Dynamics and Chaos Stabilization of Semiconductor Lasers with Optical Feedback from an Interferometer

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Abstract—This paper studies dynamical behaviors and noise properties of semiconductor lasers with optical feedback from an interferometer, i.e., two external cavities. Bifurcation diagrams versus external-cavity length and reflectivity reveal the existence of stable regions for fixed states and limit cycles. The steady-state analysis for a laser diode with two external feedbacks is performed, which shows that oscillation modes are locked at minimum excess gain modes. Oscillation frequencies of limit cycles show discrete transitions with the variation of ratios between two external-cavity lengths and reflectivities. It is proposed that such a model might be applied to stabilization of the feedback-induced chaos in external-cavity semiconductor lasers since the stable region is quite robust for wide parameter ranges. Intensity and phase noises are examined and the results indicate that the low-frequency relative intensity noise level and the linewidth can be greatly improved when the laser output is a fixed state or a limit cycle.

Index Terms—Chaos, interferometer, optical feedback, semiconductor laser.

I. INTRODUCTION

SEMICONDUCTOR lasers with external optical feedback provide one of the best physical systems for studying nonlinear dynamic phenomena. It has been verified both experimentally and numerically that the external reflectivity as well as the external-cavity length have essential effects on the dynamical behaviors of the compound-cavity lasers [1]–[3]. Recently, Fischer *et al.* [4] proposed a high-dimensional chaos model which consists of a laser diode with delayed feedback from a T-shaped cavity, as schematically shown in Fig. 1. The numerical model describing such a system includes the rate equations with two delayed feedback terms. High-dimensional chaos and Ikeda scenario have been observed in both experiments and numerical simulations when the ratios between two external-cavity lengths and reflectivities are fixed at constant values.

In this paper, we investigate the dynamical behavior of such a system with the emphasis on the variation of the laser output dynamics versus the ratios between two external-cavity lengths and reflectivities, i.e., L_2/L_1 and r_2/r_1 in Fig. 1. For appropriately chosen values of these ratios, the laser output is in fixed or periodic states. The steady-state analysis is

Fig. 1. Schematic diagram of laser diode with optical feedback from an interferometer.

performed to reveal dynamical features for stable outputs. The results demonstrate that the oscillation modes for the stable region are locked at the modes with the minimum excess gain. Oscillation frequencies of periodic states show discontinuous transitions among discrete levels upon the variations of L_2/L_1 and r_2/r_1 . Since the parameter region for stable behavior (fixed or periodic states) is quite wide, the proposed model might be applied to the stabilization of the feedback-induced instabilities and consequently to the reduction of the feedback-induced noise enhancement. We verified the effectiveness of chaos stabilization and noise reduction by including Langevin noise sources in the rate equations and investigating the variations of the relative intensity noise (RIN) level and the linewidth before and after the introduction of the second external feedback.

II. DYNAMICAL MODEL AND BIFURCATIONS

Fig. 1 shows the schematic of a laser diode with external optical feedback from an interferometer which consists of mirror 1 with the amplitude reflectivity r_1 and the length L_1 and mirror 2 with r_2 and L_2 . The internal amplitude reflectivity r_0 is assumed for both the front and rear facets of the laser diode. The dynamics of a single-mode laser diode with weak to moderate feedback can be modeled by the noise-driven equations for the amplitude $E(t)$ and the phase $\phi(t)$ of the electric field and the average carrier density $N(t)$ in the active region [1]–[4]

$$\begin{aligned} \frac{dE(t)}{dt} = & \frac{1}{2} \left\{ \frac{g[N(t) - N_0]}{\sqrt{1 + \frac{E(t)^2}{E_{\text{sat}}^2}}} - \frac{1}{\tau_p} \right\} E(t) \\ & + \kappa_1 E(t - \tau_1) \cos[\Delta\tau_1(t)] \\ & + \kappa_2 E(t - \tau_2) \cos[\Delta\tau_2(t)] + F_e(t) \end{aligned} \quad (1)$$

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TABLE I
SOME PARAMETER VALUES FOR THE LASER
DIODE USED IN THE NUMERICAL CALCULATIONS

