributions, and in the present study medium effects are considered in terms of NN amplitudes that depend on the Fermi momentum of nuclear matter. Hence, the optical potential in momentum space is described as follows

$$\begin{aligned} \hat{U}(\mathbf{p}', \mathbf{p}) = & -\frac{1}{4} \text{Tr}_2 \left\{ \int \frac{d^3k}{(2\pi)^2} \hat{M}_{pp} \right. \\ & \times \left(\mathbf{p}, \mathbf{k} - \frac{\mathbf{q}}{2} \rightarrow \mathbf{p}', \mathbf{k} + \frac{\mathbf{q}}{2} \right) \hat{\rho}_p(\mathbf{k}, \mathbf{q}) \Big\} \\ & -\frac{1}{4} \text{Tr}_2 \left\{ \int \frac{d^3k}{(2\pi)^2} \hat{M}_{pn} \right. \\ & \times \left(\mathbf{p}, \mathbf{k} - \frac{\mathbf{q}}{2} \rightarrow \mathbf{p}', \mathbf{k} + \frac{\mathbf{q}}{2} \right) \hat{\rho}_n(\mathbf{k}, \mathbf{q}) \Big\}, \quad (2) \end{aligned}$$

where $\hat{\rho}_p$ and $\hat{\rho}_n$ are density matrices for protons and neutrons, respectively. The trace is over the γ matrices with respect to the target nucleons and the subscript 2 in the trace corresponds

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to the target nucleons. To simplify the calculations, the local density approximation is used. The density matrices are given by the Fourier transforms

$$\hat{\rho}(\mathbf{k}, \mathbf{q}) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \hat{\rho}[\mathbf{r}(k)]. \quad (3)$$

Each nuclear density distribution is composed with three terms, i.e., scalar, vector, and tensor, as follows:

$$\hat{\rho}[\mathbf{r}(k)] = \rho_S[\mathbf{r}(k)] + \gamma_2^0 \rho_V[\mathbf{r}(k)] - \frac{i\boldsymbol{\alpha}_2 \cdot \hat{\mathbf{r}}}{2} \rho_T[\mathbf{r}(k)]. \quad (4)$$

The relation between the Fermi momentum k and the radius r is given via the vector density distribution,

$$k^3 = 3\pi^2 \rho_V(r). \quad (5)$$

Taking the optimal factorization into account, the optical potentials are written in the well-known $t\rho$ forms:

$$\begin{aligned} \hat{U}(\mathbf{p}', \mathbf{p}) = & -\frac{1}{4} \text{Tr}_2 \left\{ \hat{M}_{pp} \left(\mathbf{p}, \frac{\mathbf{q}}{2} \rightarrow \mathbf{p}', \frac{\mathbf{q}}{2} \right) \hat{\rho}_p(\mathbf{q}) \right\} \\ & - \frac{1}{4} \text{Tr}_2 \left\{ \hat{M}_{pn} \left(\mathbf{p}, \frac{\mathbf{q}}{2} \rightarrow \mathbf{p}', \frac{\mathbf{q}}{2} \right) \hat{\rho}_n(\mathbf{q}) \right\}. \end{aligned} \quad (6)$$

The relativistic density matrix $\hat{\rho}$ depends only on the momentum transfer $\mathbf{q}$, as follows;

$$\hat{\rho}(\mathbf{q}) = \rho_S(\mathbf{q}) + \gamma_2^0 \rho_V(\mathbf{q}) - \frac{i\boldsymbol{\alpha}_2 \cdot \mathbf{q}}{2m} \rho_T(\mathbf{q}), \quad (7)$$

where each term is a Fourier transformation of a coordinate-space density;

$$\rho_S(q) = 4\pi \int_0^\infty j_0(qr) \rho_S(r) r^2 dr, \quad (8)$$

$$\rho_V(q) = 4\pi \int_0^\infty j_0(qr) \rho_V(r) r^2 dr, \quad (9)$$

$$\rho_T(q) = -4\pi m \int_0^\infty \frac{j_1(qr)}{q} \rho_T(r) r^2 dr. \quad (10)$$

Nuclear densities, provided by the relativistic mean-field theory [2], are described in terms of upper and lower components as follows,

$$\rho_S(r) = \sum_\alpha \frac{2j+1}{4\pi} [G_\alpha^2(r) - F_\alpha^2(r)], \quad (11)$$

$$\rho_V(r) = \sum_\alpha \frac{2j+1}{4\pi} [G_\alpha^2(r) + F_\alpha^2(r)], \quad (12)$$

$$\rho_T(r) = \sum_\alpha \frac{2j+1}{4\pi} [4G_\alpha(r) \times F_\alpha(r)], \quad (13)$$

where α represents the quantum numbers of the target nucleus.

