HIE-FDTD Method for Hybrid System With Lumped Elements and Conductive Media

著者	Unno Masaki, Asai Hideki
journal or publication title	IEEE Microwave and Wireless Components Letters
volume	21
number	9
page range	453-455
year	2011-08-11
出版者	Institute of Electrical and Electronics Engineers
権利	<p>This work has been submitted to the IEEE for possible publication. Copyright may be transferred without notice, after which this version may no longer be accessible.</p> <p>(C) Copyright 2011 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other users, including reprinting/ republishing this material for advertising or promotional purposes, creating new collective works for resale or redistribution to servers or lists, or reuse of any copyrighted components of this work in other works.</p>
URL	http://hdl.handle.net/10297/6212

doi: 10.1109/LMWC.2011.2162616

HIE-FDTD Method for Hybrid System with Lumped Elements and Conductive Media

Masaki Unno, *Student Member, IEEE* and Hideki Asai, *Member, IEEE*

Abstract—In this letter, we extend the 3-dimensional (3D) HIE-FDTD method to the simulation of hybrid electromagnetic system including the linear/nonlinear lumped elements and the conductive media. First, numerical formulation of the HIE-FDTD method for the conductive media and the lumped elements are presented. Furthermore, we simulated the crosstalk effect among the coupled multi-conductor lines, and then the accuracy and effectiveness are validated by the comparison of our method with the conventional FDTD method and the ADI-FDTD method.

Index Terms—HIE-FDTD method, lumped element, conductive media

I. INTRODUCTION

THE FDTD (Finite-Difference Time-Domain) method is a very useful technique to solve a variety of electromagnetic problems[1]. However, the conventional FDTD method has the serious problem of restriction in terms of the time-step size. In other words, the CFL (Courant-Friedrich-Lewy) condition must be satisfied whenever this method is used, and the maximum time-step size depends on the minimum cell size in a computational domain. In order to cope with the restraint of the CFL condition, unconditionally stable ADI (Alternating-Direction Implicit)-FDTD method has been proposed and developed[2]. Although the ADI-FDTD method is unconditionally stable and free from CFL condition, it has been reported that the ADI-FDTD method has large truncation error[3].

On the other hand, the HIE (Hybrid Implicit-Explicit)-FDTD method which is weakly conditionally stable has been proposed and developed[4]. Additionally, stability and numerical dispersion analysis of the HIE-FDTD method have been also described[5]. The HIE-FDTD method has the advantage for the simulation of computational domain with thin unit cell such as PCB analyses compared to the conventional FDTD method. In [6], the principle of the HIE-FDTD formulation for the hybrid system with lumped elements has been presented roughly.

In this research, the formulations of various components are described strictly. In addition, we show the numerical stability and the accuracy for the example, including the linear/nonlinear elements and the conductive media in the both of time and frequency domains by comparing our method with the ADI-FDTD method.

M. Unno is with the Graduate School of Engineering, the Department of Systems Engineering, Shizuoka University, Hamamatsu, 432-8561 Japan. e-mail: koy01umk@tzasai7.sys.eng.shizuoka.ac.jp.

H. Asai is with the Department of Systems Engineering, Shizuoka University, Hamamatsu, 432-8561 Japan.

Manuscript received February 9, 2011; revised June 12, 2011.

II. THE HIE-FDTD METHOD FOR THE COMPUTATIONAL DOMAIN WITH THE CONDUCTIVE MEDIA AND LUMPED ELEMENTS

A. The HIE-FDTD Method for the Conductive Media

Here, it is assumed that the narrow side is along the z -direction. In such a case, since the explicit equations are same as the Yee's FDTD method[1], only the implicit equations of the HIE-FDTD method are described. For example, the implicit equations for the E_x and H_y are given by:

$$E_x|_{i+\frac{1}{2},j,k}^{n+1} = Ca_{i+\frac{1}{2},j,k} E_x|_{i+\frac{1}{2},j,k}^n + \frac{Cb_{i+\frac{1}{2},j,k}}{\Delta y_j} \left\{ H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}} \right\} - \frac{Cb_{i+\frac{1}{2},j,k}}{2\Delta z_k} \left\{ H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+1} - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+1} + H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^n \right\}, \quad (1)$$

$$H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+1} = H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n + \frac{Db_{i+\frac{1}{2},j,k+\frac{1}{2}}}{\Delta x_{i+\frac{1}{2}}} \left\{ E_z|_{i+1,j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} \right\} - \frac{Db_{i+\frac{1}{2},j,k+\frac{1}{2}}}{2\Delta z_{k+\frac{1}{2}}} \left\{ E_x|_{i+\frac{1}{2},j,k+1}^{n+1} - E_x|_{i+\frac{1}{2},j,k}^{n+1} + E_x|_{i+\frac{1}{2},j,k+1}^n - E_x|_{i+\frac{1}{2},j,k}^n \right\}, \quad (2)$$

