Bit-Interleaved Coded DPSK with Cyclic Delay Diversity: Design and Analysis

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Abstract—In this paper, the bit-interleaved convolutionally coded differential phase shift keying (DPSK) in combination with cyclic delay diversity (CDD) is theoretically analyzed in terms of pairwise error probability (PEP). We show that bit-interleaved coded DPSK with CDD can achieve full spatial and frequency diversity offered by the frequency selectivity of wireless channel conditioned that the minimum free distance d_{free} of the convolutional code is larger than the target diversity order. The design criterion of the considered system is presented based on the analysis of diversity and coding gains. Finally, the theoretical results obtained in this paper are confirmed by computer simulations.

Index Terms—Cyclic delay diversity (CDD), diversity order, coding gain, differential detection, differential phase shift keying (DPSK), orthogonal frequency division multiplexing (OFDM)

I. INTRODUCTION

RECENTLY, classical delay diversity techniques [1] have regained attraction for their simple structures and superior performance over fading channels [2, 3]. Delay diversity can be implemented by transmitting delayed replicas of a signal from several antennas. Since orthogonal frequency division multiplexing (OFDM) can cope with frequency selective channels with a simple receiver structure, a combination of delay diversity and OFDM is thus a promising candidate for future high data rate communications. However, a problem associated with delay diversity in combination with OFDM is that it requires a longer cyclic prefix (CP), which may reduce bandwidth efficiency. The *cyclic delay diversity* (CDD) [4], also known as *multicarrier delay diversity modulation* [3], can mitigate this drawback by transmitting cyclically delayed replicas of an OFDM signal from different antennas. Here, the frequency diversity intentionally induced by delayed replicas can be picked up by an appropriate forward error correction coding scheme such as bit-interleaved coded modulation (BICM) [5–7].

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CDD has the advantage that the overall channel can be modified as a single input channel. Thus, unlike other transmit diversity techniques such as space-time block codes (STBC) [8], this antenna diversity scheme can be implemented in existing OFDM system without changing the receiver circuits. Thus, the applications of CDD have become a vigorous area of research. In [9], it has been applied to the terrestrial digital video broadcasting (DVB-T) system and shown a remarkable performance improvement in an indoor environment. Also, the application of the CDD to the digital audio broadcasting (DAB) and digital multimedia broadcasting (DMB) can be found in [10].

Although the negative effects of fading can be reduced with CDD, increased frequency selectivity hinders the simple channel estimation. This implies that an increasing number of pilot symbols are required and thus the bandwidth efficiency is decreased. Hence, efficient channel estimation techniques for the CDD have been proposed in [11, 12]. However, these techniques still require complicated calculations and severalfold pilots compared with the single-input single output (SISO) channel.

Differential detection is an attractive alternative to the pilot-symbol assisted coherent detection since it does not require any channel state information (CSI) at the receiver to demodulate transmitted information. As pointed out in the literature, the performance of differential detection is inferior to that of coherent detection. However, since differential detection does not require any pilot signals, channel coding with increased redundancy can be employed, if the same user data rate is assumed [13]. These additional redundant bits can compensate for the performance gap between differential and coherent detections. Therefore, considering the low complexity of the receiver, the use of differential detection may be an attractive choice compared to coherent detection.

In this paper, we theoretically analyze the performance of the CDD with bit-interleaved convolutionally coded differential PSK (DPSK) modulation in terms of pairwise error probability (PEP). We also derive the design criterion for this system to obtain full diversity. In the literature of coherent detection, the full diversity criterion for an arbitrary number of transmit antennas is proposed based on PEP analysis [3, 14]. In [15], outage capacity-based parameter optimization of CDD with phase shift keying (PSK) and QAM signal constellation has been proposed. The generalized CDD using cyclic filter that minimize the PEP has been proposed in [16]. In [17], it has been shown through the PEP analysis that spatial correlations reduce the achievable diversity order of CDD systems.

The CDD with differential detection and PSK constellations in frequency domain is proposed in [4], which has less complexity than differential unitary space-time modulation [18, 19]. The performance comparison with differential STBC (DSTBC) using PSK constellations is studied in [20]. It is shown that CDD technique is an attractive alternative to DSTBC because of the following reasons.

- The CDD does not require any modification of the receiver.
- An arbitrary number of transmit antennas can be chosen and there is no need to design code matrix for specific antenna configurations.
- No additional cost is required except for the simple cyclic shifts in the time domain.

However, despite its potential advantage, the diversity gain and full diversity criterion of CDD with differential detection have not been rigorously analyzed.

The rest of the paper is organized as follows. In Section II, we describe the system model of bit-interleaved convolutionally coded DPSK in combination with CDD. In Section III, we analyze the PEP of this system over frequency selective channels. In Section IV, based on the analysis in terms of both diversity and coding gain, the design criterion of the considered system is presented. In order to justify our theoretical results, computer simulations are performed in Section V. Section VI is devoted to the performance comparison of the CDD with coherent detection and differential detection, revealing the advantage of the use of differential detection. Finally, concluding remarks are given in Section VII.

II. SYSTEM MODEL OF BIT-INTERLEAVED CODED DPSK WITH CDD

We first describe the OFDM system model with bit-interleaved convolutionally coded DPSK deploying one transmit and one receive antenna (SISO) over frequency-selective Rayleigh fading channel. We then extend the system model to that with the CDD.

A. Baseline Transmitter Model

The basic transmitter model is illustrated in Fig. 1. A BICM system can be constructed by using a bit-wise interleaver between an encoder for a binary convolutional code $\mathcal{C}$ and a mapping function over a PSK signal set $\mathcal{X} \subseteq \mathbb{C}$ of size $|\mathcal{X}| = M = 2^m$ with binary labeling map $\mu : \{0, 1\}^m \rightarrow \mathcal{X}$. During the transmission, the code sequence $\mathbf{c}$ is bit-wise interleaved and then mapped onto the signal sequence of length K , $X_k \in \mathcal{X}^K$. The bit-wise interleaver can be modeled as the reordering function $t \rightarrow (k, i)$ where t denotes the original ordering of the code sequence $\mathbf{c}$, k denotes the time ordering of the signals X_k and i indicates the position of the bit c_t in the symbol X_k where $i = 0, 1, \dots, m-1$. In this paper, we assume that the system employs the OFDM with N subcarriers and the bit-wise interleaver satisfies the following conditions; the consecutive coded bits should be mapped onto different modulated symbols and transmitted over different subcarriers such that each of coded bits is subject to statistically independent fading due to frequency

and time selectiveness of wireless channels. Moreover, *easy-to-implement* block interleavers are often adopted in practical systems such as wireless local area network (LAN) standards [21]. Thus, we also suppose that if a coded bit c_t is mapped onto the l th subcarrier, the next coded bit c_{t+1} is mapped onto the $[(l + f_s) \bmod N]$ th subcarrier, where f_s indicates the subcarrier spacing between adjacent coded bits with $f_s \geq 1$ and $l = 0, 1, \dots, N-1$.