In the generalized RIA [8] the Feynman amplitude for NN scattering is expanded in terms of covariant projection operators $\Lambda^\rho(p)$ to separate positive ($\rho = +1$) and negative ($\rho = -1$) energy sectors of the Dirac space. The invariant

amplitudes, $M_n^{\rho_1 \rho_2 \rho'_1 \rho'_2}$, and kinetic covariants, κ_n , are given by

$$\begin{aligned} \hat{M}(p_1, p_2 \rightarrow p'_1, p'_2) = & \sum_{\rho_1, \rho_2, \rho'_1, \rho'_2} \Lambda^{\rho'_1}(p'_1) \Lambda^{\rho'_2}(p'_2) \\ & \times \sum_{n=1}^{13} M_n^{\rho_1 \rho_2 \rho'_1 \rho'_2} \kappa_n \Lambda^{\rho_1}(p_1) \Lambda^{\rho_2}(p_2), \end{aligned} \quad (14)$$

where subscripts 1 and 2 correspond to the projectile and target nucleons, respectively. The covariant projection operator $\Lambda^\rho(p)$ is defined by $\Lambda^\rho(p) = \frac{1}{2m}(\rho \gamma^\mu p_\mu + m)$, and kinetic covariants κ_n are constructed from the Dirac matrices. The scalar Feynman amplitude, $M_n^{\rho_1 \rho_2 \rho'_1 \rho'_2}$, consists of the direct and exchange parts, each of which represents a sum of four Yukawa terms characterized by coupling constants, masses, and cutoff masses. In the present calculation, the IA2 parametrization of Ref. [8] is used.

The local form of the optical potential is justified at energies greater than 200 MeV. Then, the Dirac equation becomes the standard one-body equation with the optical potential calculated in the standard manner. The differential cross section and the spin observables can be calculated from the scattering amplitude obtained from the analysis of the asymptotic behavior of the scattering wave function.

To estimate multiple-scattering contributions, a relativistic analog [9] of nonrelativistic multiple-scattering theory [10] is used. The optical potential $\hat{U}(\mathbf{r})$ is expressed as follows,

$$\begin{aligned} \hat{U}(\mathbf{r}) = & \langle \Phi | \sum_i t_i | \Phi \rangle + \sum_{i \neq j} \sum_j \langle \Phi | t_i \bar{G} t_j | \Phi \rangle \\ & - \frac{A-1}{A} \langle \Phi | \sum_i t_i | \Phi \rangle \bar{G} \langle \Phi | \sum_j t_j | \Phi \rangle, \end{aligned} \quad (15)$$

where Φ , A , t_i are target nuclear wave functions, target mass number, and NN amplitudes, respectively. The first term of the expression corresponds to the usual first-order relativistic impulse approximation (RIA) optical potential while the second and third terms correspond to the second-order RIA optical potential.

In the relativistic treatment, NN amplitudes, t_i , and the target wave function, Φ , are relativistically constructed as previously discussed, and $\bar{G}$ represents the Dirac propagator for the intermediate projectile nucleon (proton in this work), which is given by

$$\bar{G} = (p - m - \gamma^0 \bar{E}_A + i\epsilon)^{-1}, \quad (16)$$

where the free projectile nucleon scattering is assumed and $\bar{E}_A$ represents an average intermediate excitation energy of the target nucleus. In the present article this energy is assumed to be zero because the incident energy is of the order of several hundred MeV and the few MeV of the average energy can be neglected.