where

$$Ca_{i+\frac{1}{2},j,k} = \frac{2\varepsilon_{i+\frac{1}{2},j,k} - \sigma_{i+\frac{1}{2},j,k}\Delta t}{2\varepsilon_{i+\frac{1}{2},j,k} + \sigma_{i+\frac{1}{2},j,k}\Delta t}, \quad Cb_{i+\frac{1}{2},j,k} = \frac{2\Delta t}{2\varepsilon_{i+\frac{1}{2},j,k} + \sigma_{i+\frac{1}{2},j,k}\Delta t},$$

$$Db_{i+\frac{1}{2},j,k+\frac{1}{2}} = \frac{\Delta t}{\mu_{i+\frac{1}{2},j,k+\frac{1}{2}}}.$$

In addition, ε , σ and μ denote the permittivity, conductivity and permeability, respectively, and i , j and k are the coordinates in 3-dimensional space.

Updating of E_x component, as shown in (1), needs the unknown H_y component at the same time step $n+1$, and thus, the E_x component has to be updated implicitly. Substituting (2) into (1), the equation for E_x field is given by (3).

$$-\frac{\alpha}{2\Delta z_{k-\frac{1}{2}}} E_x|_{i+\frac{1}{2},j,k-1}^{n+1} + \beta E_x|_{i+\frac{1}{2},j,k}^{n+1} - \frac{\gamma}{2\Delta z_{k+\frac{1}{2}}} E_x|_{i+\frac{1}{2},j,k+1}^{n+1} = \frac{Ca_{i+\frac{1}{2},j,k}}{Cb_{i+\frac{1}{2},j,k}} E_x|_{i+\frac{1}{2},j,k}^n - \frac{\alpha}{2\Delta z_{k-\frac{1}{2}}} \left\{ E_x|_{i+\frac{1}{2},j,k}^n - E_x|_{i+\frac{1}{2},j,k-1}^n \right\} + \frac{\gamma}{2\Delta z_{k+\frac{1}{2}}} \left\{ E_x|_{i+\frac{1}{2},j,k+1}^n - E_x|_{i+\frac{1}{2},j,k}^n \right\} + \frac{1}{\Delta y_j} \left\{ H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}} \right\} - \frac{1}{\Delta z_k} \left\{ H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^n \right\} + \frac{\alpha}{\Delta x_{i+\frac{1}{2}}} \left\{ E_z|_{i+1,j,k-\frac{1}{2}}^{n+\frac{1}{2}} - E_z|_{i,j,k-\frac{1}{2}}^{n+\frac{1}{2}} \right\} - \frac{\gamma}{\Delta x_{i+\frac{1}{2}}} \left\{ E_z|_{i+1,j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} \right\}, \quad (3)$$

where

$$\alpha = \frac{Db_{i+\frac{1}{2},j,k-\frac{1}{2}}}{2\Delta z_k}, \quad \gamma = \frac{Db_{i+\frac{1}{2},j,k+\frac{1}{2}}}{2\Delta z_k}, \quad \beta = \frac{1}{Cb_{i+\frac{1}{2},j,k}} + \frac{\alpha}{2\Delta z_{k-\frac{1}{2}}} + \frac{\gamma}{2\Delta z_{k+\frac{1}{2}}}.$$

Similarly, the updating equations of E_y and H_x can be obtained in the same manner. From (3), we note that E_x and E_y are updated implicitly by solving the tridiagonal matrix equations.

B. Modeling of R , L and C Elements

Since the update equation of the HIE-FDTD method is same as the one of the FDTD method as shown in [7] when the lumped element is inserted along the z -direction, it is assumed that the passive element is collocated along the x -direction. Then, (1) is replaced as follows,

$$E_x|_{i+\frac{1}{2},jk}^{n+1} = E_x|_{i+\frac{1}{2},jk}^n + \frac{\Delta t}{\epsilon_{i+\frac{1}{2},jk}} \left(\nabla \times \mathbf{H}^{n+\frac{1}{2}} \right)_x - \frac{\Delta t}{\epsilon_{i+\frac{1}{2},jk} \Delta y \Delta z} I_L|_{i+\frac{1}{2},jk}^{n+\frac{1}{2}}, \quad (4)$$

where the current $I_{L,Resistor}$ flowing through the resistor R is given by

$$I_{L,Resistor}^{n+\frac{1}{2}} = \frac{\Delta x}{R} \frac{E_x^{n+1} + E_x^n}{2}. \quad (5)$$

In addition, since every suffix of E_x for the location is $(i+1/2, j, k)$, it is abbreviated. Therefore, the coefficients Ca and Cb in (1) and (3) are respectively replaced as follows,

$$Ca = \frac{2R\epsilon\Delta y\Delta z - \Delta t\Delta x}{2R\epsilon\Delta y\Delta z + \Delta t\Delta x}, \quad Cb = \frac{2R\Delta t\Delta y\Delta z}{2R\epsilon\Delta y\Delta z + \Delta t\Delta x}.$$