Then, we denote a coded symbol sequence by $\mathbf{x} = (X_0, X_1, \dots, X_{K-1})$ and its phasor representation by

$$X_k = e^{j\Delta\phi_k}, \quad (1)$$

where $\Delta\phi$ is assigned by the mapper with Gray labeling which is essential for BICM [6]. The sequence $\mathbf{x}$ is differentially encoded and its resulting symbol sequence is denoted by $\mathbf{v} = (V_0, V_1, \dots, V_{K+N-1})$. Note that N -symbol tapped delay is used instead of one symbol delay element since differential encoding is performed across consecutive OFDM symbols, i.e., in time domain. Therefore, the first N -tuple of $\mathbf{v}$ corresponding to the first OFDM symbol is the set of initial reference symbols. In phasor notation, differentially encoded symbol V_k of the l th subcarrier in the k' th OFDM symbol $k' = 0, 1, \dots, K/N$ can be given by

$$\begin{aligned} V_{Nk'+l} &= \sqrt{E_s} e^{j\phi_{Nk'+l}} \\ V_{N(k'+1)+l} &= V_{Nk'+l} X_{Nk'+l} \\ &= \sqrt{E_s} e^{j\{\phi_{Nk'+l} + \Delta\phi_{Nk'+l}\}} \\ &= \sqrt{E_s} e^{j\phi_{N(k'+1)+l}}, \end{aligned} \quad (2)$$

where E_s is the energy per M -DPSK symbol. Each N -tuple of the differentially encoded symbol sequence $\mathbf{v}$, denoted by $\mathbf{v}_{k'} = (V_{Nk'}, V_{Nk'+1}, \dots, V_{Nk'+N-1})$, is modulated to the OFDM symbol via inverse fast Fourier transform (IFFT). The output symbols of IFFT corresponding to the k' th OFDM symbol are denoted by $(v_{Nk'}, v_{Nk'+1}, \dots, v_{Nk'+N-1})$. Finally, a cyclic prefix (CP) of appropriate length is added to each OFDM symbol.

B. Wireless Channel Model

When CP is removed and fast Fourier transform (FFT) is performed at the receiver, the received signal $Y_{Nk'+l}$ of the l th subcarrier in the k' th OFDM symbol is given by

$$Y_{Nk'+l} = H_l V_{Nk'+l} + n_{Nk'+l}, \quad (3)$$

where $n_{Nk'+l}$ is complex additive white Gaussian noise (AWGN) with zero mean and variance $2\sigma^2$ and H_l is a channel coefficient of the l th subcarrier. In this paper, a frequency selective channel is modelled as a tapped delay line with $\tau_m \in \mathbb{N}$ taps (i.e., the maximum delay spread of this channel is determined by $(\tau_m - 1)$ samples.) The channel coefficient of the l th subcarrier is then expressed as

$$H_l = \mathbf{w}_N^H(l) \mathbf{P} \mathbf{h}, \quad (4)$$

where $(\cdot)^H$ indicates transpose conjugate, $\mathbf{w}_N(l) = [1 \ W_N^l \ W_N^{2l} \ \dots \ W_N^{(\tau_m-1)l}]^H$ is a $\tau_m \times 1$ vector with $W_N \triangleq e^{-j2\pi/N}$, and $\mathbf{h} = [h_0 \ h_1 \ \dots \ h_{\tau_m-1}]^T$ is a $\tau_m \times 1$ vector of which each element is assumed to be statistically independent and modeled as a zero-mean complex

Fig. 1. Transmitter model of the bit-interleaved coded DPSK with the CDD, where N indicates the number of subcarriers. Also, SP: serial-to-parallel conversion; PS: parallel-to-serial conversion.

Gaussian random variable with unit variance. Moreover, $\mathbf{P}$ is a $\tau_m \times \tau_m$ diagonal matrix with $\sqrt{p_\tau}$, for $\tau = 0, \dots, \tau_m - 1$, on the main diagonal representing the power delay profile (PDP) of the frequency-selective channel $\mathbf{h}$. Note that all diagonal elements of $\mathbf{P}$ are real and strictly positive and normalized such that $\sum_{\tau=0}^{\tau_m-1} p_\tau = 1$. In the rest of the paper, block fading model is assumed, i.e., the fading coefficients are constant over the transmission of one frame, but independent from one transmission frame to the next.

C. Receiver Model

Let $\mathcal{X}_b^i$ denote the subset of the signal set $\mathcal{X}$ whose label has $b \in \{0, 1\}$ in the i th bit position. Then the maximum-likelihood (ML) bit metric of the coded bit c_t can be given by [22]

$$\lambda^i(Y_k, c_t) = \min_{X \in \mathcal{X}_{c_t}^i} \{ -\Re[Y_k Y_{k-N}^* X^*] \}, \quad (5)$$

where $*$ denotes the complex conjugation and $\Re$ denotes the real part of the complex number. Using (5), the ML decoder at the receiver side can make decisions according to the following rule;

$$\hat{\mathbf{c}} = \underset{\mathbf{c} \in \mathcal{C}}{\operatorname{argmin}} \sum_t \lambda^i(Y_k, c_t). \quad (6)$$

D. Cyclic Delay Diversity

Let us consider the multiple-input single-output (MISO) system with T_x transmit antennas and one receive antenna. An extension to the case with multiple receive antennas is straightforward. CDD introduces intentional cyclic delays to OFDM signals to obtain spatial diversity. Therefore, the transmit symbol from antenna n_T at time l in the k' th OFDM symbol is given by

$$v_{Nk'+l}^{(n_T)} = v_{Nk'+(l-n_T\delta)} \mod N, \quad n_T = 0, \dots, T_x - 1. \quad (7)$$

where δ is an arbitrary cyclic delay and thus $(n_T \times \delta)$ is a given cyclic delay at the n_T th transmit antenna. Note that δ is normalized to the FFT sample spacing, i.e., $\delta \in \mathbb{N}$ and $\delta \leq N$. For simplicity, we assume that the transmit energy is equally allocated to each antenna. Provided that no CSI is available at the transmitter, this energy allocation is optimum in terms of achievable coding gain, as will be shown in Section IV-B. Finally, a CP of appropriate length is added to each OFDM symbol.