Symbol	Parameter	Value
λ	Wavelength	780 nm
l	Laser cavity length	300 μm
V_c	Laser cavity volume	$1.2 \times 10^{-16} \text{ m}^3$
r_0	Facet reflectivity	0.5556
g	Gain coefficient	$7 \times 10^3 \text{ s}^{-1}$
α	Linewidth enhancement factor	3
N_0	Carrier number at transparency	1.68×10^8
N_{th}	Carrier number at threshold	2.42×10^8
τ_s	Life time of carrier	2.04 ns
τ_p	Life time of photon	1.93 ps
τ_{in}	Round-trip time of laser cavity	8 ps
R_{sp}	Spontaneous emission factor	$1.14 \times 10^{12} \text{ s}^{-1}$
E_{sat}^2	Photon number at saturation	4.80×10^6

$$\frac{d\phi(t)}{dt} = \frac{\alpha}{2} g[N(t) - N_{th}] - \frac{\kappa_1 E(t - \tau_1)}{E(t)} \sin[\Delta\tau_1(t)] - \frac{\kappa_2 E(t - \tau_2)}{E(t)} \sin[\Delta\tau_2(t)] + \frac{F_\phi(t)}{E(t)} \quad (2)$$

$$\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_s} - \frac{g[N(t) - N_0]E(t)^2}{\sqrt{1 + \frac{E(t)^2}{E_{sat}^2}}} + F_N(t) \quad (3)$$

and

$$\Delta\tau_i(t) = \omega_0 \tau_i + \phi(t) - \phi(t - \tau_i), \quad i = 1, 2. \quad (4)$$

Here, $E(t)$ is scaled so that $|E(t)|^2$ corresponds to the total photon number in the laser diode cavity, g is the modal gain coefficient, E_{sat}^2 is the photon number at saturation, N_0 is the carrier number at transparency, $N_{th} = N_0 + 1/g\tau_p$ is the carrier number at threshold, ω_0 the angular frequency of the solitary laser, and F_e , F_ϕ , and F_N are the Langevin noise terms. Also, τ_p is the photon lifetime, τ_s is the carrier lifetime, $\tau_{1,2} = 2L_{1,2}/c$ is the external-cavity round-trip time, α is the linewidth enhancement factor, and J is the injection current density. The feedback parameter κ_i is given by

$$\kappa_i = (1 - r_0^2) \frac{r_i}{\tau_{in} r_0}, \quad (i = 1, 2) \quad (5)$$

where $\tau_{in} = 2\eta l/c$ is the optical round-trip time within the laser cavity. Some numerical values adopted for the various parameters, as appropriate for a 780-nm AlGaAs semiconductor laser, are given in Table I [5].

Dynamics of delay-differential systems with two time delays have been analyzed in a few other models [6], [7].

Fig. 2. Bifurcation diagram versus ratios L_2/L_1 and r_2/r_1 for $J = 2.0J_{th}$, $L_1 = 9$ cm, and $r_1 = 4.0\%$. F: fixed states; P: periodic states; C: chaotic states; M: distinct state is difficult to recognize.

For a first-order delay-differential equation, the linear mode analysis gives a good prediction to the system's dynamical behavior [7]. In our case, a set of three equations including the nonlinear gain and spontaneous emission factors make the situation rather complicated. Therefore, a numerical calculation is necessary to study its dynamics. We numerically calculate (1)–(3) by employing a fourth-order Runge–Kutta algorithm and investigate bifurcation diagrams versus the external-cavity conditions. In order to separate deterministic chaos from stochastic noise, we did not include noises in present simulations. The noise effects will be considered in the next section with the RIN and linewidth evaluations. The contribution of spontaneous emission to the oscillation mode is included by adding $R_{sp}/2E(t)$ to the right-hand side of (1).

Fig. 2 shows bifurcation diagram versus the ratios L_2/L_1 and r_2/r_1 for $J = 2.0J_{th}$, $L_1 = 9$ cm, and $r_1 = 4.0\%$, where $J_{th} = N_{th}/\tau_s$ is the current density at threshold. Each domain is denoted with characters F (fixed state), P (periodic state), and C (chaos) to indicate its output status. The laser output shows different behaviors ranging from fixed state to chaos depending on parameter conditions. Among various dynamical behaviors of particular interest is the stable region for fixed states and periodic oscillations. The robustness of stable behaviors (fixed or periodic states) to the variation of parameters and the attractive basin for each stable state are examined.