The second-order optical potential is evaluated in the same manner of Ref. [9]. In the momentum representation it is

expressed as

$$\begin{aligned} \hat{U}^{(2)}(\mathbf{p}', \mathbf{p}) = & \int \frac{d^3k}{(2\pi)^3} \frac{1}{4} \text{Tr}_2 \left\{ \hat{M}\left(\mathbf{k}, -\frac{\mathbf{q}_b}{2} \rightarrow \mathbf{p}', \frac{\mathbf{q}_b}{2}\right) \hat{\rho}(\mathbf{q}_b) \right\} \\ & \times \hat{C}(\mathbf{q}_b, \mathbf{q}_a) \tilde{G}(\mathbf{k}) \frac{1}{4} \text{Tr}_3 \\ & \times \left\{ \hat{M}\left(\mathbf{p}, -\frac{\mathbf{q}_a}{2} \rightarrow \mathbf{k}, \frac{\mathbf{q}_a}{2}\right) \hat{\rho}(\mathbf{q}_a) \right\}, \end{aligned} \quad (17)$$

where $\mathbf{q}_a = \mathbf{p} - \mathbf{k}$ and $\mathbf{q}_b = \mathbf{k} - \mathbf{p}'$. The correlation function is defined by the expression:

$$\begin{aligned} \hat{\rho}(\mathbf{q}_b) \hat{C}(\mathbf{q}_b, \mathbf{q}_a) \hat{\rho}(\mathbf{q}_a) \\ = \int d^3r_a \int d^3r_b e^{i\mathbf{r}_a \cdot \mathbf{q}_a} e^{i\mathbf{r}_b \cdot \mathbf{q}_b} f(|\mathbf{r}_a - \mathbf{r}_b|) \hat{\rho}(\mathbf{r}_a) \hat{\rho}(\mathbf{r}_b), \end{aligned} \quad (18)$$

and the factor function is given by

$$f(r) = \sum_{\alpha=1}^3 f_{\alpha} e^{-\frac{r^2}{R_{\alpha}^2}}, \quad (19)$$

with the parameters f_{α} , R_{α} ($\alpha = 1, 2, 3$) the same as those in Ref. [9].

According to the same procedure as the first-order optical potential, the second-order terms are described in momentum space and the correlation function is used as in the well known nonrelativistic multiple scattering theory. Precise expressions and parameters have appeared in Ref. [11]. The first- and second-order optical potentials in momentum space are described as follows:

$$\begin{aligned} \hat{U}^{(1+2)}(\mathbf{p}', \mathbf{p}) = & S(\mathbf{q}) + \gamma_1^0 V(\mathbf{q}) - \frac{\boldsymbol{\alpha}_1 \cdot \mathbf{q}}{m} T(\mathbf{q}) \\ & + \left(\frac{-E\gamma_1^0 + \boldsymbol{\gamma}_1 \cdot \mathbf{p}_a + m}{m} \right) C(\mathbf{q}) \\ & + i \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q}'}{m} \{S_{LS}(\mathbf{q}) + \gamma_1^0 V_{LS}(\mathbf{q})\}, \end{aligned} \quad (20)$$

where $\mathbf{q} = \mathbf{p} - \mathbf{p}'$, $\mathbf{p}_a = (\mathbf{p} + \mathbf{p}')/2$, $\mathbf{q}' = \mathbf{q} \times \mathbf{p}_a$, respectively. The subscript 1 is with respect to the projectile nucleon and the superscript (1+2) denotes the first- and second-order terms.

The Dirac equation containing the optical potential is described in momentum space as follows:

$$\{\gamma^0 E - \boldsymbol{\gamma} \cdot \mathbf{p}' - m\} \Psi(\mathbf{p}') - \int \frac{d^3p}{(2\pi)^3} \hat{U}^{(1+2)}(\mathbf{p}', \mathbf{p}) \Psi(\mathbf{p}) = 0, \quad (21)$$

where $\Psi(\mathbf{p})$ is given by the Fourier transformation of the wave function in coordinate space:

$$\Psi(\mathbf{p}) = \int d^3r e^{-i\mathbf{p} \cdot \mathbf{r}} \Psi(\mathbf{r}). \quad (22)$$

Substituting Eq. (22) in Eq. (21) and replacing the momenta with appropriate operators, the coordinate-space Dirac equation is obtained as follows:

$$\left(\left[1 + \frac{C(r)}{m} \right] \{\gamma^0 E + i\boldsymbol{\gamma} \cdot \nabla - m\} - \hat{U}(\mathbf{r}, \nabla) \right) \Psi(\mathbf{r}) = 0. \quad (23)$$