To show the effect of introducing these intentional cyclic delays, we redefine the frequency selective channel including the virtual frequency selectivity. We firstly consider the PDP of the compound frequency selective channel. Note that we assume that the PDP's are identical for every transmit antenna in order to simplify the subsequent arguments. Let $\mathbf{P}^{(n_T)}$ denote $(\tau_m + (T_x - 1)\delta) \times (\tau_m + (T_x - 1)\delta)$ diagonal matrix of which the first $n_T\delta$ main diagonal elements are zeros, the following τ_m elements represent the PDP of the corresponding frequency-selective channel $\mathbf{h}^{(n_T)}$, and the remainders are also zeros. Namely, $\mathbf{P}^{(n_T)}$ can be given by (8) at the top of the next page, where $q_i^{(n_T)}$ represents the i th PDP corresponding to the n_T th antenna and by definition $\sum_{\tau=0}^{\tau_m-1} p_\tau = 1$. Since the sum of Gaussian random variables follows a Gaussian distribution with combined variances, the PDP of the compound frequency selective channel in (4) can be given by (9) at the top of the next page. Moreover, the channel vector in (4) corresponding to $\mathbf{P}_c$ is given by

$$\mathbf{h} = [h_0 \ h_1 \ h_2 \ \dots \ h_{(T_x-1)\delta+\tau_m-1}]^T, \quad (10)$$

where each tap is assumed to be statistically independent and modeled as a zero-mean complex Gaussian random variable with unit variance. Finally, considering that $\mathbf{w}_N(l) = [1 \ W_N^l \ W_N^{2l} \ \dots \ W_N^{((T_x-1)\delta+\tau_m-1)l}]^H$ can be given as a $((T_x-1)\delta + \tau_m) \times 1$ vector, the resulting compound wireless channel response $\mathbf{h}_l$ is also given by (4).

$$\begin{aligned}
\mathbf{P}^{(n_T)} &= \begin{bmatrix} \sqrt{q_0^{(n_T)}} & 0 & \cdots & 0 \\ 0 & \sqrt{q_1^{(n_T)}} & & \vdots \\ \vdots & & \ddots & \\ 0 & & \cdots & \sqrt{q_{(T_x-1)\delta+\tau_m-1}^{(n_T)}} \end{bmatrix} \\
&= \begin{bmatrix} 0 & \cdots & 0 & & \\ \vdots & \ddots & & & \\ 0 & 0 & & & \\ \hline & & \frac{\sqrt{p_0}}{\sqrt{T_x}} & \cdots & 0 \\ & & \vdots & \ddots & \\ & & 0 & & \frac{\sqrt{p_{\tau_m-1}}}{\sqrt{T_x}} \\ \hline & & & & 0 & \cdots & 0 \\ & & & & \vdots & \ddots & \\ & & & & 0 & & 0 \end{bmatrix}_{((T_x-1)\delta+\tau_m) \times ((T_x-1)\delta+\tau_m)} \quad (8)
\end{aligned}$$

$$\begin{aligned}
\mathbf{P}_c &= \sqrt{\sum_{n_T=0}^{T_x-1} \{\mathbf{P}^{(n_T)}\}^2} \\
&\triangleq \begin{bmatrix} \sqrt{\sum_{n_T=0}^{T_x-1} q_0^{(n_T)}} & 0 & \cdots & 0 \\ 0 & \sqrt{\sum_{n_T=0}^{T_x-1} q_1^{(n_T)}} & & \vdots \\ \vdots & & \ddots & \\ 0 & & \cdots & \sqrt{\sum_{n_T=0}^{T_x-1} q_{(T_x-1)\delta+\tau_m-1}^{(n_T)}} \end{bmatrix} \quad (9)
\end{aligned}$$

III. PAIRWISE ERROR ANALYSIS OF BIT-INTERLEAVED CODED DPSK WITH CDD

In this section, we analyze the PEP $\Pr(\mathbf{c} \rightarrow \tilde{\mathbf{c}})$ of the system described in Section II, which represents the probability of choosing the code sequence $\tilde{\mathbf{c}}$ instead of the correct code sequence $\mathbf{c}$. As mentioned in [23], exact theoretical analysis of DPSK with metric such as (5) is difficult, if not impossible. For this reason, much simpler suboptimal Gaussian metric is considered here which serves as an upper bound on bit error performance.

A code rate R convolutional encoder with minimum free distance d_{free} is assumed to generate the binary code sequence $\mathbf{c}$ at the transmitter. The PEP without CSI at the receiver can be written as [23]

$$\begin{aligned}
\Pr(\mathbf{c} \rightarrow \tilde{\mathbf{c}}) &= \Pr \left(\sum_t \left[\min_{X \in \mathcal{X}_{c_t}^i} |Y_{Nk'+l} Y_{N(k'-1)+l}^* - 2E_s X|^2 \right. \right. \\
&\quad \left. \left. \geq \min_{X \in \mathcal{X}_{\tilde{c}_t}^i} |Y_{Nk'+l} Y_{N(k'-1)+l}^* - 2E_s X|^2 \right] \right), \quad (11)
\end{aligned}$$

where $|\cdot|$ denotes the magnitude of a complex number.

Following the approach proposed in [24], we assume that the pair of two code sequences, $\mathbf{c}$ and $\tilde{\mathbf{c}}$, has the minimum free distance d_{free} . In this case, (11) can be simplified to only

d_{free} terms since the other terms in the inequality of (11) are identical. As mentioned in Section II, bit interleaver should be designed such that consecutive coded bits are mapped onto different modulated symbols and transmitted over different subcarriers. This guarantees that there exists d_{free} distinct pairs of $\mathcal{X}_{c_t}^i$ and $\mathcal{X}_{\tilde{c}_t}^i$. Let us denote

$$\hat{X} = \underset{X \in \mathcal{X}_{c_t}^i}{\text{argmin}} |Y_{Nk'+l} Y_{N(k'-1)+l}^* - 2E_s X|^2 \quad (12)$$

$$\tilde{X} = \underset{X \in \mathcal{X}_{\tilde{c}_t}^i}{\text{argmin}} |Y_{Nk'+l} Y_{N(k'-1)+l}^* - 2E_s X|^2, \quad (13)$$

where $\mathcal{X}_{c_t}^i$ and $\mathcal{X}_{\tilde{c}_t}^i$ are complementary sets of constellation points within the signal constellation set $\mathcal{X}$ for distinct d_{free} bits. Then, the PEP can be rewritten as

$$\begin{aligned}
\Pr(\mathbf{c} \rightarrow \tilde{\mathbf{c}}) &= \Pr \left(\sum_{l, d_{\text{free}}} \left[|Y_{Nk'+l} Y_{N(k'-1)+l}^* - 2E_s \tilde{X}|^2 \right. \right. \\
&\quad \left. \left. - |Y_{Nk'+l} Y_{N(k'-1)+l}^* - 2E_s \hat{X}|^2 \right] \geq 0 \right), \quad (14)
\end{aligned}$$

where $\sum_{l, d_{\text{free}}}$ indicates that the summation is taken with respect to the subcarriers (with index l) that contribute to d_{free} distinct coded bits.

