

Lattice Model for the Lotka-Volterra System

メタデータ	言語: eng 出版者: 公開日: 2022-02-18 キーワード (Ja): キーワード (En): 作成者: Tainaka, Kei-ichi メールアドレス: 所属:
URL	http://hdl.handle.net/10297/00028617

Lattice Model for the Lotka-Volterra System

Kei-ichi TAINAKA

Department of Physics, Ibaraki University, Mito 310

(Received March 23, 1988)

The spatial pattern in the lattice system composed of three competing species is studied. An individual is assumed to react with a neighbour. It is found from the stochastic simulation that the dynamics of the system strongly depend on the lattice dimension (d). When $d=2$, the system approaches the stable state regardless of initial conditions.

Many authors considered the struggle for existence following the work of Lotka and Volterra.¹⁾ In the present article, the pattern formation for the system composed of three competing species is studied. We consider the random reaction model which satisfies the following rules:

(1) There are three species, 1, 2 and 3, whose numbers of individuals (particles) are at time t , $n_1(t)$, $n_2(t)$ and $n_3(t)$, respectively. The total number $n = n_1(t) + n_2(t) + n_3(t)$ is a constant.

(2) The system constitutes n lattices, each of which is occupied by one particle. No particle ever changes its position.

(3) Each particle reacts with the neighbouring one.

(4) When a pair of particles of species i and j react, they change into two particles of species i , if $i-j=0, 1 \pmod{3}$. If $i-j=2 \pmod{3}$, they change into two particles of species j . Thus, the relation of strength between the species is cyclic.

This lattice model is an extension of the spatial freedom to the Lotka-Volterra (LV) model.^{2,3)} If rule (2) is replaced by the gas model, in other words, if the reaction takes place between any pair of particles, then our model agrees with the LV system:

$$\frac{d}{dt} n_i = c n_i (n_{i-1} - n_{i+1}), \quad (1)$$

where $n_{i+3} = n_i$, and c is the constant.

The simulation is performed by asynchronous processing; the algorithm for the evolution law is randomly processed in time

with a mean attempt rate. The time t is measured by the unit of the Monte Carlo step (MC). We call 1 MC when the random reaction occurs n times. Throughout the calculation, the periodical boundary condition is applied.

The result of the simulation reveals that the dynamics of the system strongly depend on the lattice dimension (d). At first, we consider a one-dimensional ($1d$) lattice system. The initial condition is assumed to be a random distribution. Then, the time evolution of this system is represented as the growth process of "domains", where we define domain by a lattice region occupied by the same species. In Fig. 1, the total number of domain (D) obtained by the simulation is shown against t .

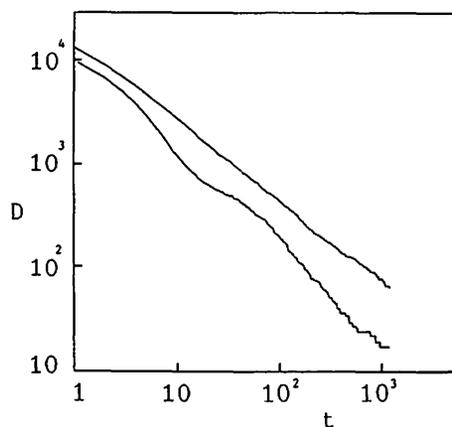


Fig. 1. The dynamics of a one-dimensional lattice system ($n=40000$). The longitudinal axis denotes the total number of domains (D). The initial condition of $n_1:n_2:n_3$ is 1:1:1 for the upper curve, and 7:2:1 for the lower curve.

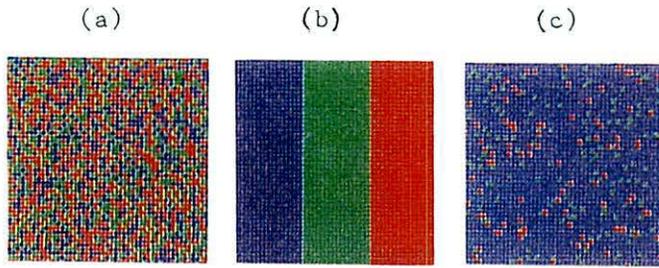


Fig. 2. Initial conditions (50×50 square lattices). The species are represented by colour: 1:blue, 2:green, 3:red.

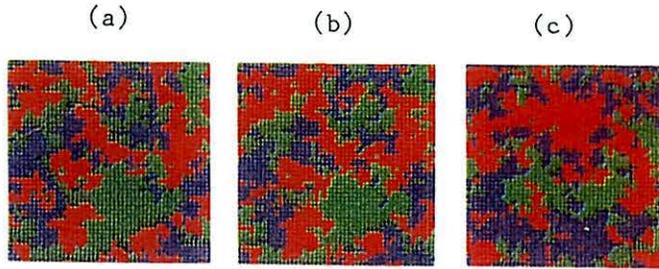


Fig. 6. Snapshots in the steady state (a): $t=200$, (b): $t=201$, (c): $t=210$. The initial condition is the pattern (a) in Fig. 2.