The localized optical potential $\hat{U}(\mathbf{r}, \nabla)$ is given by

$$\begin{aligned} \hat{U}(\mathbf{r}, \nabla) = & S(r) + \gamma^0 V(r) - \partial T(r) \left(i \frac{\boldsymbol{\alpha} \cdot \hat{\mathbf{r}}}{m} \right) \\ & - \left(i \frac{\boldsymbol{\gamma} \cdot \nabla}{m} \right) C(r) \\ & - [\partial S_{LS}(r) + \gamma^0 \partial V_{LS}(r)] \frac{\boldsymbol{\sigma} \cdot \mathbf{L}}{mr^2}. \end{aligned} \quad (24)$$

Here $\mathbf{L} = -i\mathbf{r} \times \nabla$ and local functions like $S(r)$ are obtained as follows:

$$h(r) = \frac{1}{2\pi^2} \int dq q^2 j_0(qr) h(q), \quad (25)$$

where $h(r)$ corresponds to $S(r)$, $V(r)$, and $C(r)$. The quantity $\partial h(r)$ denotes the derivative of $h(r)$ with respect to r and is given by

$$\partial h(r) = \frac{1}{2\pi^2} \int dq q^3 j_1(qr) h(q), \quad (26)$$

where $\partial h(r)$ corresponds to $\partial T(r)$, $\partial S_{LS}(r)$, and $\partial V_{LS}(r)$. In Eqs. (25) and (26) $j_0(qr)$ and $j_1(qr)$ represent spherical Bessel functions.

Using the renormalized wave function,

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{1 + \frac{C(r)}{m}}} \tilde{\Psi}(\mathbf{r}), \quad (27)$$

yields the Dirac equation in the usual form:

$$\{\gamma^0 E + i\boldsymbol{\gamma} \cdot \nabla - m - \tilde{U}(\mathbf{r})\} \tilde{\Psi}(\mathbf{r}) = 0, \quad (28)$$

where $\tilde{U}(\mathbf{r})$ has five potential terms as in Ref. [8] and is described as follows,

$$\begin{aligned} \tilde{U}(\mathbf{r}) = & \tilde{S}(r) + \gamma^0 \tilde{V}(r) - i\boldsymbol{\alpha} \cdot \hat{\mathbf{r}} \tilde{T}(r) \\ & - \{\tilde{S}_{LS}(r) + \gamma^0 \tilde{V}_{LS}(r)\} \boldsymbol{\sigma} \cdot \mathbf{L}. \end{aligned} \quad (29)$$

Each term is calculated in terms of $C(r)$ as:

$$\begin{aligned} \tilde{S}(r) &= \frac{S(r)}{1 + C(r)/m}, \\ \tilde{V}(r) &= \frac{V(r)}{1 + C(r)/m}, \\ \tilde{T}(r) &= \frac{\partial T(r)}{m + C(r)}, \\ \tilde{S}_{LS}(r) &= \frac{\partial S_{LS}(r)}{mr\{m + C(r)\}}, \\ \tilde{V}_{LS}(r) &= \frac{\partial V_{LS}(r)}{mr\{m + C(r)\}}. \end{aligned} \quad (30)$$

III. RESULTS AND DISCUSSION

Differential cross sections, analyzing powers and spin rotation functions are calculated for proton incident energies: $E_p = 100, 200, 300, 400$, and 500 MeV. To show the effect of medium contributions, calculations with the optimal factorized potentials are compared with results obtained by integrating over $\mathbf{k} \neq 0$. Results for the targets of $^{60-74}\text{Ca}$ are also compared

FIG. 1. (Color online) Density distributions for the calcium isotopes. The left side corresponds to protons and the right side to neutrons, and from the top, scalar, vector, and tensor densities are shown, respectively. The dotted lines correspond to $^{60,62,64}\text{Ca}$ and solid ones to $^{66,68,70}\text{Ca}$, and the dot-dashed ones to $^{72,74}\text{Ca}$, respectively.

with those for ^{40}Ca to show the contribution of large number of neutrons to the medium effects.

Figure 1 shows radial distributions of the densities of the calcium isotopes [2]. In the calculation of the densities the effective force TMA [12] has been used for the mean-field Lagrangian density, and the pairing correlation has been treated by means of the BCS approximation. The Ref. [2] provided a systematic phenomenological study of about 7000 nuclei where mass formulae, separation energies, and rms charge radii are in reasonable agreement with available experimental data.