Using the Chernoff bound and some manipulations [23],

$$\begin{aligned} \Pr(\mathbf{c} \rightarrow \tilde{\mathbf{c}}|\mathbf{h}') &\leq \prod_{l, d_{\text{free}}} \frac{\exp \left[-\frac{2\lambda(1-4\lambda)d_{\min}^2 \frac{E_s}{N_0} |H_l|^2}{1-(2\lambda)^2 d_{\min}^2} \right]}{1-(2\lambda)^2 d_{\min}^2} \\ &= \left(\frac{1}{1-(2\lambda)^2 d_{\min}^2} \right)^{d_{\text{free}}} \\ &\quad \times \exp \left[-\frac{\sum_{l, d_{\text{free}}} 2\lambda(1-4\lambda)d_{\min}^2 \frac{E_s}{N_0} |H_l|^2}{1-(2\lambda)^2 d_{\min}^2} \right], \end{aligned} \quad (15)$$

where λ is the Chernoff parameter to be optimized, N_0 is one-sided power spectral density of the AWGN, $d_{\min}$ denotes the minimum Euclidean distance between the two distinct symbols on the PSK constellation, and $\mathbf{h}' = (H(0), H(1), \dots, H(N-1))$. Using (4),

$$\begin{aligned} \sum_{l, d_{\text{free}}} |H_l|^2 &= \sum_{l, d_{\text{free}}} \mathbf{h}^H \mathbf{P} \mathbf{w}_N(l) \mathbf{w}_N^H(l) \mathbf{P} \mathbf{h} \\ &= \mathbf{h}^H \mathbf{P} \mathbf{A} \mathbf{P} \mathbf{h} \end{aligned} \quad (16)$$

where $\mathbf{A}$ is a $\tau_m \times \tau_m$ matrix, $\mathbf{A} = \sum_{l, d_{\text{free}}} \mathbf{A}_l$, and $\mathbf{A}_l = \mathbf{w}_N(l) \mathbf{w}_N^H(l)$. Let r denote the rank of the matrix $\mathbf{A}$, i.e., $r = \text{rank}(\mathbf{A})$. Since $\mathbf{P}$ is a nonsingular diagonal matrix, the rank of $\mathbf{P} \mathbf{A} \mathbf{P}$ should be identical with the rank of $\mathbf{A}$. Since $\mathbf{A}_l$'s, $\mathbf{A}$, and $\mathbf{P} \mathbf{A} \mathbf{P}$ are positive semidefinite Hermitian, it is expressed by singular value decomposition (SVD) as

$$\mathbf{P} \mathbf{A} \mathbf{P} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^H, \quad (17)$$

where $\mathbf{V}$ is a $r \times r$ unitary matrix, and $\mathbf{\Lambda}$ is a $r \times r$ diagonal matrix with real and nonnegative eigenvalues of $\mathbf{P} \mathbf{A} \mathbf{P}$, $\{\lambda_i(\mathbf{P} \mathbf{A} \mathbf{P})_{i=0}^{r-1}\}$ in decreasing order on the main diagonal. Let us denote the minimum element of a diagonal matrix $\mathbf{P}^2$ as $p_{\min}$, i.e., $\min_i p_i \triangleq p_{\min}$. Using the inequality $\lambda_i(\mathbf{P} \mathbf{A} \mathbf{P}) \geq p_{\min} \lambda_i(\mathbf{A})$ [24], (15) can be further rewritten as

$$\begin{aligned} \Pr(\mathbf{c} \rightarrow \tilde{\mathbf{c}}|\mathbf{h}') &\leq \left(\frac{1}{1-(2\lambda)^2 d_{\min}^2} \right)^{d_{\text{free}}} \times \\ &\quad \prod_{i=0}^{r-1} \exp \left[-\frac{\frac{2E_s}{N_0} \lambda(1-4\lambda) d_{\min}^2 p_{\min} \lambda_i(\mathbf{A}) |u_i|^2}{1-(2\lambda)^2 d_{\min}^2} \right], \end{aligned} \quad (18)$$

where u_i is the element of $\mathbf{V}^H \mathbf{h}$ for $i = 0, 1, \dots, r-1$ and its amplitude is Rayleigh distributed with probability density function $2|u_i|e^{-|u_i|^2}$.

Averaging $|u_i|$ in (18), the PEP becomes

$$\begin{aligned} \Pr(\mathbf{c} \rightarrow \tilde{\mathbf{c}}) &\leq 2 \left(\frac{1}{1-4\lambda^2 d_{\min}^2} \right)^{d_{\text{free}}} \times \\ &\quad \prod_{i=0}^{r-1} \frac{-1 + 4\lambda^2 d_{\min}^2}{-2 + 4d_{\min}^2 \lambda \{2\lambda + p_{\min} \lambda_i(\mathbf{A})(-1 + 4\lambda) \frac{E_s}{N_0}\}}. \end{aligned} \quad (19)$$

Substituting the Chernoff parameter with

$$\lambda = \frac{1}{4d_{\min}}, \quad (20)$$

we obtain the PEP as

$$\begin{aligned} \Pr(\mathbf{c} \rightarrow \tilde{\mathbf{c}}) &\leq \\ &\quad 2^{1+2\mu} \left(\frac{1}{3} \right)^\mu \prod_{i=0}^{r-1} \left[\frac{1}{\frac{3}{2} + p_{\min}(d_{\min} - 1) \lambda_i(\mathbf{A}) \frac{E_s}{N_0}} \right] \end{aligned} \quad (21)$$

$$\approx 2^{1+2\mu} \left(\frac{1}{3} \right)^\mu \left(\frac{p_{\min}(d_{\min} - 1) E_s}{N_0} \eta \right)^{-r}, \quad (22)$$

for high signal to noise ratio (SNR), where $\mu = d_{\text{free}} - r$, $0 \leq \mu \leq d_{\text{free}} - 1$, and

$$\eta = \left(\prod_{i=0}^{r-1} \lambda_i(\mathbf{A}) \right)^{1/r}. \quad (23)$$

The *diversity order* of the bit-interleaved coded DPSK with CDD can be defined as the exponent r of the upper bound in (22) and the *coding gain* is defined as

$$G_c \triangleq p_{\min}(d_{\min} - 1) \eta = p_{\min}(d_{\min} - 1) \left(\prod_{i=0}^{r-1} \lambda_i(\mathbf{A}) \right)^{1/r}. \quad (24)$$

Hence, the rank of the matrix $\mathbf{A}$ defines the diversity order of this system. Moreover, (22) guarantees that the diversity order of the system only depends on the rank of $\mathbf{A}$ regardless of the PDP of the effective wireless channel. This agrees with the result that has been proven in the case of coherent detection in [24].