Bifurcations of various conditions are investigated. It is found that the parameter regions around $L_2/L_1 \approx 0.1$ or $L_2/L_1 \approx 0.9$ especially favor the stable behaviors. Another example of bifurcation diagrams is shown in Fig. 3 for $J = 1.3J_{th}$, $L_1 = 15$ cm, and $r_1 = 2.5\%$. In some parameter regions (denoted with M), different behaviors mingle with each other and it is difficult to distinguish each domain. The reason why the parameter regions around $L_2/L_1 \approx 0.1$ or $L_2/L_1 \approx 0.9$ favor the stable states is considered in the following ways: 1) when one of the two external feedback

Fig. 3. Bifurcation diagram versus ratios L_2/L_1 and r_2/r_1 for $J = 1.3J_{th}$, $L_1 = 15$ cm, and $r_1 = 2.5\%$. Symbols are the same as in Fig. 2.

lengths is very short, i.e., $L_2/L_1 \approx 0.1$ and $L_1 = 5\text{--}15$ cm, the laser and the short length external feedback (r_2 , L_2) forms a short external-cavity laser which has more stable output behavior than the solitary laser itself when subject to another external feedback (r_1 , L_1) and 2) for $L_2/L_1 \approx 0.9$ and $L_1 = 5\text{--}15$ cm, the time delay between two external feedbacks is shorter than the relaxation time of the laser so two feedback beams interfere and, at proper parameter conditions, the interference signal results in having a smaller affect on the laser dynamics than the single external feedback (r_1 , L_1) does.

It is significant to investigate the oscillation modes for the laser diode with two external cavities. In the case of a single external feedback, the oscillation mode was examined to lock at the mode that results in the maximum linewidth reduction [8]. We discussed the problem of mode selection from the dynamical viewpoint and found that the mode with the maximum stability coefficient in the phase condition is selected as the oscillation mode for fixed or periodic outputs [9]. Following the process described in [9], we calculated oscillation modes from the steady-state analysis and compared with the numerical results. Details for the steady-state analysis for a laser diode with two external optical feedbacks are given in the Appendix. A couple of examples are depicted in Fig. 4(a) for $J = 2.0J_{th}$, $L_1 = 9$ cm, and $r_1 = 4.0\%$, and in Fig. 4(b) for $J = 1.3J_{th}$, $L_1 = 15$ cm, and $r_1 = 2.5\%$. Crosses are oscillation modes which result in the minimum excess gain and triangles represent those having the maximum stability coefficient in the phase condition. White circles represent ω_s (with the reference to ω_0) obtained from the numerical calculations and the solid bars indicate their variation ranges. Fig. 4(a) shows mode distributions versus the ratio L_2/L_1 and Fig. 4(b) shows mode distributions versus the ratio r_2/r_1 . Comparing with the bifurcation diagrams, one easily finds that, in contrast to the results of the single external feedback case, the oscillation modes in the laser diode with optical feedback from the interferometer are generally locked at modes with the minimum excess gain for fixed or periodic states.

Fig. 4. (a) Oscillation modes versus L_2/L_1 for $J = 2.0J_{th}$, $L_1 = 9$ cm, $r_1 = 4.0\%$, and $r_2/r_1 = 0.5$. (b) Oscillations modes versus r_2/r_1 for $J = 1.3J_{th}$, $L_1 = 15$ cm, $r_1 = 2.5\%$, and $L_2/L_1 = 0.1$. $\times$: mode with minimum excess gain; Δ : mode with maximum stability factor K ; $\circ$: actual oscillation mode obtained from numerical calculations. The solid bars indicate the variation range.

For a periodic output, the oscillation frequency might seem to relate with the time delay between two external cavities. The further investigation shows that the oscillation frequency depends on the ratios between two external feedbacks in a very complicated way. In Fig. 5, we show period variations versus L_2/L_1 and r_2/r_1 . The period inclines to distribute in a number of discrete levels. For a fixed ratio L_2/L_1 , the period stays in a fixed value for a certain interval of r_2/r_1 and transits to the next value. In Fig. 5(a), there are a total of four levels: one fundamental level and three high levels which are, respectively, 2-, 3-, and 6-multiples of the lowest one. The period for the lowest (fundamental) level is about 0.225 ns which is quite close to the period of relaxation oscillation (~ 0.215 ns). When L_2/L_1 is varied at a fixed value of r_2/r_1 , transitions of the period among discrete levels occur more frequently, as shown in Fig. 5(b). The situation is more or less similar to the discontinuity of mode transitions discussed by Ikeda and Mizuno [6], where competitions between two different time-delayed feedbacks in a nonlinear ring resonator cause a “frustration” in selecting the oscillation mode. However, since two delay times described there are of very different orders of magnitudes, while in our case two delays are of the same order. The investigation in frequency (period) transitions is required for further understanding.