In Fig. 1 the left side corresponds to proton densities and the right side to neutron densities, and from the top, the scalar, vector, and tensor densities are shown, respectively. The dotted lines correspond to $^{60,62,64}\text{Ca}$, and solid ones to $^{66,68,70}\text{Ca}$, and the dot-dashed ones to $^{72,74}\text{Ca}$, respectively. In the neutron distributions, some shell effects are seen in the central region for ^{72}Ca and ^{74}Ca nuclei.

Table I shows the occupation ratio of the outer shells beyond the $2p_{1/2}$ that is fully occupied in ^{60}Ca . As seen in Table I the spherically shell distributed $3s_{1/2}$ is occupied by more than

TABLE I. Occupation ratio of the outer shells in the calcium isotopes.

Outer shell	Calcium isotope (%)						
	^{62}Ca	^{64}Ca	^{66}Ca	^{68}Ca	^{70}Ca	^{72}Ca	^{74}Ca
$1g_{9/2}$	21.2	40.8	59.5	76.4	87.7	94.1	95.4
$3s_{1/2}$	0.35	1.22	3.60	12.4	45.4	86.7	93.3
$2d_{5/2}$	0.31	0.87	1.79	3.23	5.93	14.3	42.2
$2d_{3/2}$	0.04	0.10	0.21	0.36	0.60	1.00	2.46

90% in ^{72}Ca and ^{74}Ca that causes the shell effect appearing in the central region.

Proton elastic scattering in the 200- to 500-MeV region, however, provides information on the surface region of the target nucleus, and therefore inner shell structures are expected to be small. All the density parameters shown in Fig. 1 and Tables I and II are calculated by the author with the upper and lower components of the wave function [$G_\alpha(r)$ and $F_\alpha(r)$] from Ref. [2].

Figure 2 shows observables of proton elastic scattering from ^{40}Ca at the proton incident energies of 200, 300, 400, and 500 MeV, respectively. Each column is composed of (a) differential cross section, (b) analyzing power, and (c) spin rotation function. Dots are experimental data from the references appearing in Ref. [13]. Solid lines are results for IA2 parameters corresponding to the optimal factorized case, dot-dashed ones show the results with the medium effect, and the dotted lines show calculations with the multiple-scattering contribution. From Fig. 2, it is seen that IA2 calculations with the optimal factorized case give good agreement with experimental data, and the medium and multiple-scattering contributions are small at energies larger than 300 MeV as expected. At 200 MeV, however, the medium effect improves the calculations, especially for the analyzing power at forward and at large angles.

Figures 3–6 show results for $^{60,68,74}\text{Ca}$ at energies from 200 through 500 MeV, respectively. Each figure has three columns corresponding to the targets: from the left, ^{60}Ca , ^{68}Ca , and ^{74}Ca . Solid lines are calculations based on the optimal factorization, dot-dashed ones are results with the medium effect, and dotted ones for the multiple-scattering

TABLE II. Root-mean-square radius of the neutron and proton densities.

Ca isotope	$\sqrt{\langle r^2 \rangle}$		Δ_{rms} (fm)
	Neutron (fm)	Proton (fm)	
^{40}Ca	3.331	3.386	−0.055
^{60}Ca	4.124	3.531	0.593
^{62}Ca	4.190	3.551	0.639
^{64}Ca	4.253	3.571	0.682
^{66}Ca	4.317	3.591	0.726
^{68}Ca	4.397	3.610	0.787
^{70}Ca	4.537	3.625	0.912
^{72}Ca	4.697	3.634	1.063
^{74}Ca	4.828	3.642	1.186

FIG. 2. (Color online) Observables of proton elastic scattering from ^{40}Ca at 200, 300, 400, and 500 MeV. Each column is composed of (a) differential cross section, (b) analyzing power, and (c) spin rotation function. Dots are experimental data [13]. Solid lines are results for the optimal factorized case, dot-dashed lines are the results with the medium effect, and dotted lines show calculations with the multiple-scattering contribution.

contributions. As seen in the previous figures for ^{40}Ca , contributions of the medium effect are small at incident energies greater than 300 MeV, and appear slightly in the forward and large angle region, especially for spin observable at 200 MeV. The multiple scattering contributions, however, appear only in the large angle region.