IV. DESIGN OF CYCLIC DELAY DIVERSITY

In the previous section, we show that the bit-interleaved coded DPSK with the CDD can exploit the frequency selectivity efficiently and its diversity order is determined by the rank of $\mathbf{A} = \sum_{l, d_{\text{free}}} \mathbf{w}_N(l) \mathbf{w}_N^H(l)$. Based on this important result, we propose its design criterion.

A. Diversity Order Analysis

Let us consider the case $\delta \geq \tau_m$. In this case, $\mathbf{P}^{(i)}$ and $\mathbf{P}^{(j)}$ with $i \neq j$ do not have non-zero elements in common position and thus the number of non-zero elements in $\mathbf{P}_c = \sqrt{\sum_{n_T=0}^{T_x-1} \{\mathbf{P}^{(n_T)}\}^2}$ is $\tau_m T_x$. Therefore, the rank of $\mathbf{P}_c$ can be reduced to $\tau_m T_x$ since fading coefficients are assumed to be statistically independent between transmit antennas. Thus, the CDD can achieve the diversity order $r = \min(d_{\text{free}}, \tau_m T_x)$ if and only if δ satisfies $\delta \geq \tau_m$. Furthermore, the full spatial diversity can be obtained via CDD if and only if $d_{\text{free}} \geq \tau_m T_x$.

On the other hand, when $\delta < \tau_m$, the rank of $\mathbf{P}_c$ becomes $(T_x - 1)\delta + \tau_m$. Hence, the diversity order in this case can be given by $r = \min(d_{\text{free}}, \delta(T_x - 1) + \tau_m)$ and thus full spatial diversity cannot be obtained via CDD.

From the above observations, the full diversity criterion can be summarized as follows.

Full Diversity Criterion: If $d_{\text{free}} \geq \tau_m T_x$, the cyclic delay should be given by $\delta \geq \tau_m$. Since the maximum channel delay τ_m is unknown to the transmitter in practice, this suggests that the cyclic delay should be chosen as large as possible.

The PDP of each transmit antenna is determined independently in practice. In that case, the above arguments can

be applied in a straightforward manner. Let $\tau_m^{(i)}$ denote the maximum delay spread with respect to i th transmit antenna and $\tau_{\max} \triangleq \max_i \tau_m^{(i)}$. The diversity order can be given by $r = \min(d_{\text{free}}, \sum_{i=0}^{T_x-1} \tau_m^{(i)})$ if and only if $\delta \geq \tau_{\max}$.

1) *A Numerical Example:* In order to elucidate the above analysis, we show a simple numerical example in the following. The number of subcarriers is $N = 8$ for OFDM transmission, and the minimum free distance d_{free} is assumed to be 4. Without loss of generality, we only consider the case with two transmit antennas over frequency selective block Rayleigh fading channel with $\tau_m = 2$. If $\delta = \frac{N}{T_x} = 4$ is chosen, the resultant channel matrix H_l can be written as

$$H_l = \begin{bmatrix} 1 & W_8^l & \cdots & W_8^{5l} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{p_0}}{\sqrt{2}} & 0 & \cdots & 0 \\ 0 & \frac{\sqrt{p_1}}{\sqrt{2}} & & 0 \\ & 0 & & \\ \vdots & & 0 & \vdots \\ & & \frac{\sqrt{p_0}}{\sqrt{2}} & 0 \\ 0 & \cdots & 0 & \frac{\sqrt{p_1}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix}. \quad (25)$$

Obviously, the matrix $\tilde{\mathbf{P}}$ includes two zero rows. Thus, H_l can be rewritten as

$$H_l = \begin{bmatrix} 1 & W_8^l & W_8^{4l} & W_8^{5l} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{p_0}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{p_1}}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{p_0}}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{p_1}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_4 \\ h_5 \end{bmatrix}. \quad (26)$$

Without loss of generality, we assume that d_{free} distinct coded bits are mapped onto the successive subcarriers, i.e., $l = 0, \dots, d_{\text{free}} - 1$. The matrix $\mathbf{A}$ becomes

$$\mathbf{A} = \sum_{l=0}^3 \begin{bmatrix} 1 & W_8^l & W_8^{4l} & W_8^{5l} \\ W_8^{-l} & 1 & W_8^{3l} & W_8^{4l} \\ W_8^{-4l} & W_8^{-3l} & 1 & W_8^l \\ W_8^{-5l} & W_8^{-4l} & W_8^{-l} & 1 \end{bmatrix}, \quad (27)$$

and its rank is 4. On the other hand, if $\delta = 1$ is chosen, the resultant channel matrix H_l may become

$$H_l = \begin{bmatrix} 1 & W_8^l & W_8^{2l} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{p_0}}{\sqrt{2}} & 0 & 0 \\ 0 & \sqrt{\frac{p_0}{2} + \frac{p_1}{2}} & 0 \\ 0 & 0 & \frac{\sqrt{p_1}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix}, \quad (28)$$

and thus,

$$\mathbf{A} = \sum_{l=0}^3 \begin{bmatrix} 1 & W_8^l & W_8^{2l} \\ W_8^{-l} & 1 & W_8^l \\ W_8^{-2l} & W_8^{-l} & 1 \end{bmatrix}. \quad (29)$$

Apparently, the rank of the matrix $\mathbf{A}$ is 3 in this case. Thus, the full diversity gain of 4 cannot be achieved. Therefore, in this example, the optimum choice of δ is at least 2.