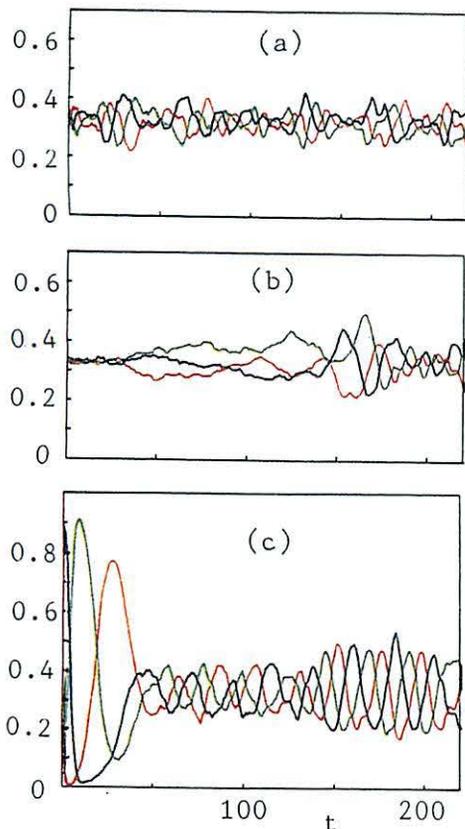


Fig. 3. Time dependence of number of each species. The colours and a, b and c represent the same meaning as in Fig. 2.

The initial condition of the ratio $n_1:n_2:n_3$ is 1:1:1 and 7:2:1 for the upper and lower curves, respectively. It can be found from this figure ($t \geq 100$) that D decreases with time as

$$D \propto t^{-\alpha},$$

where α is the constant. The value of α depends slightly on the initial condition. For example, $\alpha \sim 0.8$ and $\alpha \sim 1.2$ for the upper and lower curves, respectively.

When the lattice size is finite, the following events occur for $1d$ lattice system. After sufficient growth of domains (in the last stage of the dynamics), the large domains are aligned as the tandem repeat of three species 1, 2 and 3. At the same time as the alignment, all domains rotate in one direction. In this rotation period, the domains grow yet larger by their unification. At the final stage of the evolution process, the system necessarily attains a homogeneous pattern, that is, the whole system is occupied by one species.

When $d=2$, the simulation for the square lattice is carried out under various initial conditions. Three typical patterns for the initial condition are illustrated in Fig. 2, where the particles in (a) and (c) are randomly distributed. The ratio of the particle numbers $n_1:n_2:n_3$ is 1:1:1 for (a) and (b), and 45:3:2 for (c). In Fig.

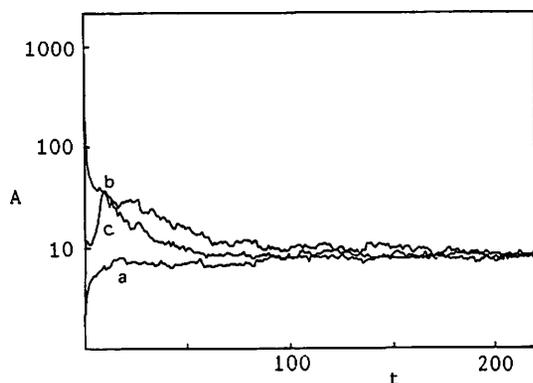


Fig. 4

Fig. 4. Time evolution of the average domain area (A). The initial conditions for curves a, b and c are the three patterns in Fig. 2.

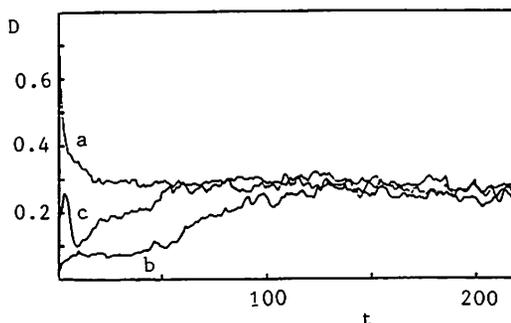


Fig. 5

Fig. 5. Time evolution of the total length of the domain boundary (D), where D is on a scale with the total lattice boundary, and a, b and c have the same meaning as in Fig. 4.

3, the time evolution of the particle number of each species n_j is shown. This figure shows that for sufficiently large t , the dynamics of the particle numbers have a similar profile regardless of the initial conditions. When the lattice size is large, n_j approaches $1/3$, and the fluctuation of n_j around $1/3$ decreases. This result is quite different from that for the LV system (gas model: eq. (1)). The dynamics of the gas model are unstable, since n_j oscillates depending on the initial condition. Accordingly, the dynamics of the particle number of each species can be stabilized by the spatial effect of the lattice model.

In Fig. 4, the time evolution of the average domain area A which is defined by the average particle number included in one domain is shown. The dynamics of the total length (D) of the domain boundary are also illustrated in Fig. 5. It is found from the time dependences of A and D that the system approaches the stationary state irrespective of the initial conditions. The values of A and D in the stationary state are 9.5 and 0.27, respectively. These values do not change in large lattice systems. The snapshots in the stationary state are illustrated in Fig. 6, where (a), (b) and (c) are the patterns at times 200, 201 and 210 MC, respectively. There are several features in these

patterns. First, many different sizes of domains are distributed, and their patterns vary greatly with time. Secondly, there is a correlation between the neighbouring domains. For example, the smallest domain composed of one green particle is surrounded not by the blue domain, but by the red domain. This fact is due to rule (4) that the green is weaker than the red, but stronger than the blue.

By the modification of rule (4), our model can easily be extended to the lattice model for the Wright system in population genetics.⁴⁾ The pattern dynamics for the Wright model show different results from that for the LV model. This result will be reported elsewhere.

Acknowledgement

The author is much indebted to Mr. Y. Ishii for help with the computer simulation and programming, and to Professor Y. Itoh for his valuable advice.

Reference

- 1) H. Haken: *Synergetics, an Introduction* (Springer, Berlin, 1977) p. 130.
- 2) Y. Itoh: *Ann. Inst. Statist. Math.* **25** (1973) 653.
- 3) Y. Itoh: *Prog. Theor. Phys.* **78** (1987) 507.
- 4) T. Maruyama: *Lecture Notes in Biomathematics* (Springer, Berlin, 1977).