To show the contributions of medium effects and multiple scattering to the Dirac optical potentials, depths of potentials are shown in Fig. 7 for the scalar potential and Fig. 8 for the vector potential. There are two columns in each figure; the left side provides the real parts of the potentials and the right side the imaginary parts. Each column has three sheets that from the top to the bottom correspond to ^{40}Ca , ^{60}Ca , and ^{74}Ca targets, respectively. In the figures solid circles correspond to the results of the first-order potentials with optimal factorization, triangles to results with the medium effect, and solid square for the multiple scattering contribution. Contributions of both the medium effect and multiple scattering to the real parts of the potentials increase slightly with decreasing incident energy. For the imaginary parts, however, the contributions show the opposite effect. In particular the energy dependence of the potentials with medium effect is significantly different from that of the factorized first- and second-order potentials. As expected these differences are small in the observables at larger

energies than 200 MeV, which are shown in Figs. 3–6, but are more significant at 100 MeV, which is not shown here because the IA is not expected to work well in this energy region.

The calcium isotopes considered here have much larger numbers of neutrons than protons and are expected to have larger neutron distributions than those of stable nuclei, e.g., ^{208}Pb . To discuss such distributions qualitatively, it is convenient to make a comparison between root-mean-square radii of the neutron and proton distributions. This kind of quantity corresponds to the size of the target nucleus and is expected to be proportional to the reaction cross section as discussed later. In Table II the root-mean-square radii of the neutron and proton distributions and the difference between them are given for the calcium isotopes under study. The numbers of neutrons and protons are the same for the stable nucleus, ^{40}Ca , the root-mean-square radius of the neutrons, however, is smaller than that of the protons. This is due to the repulsive Coulomb interaction between the protons. However, for the unstable nuclei ^{60}Ca through ^{74}Ca , the proton root-mean-square radius slightly increases with increasing neutron number because of the attractive interaction with additional neutrons. The difference between the neutron and proton root-mean-square radii increases almost linearly with respect to mass number. This inclination, however, changes

FIG. 3. (Color online) Observables of proton elastic scattering from ^{60}Ca , ^{68}Ca , and ^{74}Ca at 200 MeV. Solid lines are calculations based on the optimal factorization, dot-dashed lines are results with the medium effect, and dotted lines are results for the multiple-scattering contributions.

slightly at $A = 68$ and becomes larger for the mass numbers greater than 68 though it is not shown directly in a figure. Such a phenomenon is thought to result from shell effects. As shown in Table I, the $1g_{9/2}$ shell is solely filled with neutrons till ^{68}Ca and the outer shells are gradually occupied for $^{70-74}\text{Ca}$. This change can be seen in Fig. 9.

The purpose of this study is to obtain information on nuclear distributions, especially neutron distribution for the unstable calcium isotopes based on experimental data. Because observables are restricted for the unstable nuclei, it is necessary to determine nuclear distributions with the aid of some models.

A model analysis has already done for ^{60}Ca and ^{208}Pb nuclei in Refs. [5,7], in which the Woods-Saxon form was assumed for neutron distributions and the two parameters were determined by a pair of observables such as the reaction cross section and the first-dip position of the differential cross section. The Wood-Saxon function is given by

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-r_0)/a}}, \quad (31)$$

where r_0 and a are the radial and diffuseness parameters, respectively. The mean-square radius for this distribution is

FIG. 4. (Color online) Observables of proton elastic scattering from ^{60}Ca , ^{68}Ca , and ^{74}Ca at 300 MeV. The identification of the lines is the same as in Fig. 3.

given by

$$\langle r^2 \rangle = \frac{4\pi \int r^2 \rho(r) r^2 dr}{4\pi \int \rho(r) r^2 dr} = \frac{1}{5} [7(a\pi)^2 + 3r_0^2]. \quad (32)$$

Because the mean-square radius is well known to be proportional to the reaction cross section, one of the selected observables is able to be connected with the parameters in a direct way.

However, the differential cross section is described by the Fourier transformation of the density distribution (form

factor) on the basis of the first Born approximation and the assumption of the t - ρ form of the optical potential. The Fourier transformation of the Woods-Saxon form is given as follows:

$$\rho(q) = 4\pi \int_0^\infty j_0(qr) \frac{\rho_0}{1 + e^{(r-r_0)/a}} r^2 dr \quad (33)$$

$$\sim \frac{4\pi\rho_0}{q^3} \frac{\pi qa}{\sinh \pi qa} \{ \pi qa (\coth \pi qa) \sin qr_0 - qr_0 \cos qr_0 \}, \quad (34)$$

FIG. 5. (Color online) Observables of proton elastic scattering from ^{60}Ca , ^{68}Ca , and ^{74}Ca at 400 MeV. The identification of the lines is the same as in Fig. 3.

where $j_0(qr)$ denotes the spherical Bessel function of the 0th order. The first-dip position of the differential cross section is evaluated with $\rho(q) = 0$, indirectly giving another relation between the observable and the parameters.