Next, $I_{L,Capacitor}$, which is defined as the current through the capacitor, is given by

$$I_{L,Capacitor}^{n+\frac{1}{2}} = \frac{C\Delta z}{\Delta t} (E_x^{n+1} - E_x^n), \quad (6)$$

where C is the capacitance. Therefore, the coefficients Ca and Cb in (1) and (3) are revised respectively as follows,

$$Ca = 1, \quad Cb = \frac{\Delta t\Delta y\Delta z}{\epsilon\Delta y\Delta z + C\Delta x}.$$

Finally, $I_{L,Inductor}$, which is defined as the current through the inductor, is derived by

$$I_{L,Inductor}^{n+\frac{1}{2}} = \frac{\Delta t\Delta x}{L} \sum_{m=0}^{n-1} E_x^m, \quad (7)$$

where L is the inductance. Therefore, the coefficients Ca and Cb in (1) and (3) are modified respectively as follows,

$$Ca = 1, \quad Cb = \frac{\Delta t}{\epsilon}.$$

In addition, the term of Σ in (7) must be added to the right-hand side (RHS) of (3). The RHS is replaced by

$$RHS = V_{RHS} - \frac{\Delta x\Delta t}{\Delta y\Delta zL} \sum_{m=0}^{n-1} E_x^m,$$

where V_{RHS} is same as the RHS of (3).

Similarly, the updating equation for E_y is also derived in the same manner.

C. Modeling of Discrete Source

When a discrete source such as voltage source V_s with internal series resistance R_s is included in the x -direction in the cell, $I_{L,Source}$, which is defined as the current through the voltage source, is derived as

$$I_{L,Source}^{n+\frac{1}{2}} = \frac{(E_x^{n+1} + E_x^n)\Delta x + 2V_s^{n+\frac{1}{2}}}{2R_s}. \quad (8)$$

Then substituting (8) into the iteration equation of HIE-FDTD method, the coefficients such as Ca and Cb and the RHS of (3) are represented, respectively, by

$$Ca = \frac{2R\epsilon\Delta y\Delta z - \Delta t\Delta x}{2R\epsilon\Delta y\Delta z + \Delta t\Delta x}, \quad Cb = \frac{2R\Delta t\Delta y\Delta z}{2R\epsilon\Delta y\Delta z + \Delta t\Delta x},$$

$$RHS = V_{RHS} + \frac{1}{\Delta y\Delta zR_s} V_s^{n+\frac{1}{2}}. \quad (9)$$

In the case of modeling the ideal voltage source, the series resistor R_s is set to zero. When it is impossible to divide the coefficient by $R_s = 0$ like the last term in (9), R_s is approximated by a very small value.

D. Modeling of Nonlinear Device

The current $I_{L,Diode}$ flowing through the diode is derived as follows,

$$I_{L,Diode}^{n+\frac{1}{2}} = I_o \left[\exp \left\{ \frac{q(E_x^{n+1} + E_x^n)\Delta x}{2kT} \right\} - 1 \right], \quad (10)$$

where k is Boltzmann's constant, q is the charge of an electron, T is temperature in Kelvin, and I_o is saturation current. Therefore, the coefficients Ca and Cb in (1) and (3) and the RHS of (3) can be given, respectively, as

$$Ca = 1, \quad Cb = \frac{\Delta t}{\epsilon},$$

$$RHS = V_{RHS} - \frac{1}{\Delta y\Delta z} I_o \left[\exp \left\{ \frac{q(E_x^{n+1} + E_x^n)\Delta x}{2kT} \right\} - 1 \right]. \quad (11)$$

It is necessary to solve the nonlinear simultaneous equation based on (3) of which the RHS is replaced by (11).

In the HIE-FDTD method, when the linear/nonlinear lumped elements are included in the unit cell, only the simple operations, namely, replacing the coefficients and adding the term of Σ , have to be done against the object cell.

III. NUMERICAL RESULTS

In order to verify the accuracy and the efficiency of the HIE-FDTD method with the lumped elements and conductive media, the simulation has been done for the multi-conductor transmission lines illustrated in Fig. 1. Also, Fig. 1 shows the the number of cells and minimum cell. The diode is inserted along the y -direction, and the Newton-Raphson method is used to solve the nonlinear simultaneous equation. In Fig. 1, Mur's first-order absorbing boundary condition (ABC) has been adopted on the outer surface. Because of applying the implicit-difference method for z -direction, the implementation of the Mur's first-order ABC for the x - y planes at $k = 0$ and $k = NZ$ should be applied inside the tridiagonal matrix.