B. Coding Gain Analysis

In the previous subsections, we have derived the optimum cyclic delay capable of achieving the full diversity. If $d_{\text{free}} \geq \tau_m T_x$ and $\delta = \tau_m$, the matrix $\mathbf{A}$ has full rank and thus full diversity can be achieved. However, the coding gain G_c can be very small due to the infinitesimally small eigenvalue $\lambda_i(\mathbf{A})$. If $\lambda_i(\mathbf{A})$ is sufficiently small such that $p_{\min}(d_{\min} - 1)\lambda_i(\mathbf{A})E_s/N_0 \ll 1$ in (21), then we have

$$\left(\frac{1}{\frac{3}{2} + p_{\min}(d_{\min} - 1)\lambda_i(\mathbf{A})\frac{E_s}{N_0}} \right) \approx \frac{2}{3}. \quad (30)$$

Consequently, the eigenvalue $\lambda_i(\mathbf{A})$ fails to contribute to a diversity gain. Hence, in low SNR region, the diversity gain is actually invisible due to the small nonzero eigenvalue. In high SNR region, this negative effect of the nonzero eigenvalue is decreased and thus full diversity effect becomes visible even for a small eigenvalue. Thus, the additional condition of the cyclic delay δ should be studied so that the maximum coding gain G_c is obtained.

1) *Upper Bounds of Coding Gain:* We first derive an upper bound of the coding gain for given eigenvalues.

If $d_{\text{free}} \geq \tau_m T_x$ and $\delta \geq \tau_m$ are assumed, $\mathbf{A}$ has full rank and also is a Toeplitz matrix. Hence, it follows that

$$\prod_{i=0}^{r-1} \lambda_i(\mathbf{A}) = \det(\mathbf{A}), \quad (31)$$

and

$$\sum_{i=0}^{r-1} \lambda_i(\mathbf{A}) = \text{Tr}(\mathbf{A}) = d_{\text{free}} \times \tau_m T_x. \quad (32)$$

The arithmetic mean and geometric mean of $\lambda_i(\mathbf{A})$ satisfy the inequality

$$\frac{1}{r} \sum_{i=0}^{r-1} \lambda_i(\mathbf{A}) \geq \left(\prod_{i=0}^{r-1} \lambda_i(\mathbf{A}) \right)^{1/r}. \quad (33)$$

From (31) to (33), the inequality may be rewritten as

$$\frac{d_{\text{free}} \times \tau_m T_x}{r} \geq \det(\mathbf{A})^{1/r}. \quad (34)$$

Considering $r = \min(d_{\text{free}}, \tau_m T_x)$ and $d_{\text{free}} \geq \tau_m T_x$, we may obtain

$$d_{\text{free}} \geq \det(\mathbf{A})^{1/r}. \quad (35)$$

Equality in (35) holds only when all eigenvalues $\lambda_i(\mathbf{A})$ are equal to d_{free} . If all off-diagonal elements of $\mathbf{A}$ become zeros, all eigenvalues are equal to d_{free} and thus maximum coding gain can be achieved.

On the other hand, if $\tau_m T_x > d_{\text{free}}$, $\mathbf{A}$ is still Toeplitz matrix and thus the trace of $\mathbf{A}$ can be given by (32). Hence, considering (32) and (33), we get

$$\tau_m T_x > \left(\prod_{i=0}^{r-1} \lambda_i(\mathbf{A}) \right)^{1/r}, \quad (36)$$

where the condition satisfying the equality cannot be met and thus the equality is omitted out. This upper bound indicates that the coding gain is proportional to the number of transmit antennas T_x even in the case of $\tau_m T_x > d_{\text{free}}$. Thus, all available transmit antennas should be used for the transmission with CDD to obtain the maximum coding gain even if $\tau_m T_x > d_{\text{free}}$.

The upper bound of coding gain can be derived as follows. Let $\mathcal{E}_{n_T}$ denote the transmit energy of the n_T th antenna and $\mathcal{E}_{\min}$ indicates the minimum allocated energy of all transmit antennas, where $\sum_{n_T=0}^{T_x-1} \mathcal{E}_{n_T} = 1$. Then, from (24), the coding gain can be rewritten as

$$G_c = \mathcal{E}_{\min} p_{\min} (d_{\min} - 1) \left(\prod_{i=0}^{r-1} \lambda_i(\mathbf{A}) \right)^{\frac{1}{r}}. \quad (37)$$

From (35) and (36), the coding gain with maximum eigenvalues may be given as

$$G_c \leq \mathcal{E}_{\min} p_{\min} (d_{\min} - 1) \times \max(d_{\text{free}}, \tau_m T_x), \quad (38)$$

The above equation suggests that the maximum coding gain is given by maximizing $\mathcal{E}_{\min}$ so that the transmit energy should be allocated equally to each antenna, i.e., $\mathcal{E}_0 = \mathcal{E}_1 = \dots = \mathcal{E}_{T_x-1}$.

2) *Coding Gain Design*: From the above analyses, the following conditions are required in order to achieve the maximum coding gain.

- The cyclic delay δ should be chosen to make all eigenvalues equal.
- The transmit energy should be equally allocated to each transmit antenna.

Our derived PEP is effective only for high SNR region and thus the effect of the eigenvalues may be relatively small. Hence, the performance improvement by optimizing the coding gain may be trivial. Even so, if the infinitesimally small eigenvalue is occurred, the diversity order is decreased even in the high SNR region. Therefore, the condition of the cyclic delay is discussed here to avoid losing the diversity gain. Note that $d_{\text{free}} \geq \tau_m T_x$ is assumed in the following since our interest is to obtain the full diversity by using the CDD. Moreover, the frequency non-selective block Rayleigh fading channel, i.e., $\tau_m = 1$, is also assumed since the frequency non-selective channel is the worst case in terms of the diversity gain and thus the CDD should exploit the frequency selectivity most efficiently when $\tau_m = 1$. If the system has the infinitesimally small eigenvalue, $\det(\mathbf{A})$ becomes zero. This condition apparently depends on off-diagonal elements of $\mathbf{A}$. Then, we easily obtain the condition of avoiding the infinitesimally small eigenvalues as follows:

$$\frac{2f_s \delta}{N} \times n_T \neq 2\mathbb{N}, \quad (39)$$

for any $n_T = 1, \dots, T_x - 1$, where $\mathbb{N}$ is a natural number. Obviously, if δ does not meet this condition, $\det(\mathbf{A})$ becomes zero and thus the full diversity cannot be ensured even if δ meets the full diversity criterion.

C. Design of Bit-interleaved Coded DPSK with Cyclic Delay Diversity

From the analysis of full diversity and coding gain, the design of bit-interleaved coded DPSK with cyclic delay diversity can be summarized as follows.