The upper graphs in Fig. 9 show the reaction cross sections and the first-dip positions with respect to the difference between the rms radii of protons and neutrons (Δ_{rms} in Table II). The shell effects mentioned above are seen in the reaction cross sections in the large Δ_{rms} region. In Fig. 9 the solid circles, triangles and squares correspond to the results for

the first-order, including medium effects, and the second-order Dirac optical potentials, respectively. The results for ^{48}Ca are also shown.

As seen in the lower graphs of Fig. 9, the reaction cross sections increase with increasing the mass number and with decreasing the incident energy up to 200 MeV and come to be almost the same at energies greater than 300 MeV. The contribution of the multiple scattering to the reaction cross section is rather small in comparison with that to the potentials themselves in which the contribution appears as

FIG. 6. (Color online) Observables of proton elastic scattering from ^{60}Ca , ^{68}Ca , and ^{74}Ca at 500 MeV. The identification of the lines is the same as in Fig. 3.

a significant increase of the absolute value of the potential depth. The contribution of medium effects, however, is rather large at energies of 100 and 200 MeV but is small at energies greater than 300 MeV. In other words, at energies above 300 MeV the reaction cross section is energy independent with respect to contributions from medium effects and multiple scattering.

The first-dip position of the differential cross section increases with decreasing the incident energy and decreases with increasing mass number or Δ_{rms} . The latter phenomenon

is the opposite of that of the reaction cross section. As seen in the case of the reaction cross section, the contribution of medium effects is large at 100 MeV but that of multiple scattering is rather small at all energies studied here. Such results enable us to use the simple form of the first-order analysis when we determine the nuclear distributions of unstable nuclei from the observables around 300 MeV.

To show qualitatively how the nuclear distribution is determined from these observables based on the model analysis, contour maps of mean-square radius given by Eq. (32)

FIG. 7. (Color online) Depth of the Dirac scalar optical potentials for the targets: ^{40}Ca , ^{60}Ca , and ^{74}Ca . The left side shows the real parts of the potentials, and the right side the imaginary parts. Each column has three sheets which from the top to the bottom correspond to ^{40}Ca , ^{60}Ca , and ^{74}Ca targets, respectively. Solid circles correspond to the results of the first-order potentials with optimal factorization, triangles to results with the medium effect, and solid squares to results for the multiple-scattering contribution.

and angle satisfying the equation $\rho(q) = 0$ are provided in Fig. 10, with respect to the Woods-Saxon parameters: radius, r_0 , and diffuseness, a . Fifty contour lines are drawn between 2 and 7 fm² in the left graph for the mean-square radius, and 35 lines between 10° and 17° in the right graph for the first-dip position of differential cross section. Therefore, the intervals of the contour lines are 0.1 fm² and 0.2°, respectively. As seen in Fig. 9 the mean-square radius increases and the dip angle decreases with increasing parameters. Because these two observables show different dependence on the two parameters, the parameters may be constrained by superposing them.

Figure 11 shows contour maps of the reaction cross section and the first-dip angle of the differential cross section for ^{60}Ca at 300 MeV, which are calculated by the first-order RIA optical potentials without the medium effect. Seventy contour lines are drawn between 40 and 110 fm² in the left sheet for the reaction cross section and 50 lines between 10° and 15° in the right sheet for the first-dip position of differential cross section, and therefore the intervals of the contour lines are 1.0 fm² and 0.1°, respectively. In comparison with the mean-square-radius of Fig. 10 it is seen that the reaction cross section is directly proportional to the mean-square radius in almost all regions of the parameter space. However, the angle

FIG. 8. (Color online) Depth of the Dirac vector optical potentials for the targets: ^{40}Ca , ^{60}Ca , and ^{74}Ca . Indication of the lines and legends is the same as in Fig. 7.

