Growth model with phase transition: Drift-diffusion-limited aggregation

Takashi Nagatani

College of Engineering, Shizuoka University, Hamamatsu 432, Japan

(Received 30 August 1988)

A growth model is presented to show a phase transition. The drift-diffusion-limited aggregation model with a positive drift proposed by Meakin [Phys. Rev. B **28**, 5221 (1983)] is extended to that with a negative drift. When the drift changes from a positive value to a negative value, the structure of the aggregate varies from a dense structure, through the diffusion-limited aggregation (DLA) fractal, to a needlelike structure. In contrast with the crossover from the DLA fractal to the dense structure at a positive drift, at a negative drift the crossover is shown from the fractal structure at short length scales to a one-dimensional structure at longer length scales.

Recently, there has been increasing interest in the problem of the geometrical structure in the kinetic growth model.¹⁻⁶ A number of models for cluster formation have been proposed and a variety of computer simulations have been carried out to investigate the relationships between the cluster geometry and the growth mechanism.⁷⁻¹³ The structure of the aggregates strongly depends on the dynamics of the growth process. Meakin¹⁴ investigated the effect of particle drift on diffusion-limited aggregation (DLA) by using biased random walkers. He found a crossover from the DLA fractal on short length scales to a dense structure on longer length scales, with the use of Monte Carlo simulation. Even though there is not a drift of particles, convection is induced in the diffusion field by the moving interface. The convective diffusion field is also simulated by the diffusion process with a drift. Sarkar and Jensen¹⁵ studied the evolution of the interface for a one-sided model of directional solidification on a lattice with biased random walkers. Nagatani¹⁶ showed that the DLA fractal crossed over to one of the two distinct nonfractal structures, depending on the direction of the heat flow in the solidification process. With use of the real-space renormalization-group method,¹⁷ the DLA process was formulated as a critical phenomenon.

In this Brief Report, we present a growth model where a phase transition occurs. Especially, we give attention to the diffusive system with a negative drift where a particle follows a biased random walk away from the seed particle. The drift-diffusion-limited aggregation with a positive drift was analyzed by Meakin.¹⁴ However, the diffusion-limited aggregation with a negative drift was never analyzed. We study what does the structure of the aggregate change with changing direction of the drift. We show whether or not a phase transition occurs in the drift-diffusion-limited aggregation model.

We consider the diffusive system with the radial drift. The simulation of the drift-diffusion-limited cluster formation in the presence of the radial drift of particles is carried out with the use of a simple square lattice. We consider a subset of the square lattice enclosed by a circle with a radius R ($\gg 1$). We start out with a single occu-

pied lattice site (shown by the black square in Fig. 1) on the center of the circle. Particles are introduced one at a time at a randomly chosen point on the outer circular boundary. We allow particles to follow radial-biased random trajectories in the vicinity of the original occupied site or seed. For a positive drift, particles perform a radial-biased random walk with probability P toward the center (or seed particle). For a negative drift, particles perform a biased Brownian motion with probability $-P$ away from the center. A particle continues to move until it either reaches a point adjacent to a site already occupied by a particle, or until it reaches the outer boundary. Figure 1 shows a schematic representation of the model used to stimulate the effects of the radial particle drift on the diffusion-limited aggregation. We have carried out calculations for a range of P values ($-0.02 < P < 1$). For a positive P , Meakin¹⁴ and Nagatani¹⁸ have studied the

FIG. 1. The model used to simulate the effect of negative drift on the diffusion-limited aggregation. The seed particle positioned on the center is indicated by the black square. A single particle is introduced at a time at a randomly chosen point on the outer circular boundary, and performs a radial-biased random walk with probability P away from the center.

pattern formed by the drift-diffusion-limited aggregation model. The crossovers have been found from the DLA fractal to the dense aggregation. We restrict ourselves to the case with a negative drift. Figure 2 shows results obtained for a few clusters using the procedures outlined

above. The clusters are grown until the radius reaches 120 units. The cluster shown in Fig. 2(a) was obtained using a radial drift probability $P=0.0$. The cluster is the DLA fractal. Figures 2(b), 2(c), 2(d), and 2(e) show, respectively, the clusters for $P=-0.005$, -0.009 , -0.016 ,

FIG. 2. Typical clusters grown at a negative drift with the use of the model shown in Fig. 1. (a) DLA cluster of 4192 particles. (b) Cluster of 3278 particles grown with the use of a drift probability $P=-0.005$. (c) Cluster of 2972 particles for $P=-0.009$. (d) Cluster of 2130 particles for $P=-0.016$. (e) Cluster of 1195 particles for $P=-0.018$.

and -0.018 . As the radial-drift probability P is lower, the overall shape of the cluster becomes a less ramified structure. Even a very small value of the negative drift becomes effective for the DLA fractal. For comparison, we show the clusters obtained for a positive drift in Fig. 3. As the positive drift is higher, the overall shape of the cluster becomes closer to a circular-dense pattern. On the other hand, for the negative drift, the overall shape of the cluster becomes a more eccentric structure with the increase of the negative drift. The shape of the cluster is different for each sample of the simulation and is very unstable. Figure 4 shows the plot of the particle number against the radius. The curves a , b , c , d , and e in Fig. 4

FIG. 3. Typical clusters grown at a positive drift. (a) Cluster of 7000 particles grown with the use of a drift probability $P=0.1$. (b) Cluster of 8600 particles for $P=0.2$. (c) Cluster of 10250 particles for $P=0.6$.