To obtain the full diversity gain, the choice of cyclic delay should meet the condition that $\delta \geq \tau_m$. However, in practical situations with differential detection, the knowledge of the maximum delay spread τ_m is not available at both transmitter and receiver side. Therefore, the cyclic delay δ should be chosen as large as possible except for cases violating (39). For instance, an example choice of δ for $f_s = 8$ is given by

$$\delta = \lfloor \frac{N}{T_x} \rfloor - 1. \quad (40)$$

Moreover, the transmit energy should be allocated equally to each antenna to maximize the coding gain for given δ and T_x .

Note that this design criterion is also valid for the CDD with coherent detection. In [24], the PEP of the coherent detection is given as

$$\Pr(\mathbf{c} \rightarrow \tilde{\mathbf{c}}) \leq \frac{1}{2} \prod_{i=0}^{r-1} \left(1 + \frac{d_{\min}^2 p_{\min} \lambda_i(\mathbf{A}) E_s}{4N_0} \right)^{-1}, \quad (41)$$

$$\approx \left(\frac{d_{\min}^2 p_{\min} E_s}{4N_0} \eta \right)^{-r}. \quad (42)$$

Thus, the diversity order can be defined as the exponent of the above equation and also the coding gain is defined as

$$G_c \triangleq d_{\min}^2 p_{\min} \eta = d_{\min}^2 p_{\min} \left(\prod_{i=0}^{r-1} \lambda_i(\mathbf{A}) \right)^{1/r}. \quad (43)$$

Obviously, the above results on both diversity and coding gain with differential detection can be applied to the case of coherent detection in a straight forward manner (cf., [3, 14]).

V. SIMULATION RESULTS

In this section, computer simulations are performed to confirm our theoretical analysis and show the performance of the bit-interleaved convolutionally coded DPSK with the CDD. In the following, differential quadrature-PSK (DQPSK), i.e., $M = 4$, is used, the number of subcarriers N is 128 for OFDM transmission, and the length of CP is 16. The bit interleaver proposed in [21] is used in the simulations and thus the subcarrier spacing is $f_s = 8$. Note that this interleaver is performed within one OFDM symbol to avoid an extra delay requirement to start decoding at the receiver. Also, the block fading is assumed.

A. Diversity Order of Bit-interleaved Coded DPSK with CDD

Fig. 2 shows the bit error rate (BER) performance of the bit-interleaved coded DPSK with CDD using various numbers of antennas over frequency non-selective block Rayleigh fading channel. The rate-1/2 convolutional code with Viterbi decoding is used in computer simulations with information size 8192 bits, generation matrix $(133, 171)_8$ in octal form, and constraint length 7. Also, from the design criterion derived

Fig. 2. Comparison of the bit error rate performance of the bit-interleaved coded DPSK with CDD using various number of transmit antennas over frequency nonselective block Rayleigh fading channels. Note that solid lines indicate theoretical BER performance of DBPSK with L transmit diversity branches.

Fig. 3. The effect of the choice of δ to the diversity gain where $T_x = 2$ and $\tau_m = 2$. Note that solid lines indicate theoretical BER performance of DBPSK with L transmit diversity branches.

in Section IV-C, $\delta = N/T_x - 1$ is used in the simulations. To evaluate the diversity order of the bit-interleaved coded DQPSK with OFDM, we also show the BER performance of DBPSK with L transmit diversity branches, which is given by [25]

$$P_b = \left[\frac{1}{2}(1 - \mu) \right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1 + \mu) \right]^k, \quad (44)$$

where

$$\mu = \frac{E_s/N_0}{1 + E_s/N_0}. \quad (45)$$

From the figure, as the number of antennas increases, the diversity order increases up to the maximum diversity $r = \min(d_{\text{free}}, T_x)$. If the number of antennas exceeds the minimum free distance d_{free} , no further diversity gain can be obtained for the reason mentioned in Section IV-B. However, the coding gain is still obtained.

Fig. 3 shows the effect of the choice of δ over frequency selective channel with equal power taps and exponential power

Fig. 4. The effect of the choice of δ in terms of BER performance when $T_x = 2$ at $E_b/N_0 = 21\text{dB}$.

taps. It is apparent that the BER performance with two transmit antennas and $\delta = 1$ over frequency selective channel with $\tau_m = 2$ cannot achieve the maximum diversity $\tau_m T_x = 4$. In this case, its diversity order is $\delta(T_x - 1) + \tau_m = 3$. Thus, the slope of the ideal transmit diversity with $L = 3$ is identical with that of two transmit antennas with $\delta = 1$ and $\tau_m = 2$. Also, when the conditions, $\delta > \tau_m$ and $d_{\text{free}} > \tau_m T_x$, are satisfied, the diversity order of the system is given by $\tau_m T_x = 4$ regardless of the PDP. Therefore, in this case, the slopes of two transmit antennas with $\tau_m = 2$ are identical with that of the ideal transmit diversity with $L = 4$ as seen in the figure. However, the coding gain of the channel with the exponential PDP is degraded compared with that of the equal power taps since the coding gain can be improved by increasing p_{min} in (22). This fact may confirm the design criteria of the CDD, which can be seen as the virtual frequency selective channel with arbitrary PDP.

B. Effects of Cyclic Delay δ on Coding Gain

In order to justify the analytical results on coding gain, the effects of cyclic delay δ are quantitatively evaluated here. The rate-1/2 convolutional code with $d_{\text{free}} = 5$ and generator matrix $(5, 7)_8$ is used for the simulations. Also, wireless channel is modeled as the frequency non-selective Rayleigh fading channel, i.e., $\tau_m = 1$. In the case of two transmit antennas, the possible maximum cyclic delay is given by $N/T_x = 64$. Then, considering (39), the injudicious choices of δ are given as $\delta = \{16, 32, 48, 64\}$. Also, in the case of four transmit antennas, the possible maximum cyclic delay is $N/T_x = 32$. Then, the injudicious choices of δ are given as $\delta = \{16, 32\}$ for both $n_T = 1$ and 3, and $\delta = \{8, 16, 24, 32\}$ for $n_T = 2$, respectively.

