Starting functions in representation theory of algebras

メタデータ	言語: en
	出版者: Shizuoka University
	公開日: 2017-06-07
	キーワード (Ja):
	キーワード (En):
	作成者: Nakashima, Ken
	メールアドレス:
	所属:
URL	https://doi.org/10.14945/00010201

（課程博士・様式７）(Doctoral qualification by coursework,Form 7)

学 位 論 文 要 旨

Abstract of Doctoral Thesis

専　　攻：情報科学　　　　　　　　　　　　　氏　　名：中島 健

論文題目：Starting functions in representation theory of algebras

論文要旨：Throughout this paper $\Bbbk$ is an algebraically closed field, and all vector spaces (resp. algebras, linear maps) are assumed to be finite-dimensional $\Bbbk$-vector spaces (resp. basic connected finite-dimensional $\Bbbk$-algebras, $\Bbbk$-linear maps). Further all modules over an algebra considered here are assumed to be finite-dimensional modules.

Starting functions are an important tool in representation theory of algebras. They were introduced by Gabriel to compute AR-quivers in [5], and were developed such as in [6],[7],[4],[3],[2, 6.6] and [1]. In this paper, we give two results obtained by using starting functions.

The first result gives an easier way to calculate a bijection from the set of isoclasses of tilted algebras of Dynkin type Δ to the set of configurations on the translation quiver $\mathbb{Z}\Delta$, where Δ is a Dynkin graph of type A, D, E. By Riedtmann [7, 2.5] the computation of the Auslander-Reiten quiver (AR-quiver for short) Γ_Λ of a representation-finite standard self-injective algebra Λ of type Δ is reduced to that of stable AR-quiver ${}_s\Gamma_\Lambda$ of Λ and the configuration $\mathcal{C}_\Lambda$ of Λ as the isomorphism $\Gamma_\Lambda \cong ({}_s\Gamma_\Lambda)_{\mathcal{C}_\Lambda}$ shows. The stable AR-quiver ${}_s\Gamma_\Lambda$ is given by the orbit category presentation of Λ, namely if $\Lambda \cong \hat{A}/G$ for some tilted algebra A of type Δ, then ${}_s\Gamma_\Lambda \cong \mathbb{Z}\Delta/G$. Therefore to recover the AR-quiver Γ_Λ it suffices to compute the configuration $\mathcal{C}_\Lambda$ by using information of A. Set $\mathbf{C}(\Delta)$ to be the set of configurations on the translation quiver $\mathbb{Z}\Delta$ (see Definition 1.6), and $\mathbf{T}(\Delta)$ to be the set of isoclasses of tilted algebras of type Δ. Bretscher, Läser and Riedtmann gave a bijection $c\colon \mathbf{T}(\Delta) \to \mathbf{C}(\Delta)$ in [4], which makes it possible to compute $\mathcal{C}_\Lambda$ as the equivalence class of $c(A)$. Hence we can compute Γ_Λ using these data. But the map c is not given in a direct way, it needs a long computation of a function on $\mathbb{Z}\Delta$. In this paper we will give an easier way to calculate the map c by giving a map sending each projective indecomposable A-module over a tilted algebra A in $\mathbf{T}(\Delta)$ to an element of the configuration $c(A)$ in $\mathbf{C}(\Delta)$ by using starting functions.

The second result gives a general formula that computes the indecomposable decomposition of any finite-dimensional module over any finite-dimensional algebra. Let A be an algebra, $\mathcal{L}$ a complete set of representatives of isoclasses of indecomposable A-modules. Then the Krull-Schmidt theorem states the following. For each A-module M, there exists a unique map $\boldsymbol{d}_M\colon \mathcal{L} \to \mathbb{N}_0$ such that

$$M \cong \bigoplus_{L\in\mathcal{L}} L^{(\boldsymbol{d}_M(L))},$$

which is called an *indecomposable decomposition* of M. Therefore, $M \cong N$ if and only if $\boldsymbol{d}_M = \boldsymbol{d}_N$ for all A-modules M and N, i.e., the map $\boldsymbol{d}_M$ is a complete invariant of M under isomorphisms. Note that since M is finite-dimensional, the support $\operatorname{supp}(\boldsymbol{d}_M) := \{L \in \mathcal{L} \mid \boldsymbol{d}_M(L) \neq 0\}$ of $\boldsymbol{d}_M$ is a finite set. We call such a theory a *decomposition theory* that computes the indecomposable decomposition of a module. The purpose of this part is to develop a

decomposition theory by using the knowledge of AR-quivers. Thus in the case that $\mathcal{L}$ is already computed and all almost split sequences are known, we aim to compute

(I) $\boldsymbol{d}_M$ and

(II) a finite set S_M such that $\mathrm{supp}(\boldsymbol{d}_M) \subseteq S_M \subseteq \mathcal{L}$

for all A-modules M. Note that (II) is needed to give a finite algorithm. If A is representation-finite (i.e., if the set $\mathcal{L}$ is finite), then the problem (II) is trivial because we can take $S_M := \mathcal{L}$. First, we give a general solution of the problem (I) by using starting functions, and we apply it to the Kronecker algebra. Further, we consider problem (II) for the Kronecker algebra, and give an example of S_M.

References

[1] Bongartz, K.: *Critical Simply Connectev Algebras*, Manuscripta Math., **46** (1984), 117–136.

[2] Bongartz, K. Gabriel, P.: *Covering Spaces in representation theory*, Invent. Math., **65** (1982), 331–378.

[3] Bongartz, K.: *Treue einfach zusammenhängende Algebren. I.* Comment. Math. Helv. **57** (1982), no. 2, 282–330.

[4] Brescher, O. Läser, C. Riedtmann, C.: *Selfinjective and Simply Connected Algebras*, Manuscripta Math., **36** (1981), 253–307.

[5] Gabriel, P.: *Auslander-Reiten sequences and representation-finite algebras. Representation theory, I* (Proc. Workshop, Carleton Univ., Ottawa, Ont., 1979), pp. 1–71, Lecture Notes in Math.,**831**, Springer, Berlin, 1980.

[6] Riedtmann, C.: *Algebren, Darstellungsköcher, Überlagerungen und zurück*, Comment. Math. Helv. **55** (1980), no.2, 199–224.

[7] Riedtmann, C.: *Representation-finite self-injective algebras of class A_n*, Representation theory, II (Proc. Second Internat. Conf., Carleton Univ., Ottawa, Ont., 1979), 449–520, Lecture Notes in Math., 832, Springer, Berlin, 1980.