III. APPLICATIONS TO STABILIZATION OF FEEDBACK-INDUCED CHAOS

Since the parameter regions and attractive basin for fixed or periodic states are quite robust, we consider the double external feedback model as a means to control the dynamics of laser diodes with the external feedback. The introduction of the second feedback loop can be readily implemented

Fig. 5. (a) Periods for periodic oscillations versus r_2/r_1 ; $\circ$: $L_2/L_1 = 0.05$; $\times$: $L_2/L_1 = 0.35$; $\triangle$: $L_2/L_1 = 0.70$. (b) Periods versus L_2/L_1 ; $\circ$: $r_2/r_1 = 0.6$; $\times$: $r_2/r_1 = 0.8$. Other parameters are $J = 2.0J_{th}$, $L_1 = 9$ cm, and $r_1 = 4.0\%$.

either by making an interferometer like Fig. 1 or simply by inserting a half mirror in the feedback loop. By adjusting feedback length or reflectivity of the second feedback mirror as a parameter, we can stabilize the feedback-induced chaos. Figs. 2 and 3 indicate there are wide parameter ranges of feedback reflectivity or length for stable outputs (fixed or periodic states). Fig. 6(a) shows an example of time variations for a laser with single external feedback. Clearly there exists feedback-induced chaos in time variations of the laser output. Fig. 6(b) shows the results when the second external feedback is introduced. The solid and dotted lines correspond to periodic oscillation and fixed state, respectively.

In conventional chaos-control algorithms, the variation range of the control parameter is usually much smaller than its nominal value and thus the stabilized trajectories are regarded as unstable periodic orbits which were embedded in the original chaotic attractor. Since the reflectivity of the second feedback is of the same order of the first one in our case, the dynamics of the laser with two external cavities certainly differs from that with only one external-cavity, and the obtained fixed or periodic states cannot be simply explained as the results of stabilizing feedback-induced chaos from the conventional chaos-control concept. Here we use the terminology "chaos stabilization," meaning that, with the introduction of the second feedback, we can get stable behavior or avoid the feedback-induced chaos and therefore suppress the noise enhancement induced by the first external feedback.

Feedback-induced chaos is considered to be one of the main causes of the feedback-induced noise enhancement [5], [10].

Fig. 6. Time variations of the laser outputs for $J = 2.0J_{th}$, $L_1 = 9$ cm, and $r_1 = 4.0\%$. (a) $r_2 = 0$; (b) solid line: $L_2/L_1 = 0.2$, $r_2/r_1 = 0.5$, and broken line: $L_2/L_1 = 0.4$, $r_2/r_1 = 0.5$.

In a laser data recording system, it is especially demanded that the feedback-induced intensity noise be reduced. A direct modulation of injection current of the laser diode known as the high-frequency injection (HFI) technique is conventionally employed for the purpose of noise reduction. In our previous researches, we handled the problem from the chaos stabilization viewpoint and numerically demonstrated the relevance between the chaos stabilization and the noise reduction regarding the HFI algorithm [10], [11]. Here, we show the effectiveness of the chaos-stabilization algorithm through the second feedback by calculating noise properties before and after the introduction of the second external feedback.

The noise sources can be included in the rate equation through the Langevin noise terms $F_e(t)$, $F_\phi(t)$, and $F_N(t)$. Noise levels are determined from the following autocorrelation relations [12]:

$$\langle F_i(t) \rangle = 0 \quad (6)$$

$$\langle F_i(t) F_j(u) \rangle = 2D_{ij} \delta(t - u) \quad (7)$$

with

$$\begin{aligned} 2D_{ee} &= 2D_{\phi\phi} = \frac{R_{sp}}{2} \\ 2D_{NN} &= 2 \left(R_{sp} I + \frac{N}{\tau_s} \right) \\ 2D_{e\phi} &= 0 \\ 2D_{eN} &= -R_{sp} I^{1/2} \end{aligned} \quad (8)$$

where $\langle \rangle$ denotes the time average, δ is the delta function, and I and N are, respectively, the average photon number and the carrier number within the laser cavity.