Fig. 4 shows the effects of the choice of δ with two transmit antennas in terms of the BER performance at $E_b/N_0 = 21\text{dB}$. Obviously, the performance with injudicious cyclic delays is inferior to that with the other cyclic delays since the infinitesimally small eigenvalues $\lambda_i(\mathbf{A})$ fails to contribute to diversity gain. In other cases, the performance is almost identical regardless of the choice of different δ . Fig. 5 shows

Fig. 5. The effect of the choice of δ in terms of BER performance when $T_x = 4$ at $E_b/N_0 = 17\text{dB}$.

the effect of the choice of δ with four transmit antennas at $E_b/N_0 = 17\text{dB}$. Similar to the case with two transmit antennas, the performance with injudicious cyclic delays is apparently degraded depending on the number of n_T violating (39) with the given δ . The performance with $\delta = \{16, 32\}$ is inferior to that with $\delta = \{8, 24\}$ since $\delta = \{16, 32\}$ violate (39) for every n_T while $\delta = \{8, 24\}$ only for $n_T = 2$. These results confirm theoretical results on the design of cyclic delay δ .

VI. PERFORMANCE COMPARISON OF COHERENT DETECTION AND DIFFERENTIAL DETECTION

The performance of the bit-interleaved DPSK with the CDD has been discussed in the previous sections. However, as pointed out in the literature, the performance of differential detection is essentially inferior to that of coherent detection. In this section, we compare the performance of the CDD with differential detection with that of coherent detection to show the advantage of differential detection in practical scenarios.

In practice, coherent detection requires the accurate CSI at the receiver side and thus the pilot symbol-aided or decision-directed estimation must be used to track channel variations in time and frequency domain. Channel estimators for OFDM systems with multiple antennas have been developed in [26, 27] by using the correlation of the channel parameters at different frequencies. However, increased frequency selectivity induced by the CDD hinders these conventional channel estimators. This fact implies that the overhead due to pilots will increase in one data frame and thus the bandwidth efficiency will decrease compared with the SISO systems. Although efficient channel estimation techniques for the CDD have been developed in [11, 12], these techniques still require both severalfold pilot symbols and complex estimation circuits compared with the SISO systems. For example, in [12], a simple and efficient channel estimation technique for the CDD has been presented whereas the application of that is limited to maximum cyclic delays. However, the pilot overhead is still

Fig. 6. Comparison of the bit error rate performance of the bit-interleaved coded QPSK and DQPSK with the CDD over frequency nonselective block Rayleigh fading channels. The rate-3/4 code convolutional code is used for QPSK and the rate-1/2 is used for DQPSK.

T_x times the overhead of a conventional SISO system¹. On the other hand, differential detection does not require pilot signals so that coding with increased redundancy can be employed instead, if the same user data rate is to be transmitted [13]. These additional redundant bits can compensate for the performance gap between differential and coherent detections.

If 33% of the subcarriers are needed for the transmission of pilot symbols in the case of quadrature PSK (QPSK), the code rate $R = 3/4$ can be used for PSK and the code rate $R = 1/2$ can be used for DPSK modulation. Also, if 25% of the subcarriers are needed for the transmission of pilot symbols, the code rate $R = 2/3$ can be used for PSK and the code rate $R = 1/2$ can be used for DPSK modulation.

Fig. 6 shows the bit error performance of the bit-interleaved coded QPSK and DQPSK with the CDD over frequency nonselective block Rayleigh fading channels. Note that parameters given in the previous section are used for simulations. The rate-3/4 punctured convolutional code is used for QPSK and the rate-1/2 is used for DQPSK, where the maximum free distance of the rate-3/4 punctured convolutional code is $d_{\text{free}} = 5$ [25]. For the coherent detection, the perfect CSI is assumed to be available at the receiver side and thus this performance can be considered as the ideal performance of the pilot symbol-aided coherent detection. The performance loss of differential detection with a single transmit antenna with respect to coherent detection is about 2 dB at a BER of 10^{-1} . However, as observed, the gap between differential and coherent detections becomes less than 1 dB at a BER of 10^{-2} in the case of using two transmit antennas. Moreover, in the case of four transmit antennas, the performance of differential detection is superior to that of coherent detection in high SNR region.

Fig. 7 shows the bit error performance comparison between the rate-2/3 punctured convolutional coded QPSK and rate-1/2 convolutional coded DQPSK. Also, the maximum

¹For example, the channel estimator proposed in [12] requires at least 20% pilot symbols in an OFDM symbol for the reliable data transmission with $T_x = 2$.

Fig. 7. Comparison of the bit error rate performance of the bit-interleaved coded QPSK and DQPSK with the CDD over frequency nonselective block Rayleigh fading channels. The rate-2/3 code convolutional code is used for QPSK and the rate-1/2 is used for DQPSK.

free distance of the rate-2/3 punctured convolutional code is $d_{\text{free}} = 6$. The gap between the coherent and differential detections is about 2.5 dB at a BER of 10^{-1} in the case of SISO. This gap would be compensated by using multiple antennas close to d_{free} as similar to the above. For instance, with four transmit antennas, the gap becomes about 1.5 dB at a BER of 10^{-3} . Therefore, considering the low complexity of the receiver, the bit-interleaved coded DPSK may be an attractive choice compared to coherent detection if more than 20% pilot symbols are required for the pilot-aided coherent detection with the CDD in order to achieve an accurate channel estimation. Note that the performance of coherent detection can be improved by using the non-punctured convolutional code with the larger d_{free} since the larger d_{free} can provide the higher coding gain as observed in (43). However, in this case, much higher decoding complexity should be paid for coherent detection compared with differential detection.

VII. CONCLUSIONS

In this paper, we analyzed the PEP of the bit-interleaved coded DPSK with CDD and showed that it can achieve the diversity order $\min(d_{\text{free}}, \tau_m T_x)$ if and only if $\delta > \tau_m$ and $d_{\text{free}} \geq \tau_m T_x$ similar to coherent BICM system with CDD. Moreover, the injudicious choices of δ cause the infinitesimally small coding gain and thus the performance is degraded significantly even if the full diversity criterion is satisfied. Based on these analyses, we presented the design criterion of the bit-interleaved coded DPSK with CDD. Note that this design criterion is valid for the CDD with coherent detection. Computer simulations confirmed these theoretical results. Also, numerical results showed that the performance gap with respect to the ideal coherent detection could be compensated for if more than 20% pilot symbols were required for the pilot-aided coherent detection with CDD. Hence, the bit-interleaved coded DPSK may be an attractive choice compared to coherent detection in this case.

Finally, throughout this paper, we have focused on the traditional point-to-point communications. However, our re-

sults can readily be extended to collaborative communications such as distributed space-time coded protocols [28]. The bit-interleaved coded DPSK with CDD can achieve the full diversity gain with an arbitrary number of transmit antennas and without the knowledge of transmit antennas at the receiver side. This aspect may result in significant advantages compared with the coherent-detection-based collaborative protocols using the STBC or CDD in the collaborative diversity paradigm.

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