In order to check the stability condition, the time step sizes were set to $7.1 \Delta t_{CFL}$ and $7.2 \Delta t_{CFL}$. Here, $7.1 \Delta t_{CFL}$ is satisfied with the condition shown in [4]. On the other hand, $7.2 \Delta t_{CFL}$ is not satisfied. Only in the case of $7.2 \Delta t_{CFL}$, the modified method was not stable. Thus, the stability condition of the modified method is regarded to be same as the one of the original HIE-FDTD method. The waveform results at the nodes $n2, n3, n4$ and $n6$ by using $7.1 \Delta t_{CFL}$ are shown in Fig. 2. For the sake of comparison, the ADI-FDTD method was also implemented, and the time-step size was set to $7\Delta t_{CFL}$ and $17\Delta t_{CFL}$. From Fig. 2, it is confirmed that the accuracy of the ADI-FDTD method degrades as the time-step size becomes large. In contrast, it has been confirmed that the waveforms obtained by the HIE-FDTD method are equivalent to the ones by the conventional FDTD method.

Fig. 1. Computational domain: (a) an example with multi-conductor lines and lumped elements, (b) equivalent circuit model and (c) cross section.

TABLE I
CPU* TIMES AND MEMORY REQUIREMENT

	Δt	Steps	CPU Time	Memory
FDTD	3.30×10^{-14} s	212160	3445.9 s	36.532 KB
ADI-FDTD x7	2.31×10^{-13} s	30308	2370.5 s	57.684 KB
ADI-FDTD x17	5.61×10^{-13} s	12480	978.6 s	57.684 KB
HIE-FDTD x7	2.36×10^{-13} s	29708	994.2 s	40.748 KB

*Note: Intel Xeon (CPU : 5482 @ 3.20GHz; 3.19GHz, 32G RAM) is used.

In order to quantify the difference between the HIE-FDTD method and the ADI-FDTD method, the relative errors of these methods to the FDTD method are calculated at the frequency sampling points. Figure 3 shows the relative error at the node $n3$, where the time step size of the HIE-FDTD method is set to $7 \Delta t_{CFL}$. From Fig. 3, the relative error of the ADI-FDTD method becomes large at the high-frequency range. On the contrary, the relative error of the HIE-FDTD method is below 1% for the wide-frequency range.

The CPU time and memory requirement with respect to a variety of time step sizes are shown in Table I. From Table I, it is confirmed that the CPU time is reduced to 1/3.46 and the memory requirements for the conventional FDTD method and HIE-FDTD method are almost the same.

IV. CONCLUSION

This letter described the extension of HIE-FDTD method to the hybrid electromagnetic simulation with the lumped elements and the conductive media. From the numerical results, it has been confirmed that the waveform results between the conventional FDTD method and the HIE-FDTD method show the good agreements and the HIE-FDTD method is 3.5 times faster than the conventional FDTD method when the time step size for the HIE-FDTD method is 7.1 times larger than the conventional one.

REFERENCES

[1] K. S. Yee, "Numerical solution of initial boundary value problems involving maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 302–307, May 1966.

Fig. 2. Simulation results: voltage outputs at (a) node $n2$, (b) node $n3$, (c) node $n4$ and (d) node $n6$.

Fig. 3. Relative errors of HIE-FDTD method and ADI-FDTD method to the FDTD method are calculated at the frequency sampling points.

- [2] T. Namiki, "3-D ADI-FDTD method-unconditionally stable time-domain algorithm for solving full vector maxwell's equations," *IEEE Trans. Microwave Theory Tech*, vol. 48, no. 10, pp. 1743–1748, Oct. 2000.
- [3] S. Garcia, T. Lee, and S. Hagness, "On the accuracy of the ADI-FDTD method," *IEEE Antennas Wireless Propagat Lett*, vol. 1, pp. 31–34, 2002.
- [4] J. Chen and J. Wang, "A three-dimensional semi-implicit FDTD scheme for calculation of shielding effectiveness of enclosure with thin slots," *IEEE Trans. Electromag. Compat.*, pp. 354–360, May 2007.
- [5] F. Xiao, X. Tang, and L. Wang, "Stability and numerical dispersion analysis of a 3D hybrid implicit-explicit FDTD method," *IEEE Trans. Antennas Propagat.*, vol. 56, pp. 3346–3350, 2008.
- [6] M. Unno and H. Asai, "3D HIE-FDTD Method for the Network Including Lumped Elements and Its Estimation," *Technical report of IEICE, EMCJ2010-57*, pp. 31–36, Oct. 2010.
- [7] W. Sui, D. A. Christensen, and C. H. Durney, "Extending the two-dimensional FDTD method to hybrid electromagnetic system with active and passive lumped elements," *IEEE Trans. Microwave Theory Tech*, vol. 40, no. 4, pp. 724–730, 1991.