The RIN is usually employed to evaluate the noise properties in some applications of laser diodes such as data recording systems. Its value can be obtained by calculating the spectrum of intensity fluctuations. Specifically, if $P(t) = |E(t)|^2$ is the

Fig. 7. RIN spectra corresponding to Fig. 6. Solid line: $r_2 = 0$, dashed line: $L_2/L_1 = 0.2$, $r_2/r_1 = 0.5$, and broken line: $L_2/L_1 = 0.4$, $r_2/r_1 = 0.5$.

photon number, the RIN spectrum is defined as the Fourier transform of the autocorrelation function according to the relations [5], [10]

$$S(\omega) = \int_{-\infty}^{\infty} \langle \delta P(t) \delta P(t+t') \rangle \exp(-i\omega t') dt' \quad (9)$$

$$\text{RIN} = \frac{S(\omega)}{I^2 \cdot \Delta f} \quad (10)$$

where $dP(t) = P(t) - I$ is the fluctuation of the photon number at time t and Δf is the bandwidth for measurement. Fig. 7 shows the RIN spectra corresponding to the outputs in Fig. 6. The solid line corresponds to Fig. 6(a) while broken and dashed lines correspond to periodic and fixed states shown in Fig. 6(b). The low-frequency RIN level is greatly reduced when the feedback-induced chaos is stabilized to periodic or fixed states.

Figs. 8(a) and 9(a) show the variation of the low-frequency RIN (average over the 0–100-MHz range) as a function of the ratios between the two external feedbacks. The parameters are fixed at $J = 2.0J_{th}$, $L_1 = 9$ cm, $r_1 = 4.0\%$, and $r_2 = 2.0\%$ for Fig. 8, and $J = 1.3J_{th}$, $L_1 = 15$ cm, $r_1 = 2.5\%$, and $L_1 = 1.5$ cm for Fig. 9, respectively. Circles are calculation results while dashed and broken lines represent the RIN level for the solitary laser ($r_1 = r_2 = 0$) and the laser with only one external feedback ($r_2 = 0$), respectively. Comparing the results with those of Figs. 2 and 3, one finds that the RIN can be reduced to the solitary laser level for fixed or periodic states.

As an evaluation of the coherence of the laser, we also calculate the linewidth. The linewidth is defined as the FWHM of the spectrum calculated from the Fourier transform of the autocorrelation function of the laser field [12], i.e.,

$$G(\omega) = \int_{-\infty}^{\infty} \langle E(t)E(t+t') \rangle \exp(-i\omega t') dt'. \quad (11)$$

In Figs. 8(b) and 9(b), the linewidth obtained from the numerical results are shown as circles. Dashed and broken lines represent the linewidth for the solitary laser ($r_1 = r_2 = 0$) and the laser with only one external feedback ($r_2 = 0$), respectively. While the single external feedback results into the drastical linewidth broadening, the second feedback can remarkably reduce such broadening. Actually, the linewidth

Fig. 8. (a) Low-frequency RIN level and (b) linewidth variations versus L_2/L_1 . $J = 2.0J_{th}$, $L_1 = 9$ cm, $r_1 = 4.0\%$, and $r_2/r_1 = 0.5$. Circles represent the calculation results of RIN or linewidth. Dashed and broken lines represent levels for a solitary laser and a laser with one external feedback, respectively.

Fig. 9. (a) Low-frequency RIN level and (b) linewidth variations versus r_2/r_1 . $J = 1.3J_{th}$, $L_1 = 15$ cm, $r_1 = 2.5\%$, and $L_2/L_1 = 0.1$. Circles represent the calculation results of RIN or linewidth. Dashed and broken lines represent levels for a solitary laser and a laser with one external feedback, respectively.

becomes narrow compared with the solitary laser when the feedback-induced chaos is stabilized to fixed or periodic states.