of the first-dip position of the differential cross section is much different from the corresponding result given in Fig. 10. Similar structure is, nevertheless, seen in the region of the diffuseness parameter, a , smaller than 0.4 fm. The most different feature is that the valley structure appears in the region of a larger than 0.8 fm, which is expected for this unstable nucleus. Such a characteristic structure is expected to arise from the interference of the incident proton, which is not included in the equation, $\rho(q) = 0$. For a fixed radius parameter, r_0 , increasing diffuseness parameter, a , up to about 0.8 fm seems to give similar contribution to the dip position as increasing radius parameter. Beyond 0.8 fm the

contribution of increasing a , however, becomes similar as decreasing r_0 . When the target density is extremely low, the interaction between projectile proton and the target nucleons becomes so small that no contributions are appeared in the differential cross section. Hence for very large a , the radius parameter, r_0 , that is smaller than the fixed one gives effective contributions, and the valley structure appears in the contour map.

For ^{60}Ca at 300 MeV the analysis was performed in Ref. [7], and two observables have been obtained using the density provided by the relativistic mean-field theory, namely $\sigma_r = 75.34$ fm and $\theta = 12.22^\circ$. According to Fig. 11 this

FIG. 9. (Color online) Reaction cross sections and the first dip positions. The upper sheets show the reaction cross sections (left) and the first-dip positions (right) with respect to the difference between mean radii of protons and neutrons (Δ_{rms} in Table II). The lower graphs are with respect to the mass number in lower side. Solid circles, triangles, and squares correspond to the results for first-order, medium effect, and the second-order Dirac optical potentials, respectively.

pair of observables gives in turn $r_0 = 4.32$ fm and $a = 0.64$ fm. Comparisons of the Woods-Saxon density distribution with the original relativistic mean-field one, and quantitative analysis of experimental errors have been done in Ref. [7]. The

same procedure can be applied to unstable nuclei in addition to the calcium isotopes considered here, and distributions, especially neutron ones, can be obtained when experimental data becomes available.

FIG. 10. (Color) Contour maps of analytic calculations for Woods-Saxon density parameters for ^{60}Ca . The left side shows the mean-square radius given by Eq. (32) and the right side shows the angle satisfying the equation $\rho(q) = 0$, with respect to the two Woods-Saxon parameters of Eq. (31); radius, r_0 , and diffuseness, a .

FIG. 11. (Color) Contour maps of calculated observables at 300 MeV for Woods-Saxon density parameters for ^{60}Ca . The left side shows the reaction cross section and the right side shows the first-dip angle of the differential cross section.

IV. SUMMARY

The present study presented calculations of observables for the elastic scattering of protons from the calcium isotopes in the mass range from 60 through 74 at energies of 100–500 MeV using the RIA with the nuclear densities given by the relativistic mean-field theory. Medium effects and multiple scattering contribution have been included in the optical potentials by integrating over the Fermi momentum of target nucleons and including the second-order potentials, respectively. The contributions of these effects have been shown to be significant at low energies but small at energies higher than 300 MeV. These results have enabled us to analyze the nuclear distributions, especially the neutron distributions for the unstable calcium nuclei.

Two observables have been chosen to determine the parameters of the assumed Woods-Saxon distribution. The mathematical relation between the reaction cross section and the dip position of the differential cross section, and the parameters of the Woods-Saxon distribution was given. The mean-square radius, $\langle r^2 \rangle$, showed almost the same contour structure as the reaction cross section for ^{60}Ca at 300 MeV with respect to the two Woods-Saxon parameters. However, the evaluation of the Born approximation result, $\rho(q) = 0$, showed different structure from the corresponding dip position with respect to large value of parameters, especially radius, r_0 , because the diffraction had been ignored when only the first

Born approximation was used. With respect to small radius parameters, the similar structures were seen even though the diffuseness parameter, a , was large. It has been shown that, like the previous ^{60}Ca study, the parameters of the neutron distributions for unstable nuclei including the calcium isotopes in addition to ^{60}Ca could be determined using the reaction cross section and the first dip position of the differential cross section for the proton-elastic scattering, based on an RIA analysis at energies greater than 300 MeV.

On the basis of the results shown in the present article, studies of the neutron distributions for nickel isotopes whose mass numbers are from 48 through 82 are ongoing and results will appear in the near future. To confirm the validity of the procedure discussed here it is necessary to apply it to the well-investigated ^{48}Ca nucleus for which there are much experimental data. Such a study at 300 MeV is under preparation.

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