FIG. 4. The number of particles plotted against the radius shown in the form of log-log plots for five clusters grown with the use of different drift probabilities. The curves a , b , c , d , and e indicate the cases obtained by Fig. 2. At small distances the slope of curves is close to $\frac{5}{3}$.

indicate, respectively, the cases for $P=0.0$, -0.005 , -0.009 , -0.016 , and -0.018 . The crossover in the fractal dimensionality of the cluster is found from $D \sim \frac{5}{3}$ on short length scales to $D \sim 1$ on long length scales. The fractal dimension of the cluster on long length scales ranges from 0.7 to 1.2. The fractal dimension is approximated by $D \sim 1$. On the other hand, for the positive drift, Meakin has found the crossover from the DLA fractal on short length scales to the dense structure on long length scales. When the drift changes from a positive value to a negative value, the structure of the aggregate varies from a dense structure, through the DLA fractal, to a needle-like structure. It is found that the phase transition can occur in the drift-diffusion-limited aggregation. We show schematically the phase transition by the plot of the particle number against the cluster radius in Fig. 5. When

FIG. 5. Schematic behavior of the particle number against the radius in the form of log-log plots. At a positive drift, the crossover occurs from the DLA fractal at small length scales to the dense structure at long length scales. At a negative drift, the crossover occurs from the DLA fractal at small length scales to the one-dimensional structure at long length scales.

the particle drift changes from a positive value to a negative, the slope of the particle number on long length scales varies from $D \sim 2$, through $D \sim \frac{5}{3}$, to $D \sim 1$.

In summary, we present the drift-diffusion-limited aggregation model with changing direction of particle drift. We find that the model shows the phase transition from the dense structure, through the DLA fractal, to the needle, accordingly, as the particle drift varies from a positive value to a negative. The phase transition may be re-

lated to the Saffman-Taylor instability.¹⁹ The instability can occur with changing direction of flow in the viscous fingering. At a positive drift Meakin found the crossover from the fractal structure at short length scales to a dense structure at longer length scales. On the other hand, at a negative drift, we find the crossover from the DLA fractal at short length scales to a needle at longer length scales.

¹*Kinetics of Aggregation and Gelation*, edited by F. Family and D. P. Landau (North-Holland, Amsterdam, 1984).

²*On Growth and Form*, edited by H. E. Stanley and N. Ostrowsky (Martinus Nijhoff, The Hague, 1985).

³*Fractal in Physics*, edited by L. Pietronero and E. Tosatti (North-Holland, Amsterdam, 1986).

⁴*Statistical Physics*, edited by H. E. Stanley (North-Holland, Amsterdam, 1986); *Physica A* **140**, special issue (1986).

⁵*The Physics of Structure Formation*, edited by W. Guttinger and G. Dangelmayr (Springer, Berlin, 1987).

⁶H. J. Herrmann, *Phys. Rep.* **136**, 153 (1986).

⁷T. A. Witten and L. M. Sander, *Phys. Rev. Lett.* **47**, 1400 (1981); *Phys. Rev. B* **27**, 5686 (1983).

⁸P. Meakin, *Phys. Rev. A* **27**, 1495 (1983); *A* **27**, 2616 (1983).

⁹R. Kapral, S. G. Whittington, and R. C. Desai, *J. Phys. A* **19**,

1727 (1986).

¹⁰F. Family, D. E. Platt, and T. Vicsek, *J. Phys. A* **20**, L1177 (1987).

¹¹J. Nittmann and H. E. Stanley, *J. Phys. A* **20**, L1185 (1987).

¹²E. Ben-Jacob, R. Godbey, N. D. Goldenfeld, J. Koplik, H. Levine, T. Mueller, and L. M. Sander, *Phys. Rev. Lett.* **55**, 1315 (1985).

¹³T. Vicsek, *Phys. Rev. A* **32**, 3084 (1985).

¹⁴P. Meakin, *Phys. Rev. B* **28**, 5221 (1983).

¹⁵S. K. Sarkar and M. H. Jensen, *Phys. Rev. A* **35**, 1877 (1987).

¹⁶T. Nagatani, *Phys. Rev. A* **37**, 4461 (1988).

¹⁷T. Nagatani, *Phys. Rev. A* **36**, 5812 (1987); **37**, 3514 (1988).

¹⁸T. Nagatani (unpublished).

¹⁹P. G. Saffman and S. G. Taylor, *Proc. R. Soc. London, Ser. A* **245**, 312 (1958).