IV. CONCLUSION

Semiconductor lasers with optical feedback from an interferometer consisting of two external feedbacks are studied by numerical calculations. The ratios between two exter-

$$\frac{g[N_s - N_0]}{\sqrt{1 + \frac{E_s^2}{E_{\text{sat}}^2}}} - \frac{1}{\tau_p} + 2\kappa_1 \cos[\omega_s \tau_1] + 2\kappa_2 \cos[\omega_s \tau_2] + \frac{R_{\text{sp}}}{E_s^2} = 0 \quad (\text{A2})$$

nal feedback reflectivities and lengths are varied to obtain the bifurcation diagram which shows that there exist robust parameter regions for stable behaviors like fixed states or periodic oscillations. The steady-state analysis for the laser diode with two external cavities is performed and the results show that the oscillation modes are locked at minimum excess gain ones, a conclusion which is in contrast to the model with single external feedback. The frequency for the periodic oscillation depends on the ratios L_2/L_1 and r_2/r_1 in a complicated way and the fundamental frequency is close to the relaxation oscillations. Since the parameter region is quite robust for stable behaviors, we propose it as a means to stabilize the feedback-induced instabilities. The RIN spectra and the linewidth are calculated and the results demonstrate that the low-frequency RIN is reduced to the solitary laser level and the linewidth is greatly narrowed when fixed or periodic states are realized in the laser output with the introduction of the second feedback.

APPENDIX STEADY-STATE ANALYSIS

The steady-state solutions to (1)–(3) are written as [13]

$$E(t) = E_s \quad (\text{A1a})$$

$$\phi(t) = (\omega_s - \omega_0)t \quad (\text{A1b})$$

$$N(t) = N_s. \quad (\text{A1c})$$

Here, ω_s is the oscillation frequency for the composite laser system. By inserting the above equations into (1)–(3), we obtain the following equations, including (A2), shown at the top of the page, for the steady-state solutions:

$$\omega_s - \omega_0 = \frac{\alpha}{2} g(N_s - N_{\text{th}}) - \kappa_1 \sin[\omega_s \tau_1] - \kappa_2 \sin[\omega_s \tau_2] \quad (\text{A3})$$

$$J - \frac{N_s}{\tau_s} - \frac{g[N_s - N_0]E_s^2}{\sqrt{1 + \frac{E_s^2}{E_{\text{sat}}^2}}} = 0. \quad (\text{A4})$$

The excess gain is defined as

$$\Delta G = \frac{g[N_s - N_0]}{\sqrt{1 + \frac{E_s^2}{E_{\text{sat}}^2}}} - \frac{1}{\tau_p} = -2\kappa_1 \cos[\omega_s \tau_1] - 2\kappa_2 \cos[\omega_s \tau_2] - \frac{R_{\text{sp}}}{E_s^2}. \quad (\text{A5})$$

Another important measure for the oscillation mode, namely the stability coefficient of the phase condition, is defined as a

factor K . From (A2) to (A4), one obtains

$$K = \frac{d\omega_0}{d\omega_s} = 1 + \kappa_1 \tau_1 \cos(\omega_s \tau_1) + \kappa_2 \tau_2 \cos(\omega_s \tau_2) - \frac{\alpha \xi}{B} [\kappa_1 \tau_1 \sin(\omega_s \tau_1) + \kappa_2 \tau_2 \sin(\omega_s \tau_2)] \quad (\text{A6})$$

where

$$B = \frac{2\left(1 + \frac{\xi}{\eta}\right)[(J + R_{\text{sp}})\tau_s + N_{\text{th}}]}{(N_s - N_0)(1 + \eta^2)\frac{\xi}{\eta^2}} - 1 \quad (\text{A7})$$

and

$$\xi = g\tau_s E_s^2 \quad (\text{A8})$$

$$\eta = \sqrt{1 + \frac{E_s^2}{E_{\text{sat}}^2}}. \quad (\text{A9})$$

When the gain saturation and the spontaneous factors are neglected, i.e., $E_{\text{sat}} = \infty$ and $R_{\text{sp}} = 0$, then $\eta = 1$ and $B = \xi$. Consequently, (A5) and (A6) turn out to be

$$\Delta G = -2\kappa_1 \cos[\omega_s \tau_1] - 2\kappa_2 \cos[\omega_s \tau_2] \quad (\text{A10})$$

and

$$K = 1 + \sqrt{1 + \alpha^2} [\kappa_1 \tau_1 \cos(\omega_s \tau_1 + \tan^{-1} \alpha) + \kappa_2 \tau_2 \cos(\omega_s \tau_2 + \tan^{-1} \alpha)] \quad (\text{A11})$$

which reduce to the simple forms similar to the results discussed in [13].

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