Earthquake size : An example of a statistical distribution that lacks a well-defined mean

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Power-law distributions are observed to describe many physical phenomena with remarkable accuracy. In some cases, the distribution gives no indication of a cutoff in the tail, which poses interesting theoretical problems, because its average is then infinite. It is also known that the averages of samples of such data do not approach a normal distribution, even if the sample size increases. These problems have previously been studied in the context of random walks. Here, we present another example in which the sample average increases with the sample size. In the Gutenberg–Richter law for earthquakes, we show that the cumulative energy released by earthquakes grows faster than linearly with time. Here, increasing the time span of observation corresponds to increasing the sample size. While the mean of released energy is not well defined, its distribution obeys a non-trivial scaling law.

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I. INTRODUCTION

Very diverse physical, biological, and social phenomena display distributions that obey a power law with remarkable accuracy and over several orders of magnitude.[1] In some cases, the power law holds true without any sign of a cutoff, which would indicate that the distribution has no finite mean. From a purely theoretical perspective, theorems and limit distributions for such cases are known in mathematics.[2–5] From a practical or physical standpoint, however, measurable implications caused by the peculiar properties of power law distributions have attracted little attention until recently.[6] The mathematics of probability distributions with infinite means was investigated by Paul Lévy in the 1930s.[2,4] Several decades later, Lévy's results were applied in physics to explain phenomena related to random walks, e.g., anomalous diffusion in disordered conductors.[7–10,15] In recent years, this subject has become popular and found applications to numerous physical systems from laser cooling to aging.[11–14]

The absence of a well-defined average requires that the probability of a variable x behaves as $P(x) \propto 1/(x^{1+\alpha})$ with $\alpha > 0$ in the limit of $x \to \infty$. In laser cooling of atomic gases, for instance, the lifetime of atoms in a given momentum state follows this type of distribution.[14] A divergent lifetime signifies that the system is experimentally unable to reach a steady state despite the experimental time scale being significantly larger than the microscopic one.

Another example is the law of large numbers. For the sum S_N of N independent and identically distributed random variables, it is often taken for granted that the average S_N/N approaches a finite value $\bar{\mu}$ as N increases. This is the (weak) law of large numbers, which underlies the inference in physics experiments that the true value of the observable is $\bar{\mu}$. If the variable has a finite variance, σ^2, then the sum S_N approaches the normal distribution with mean $N\bar{\mu}$ and variance $N\sigma^2$. This is the central limit theorem.

However, it is possible to imagine anomalous cases for which the limit $\bar{\mu}$ does not exist or is ill-defined. To deal with these phenomena, the usual method, which underlies the mean field theory, of considering the average value of a physical observable separately from its fluctuations is of no help. Lévy derived the probability distribution for such cases in terms of the Fourier transform of the probability density function.[2] Phenomena such as random jumps of a state variable, e.g., position or displacement in a random walk, are natural applications of Lévy's results. Atomic momentum in subrecoil laser cooling is also a good example.[14] However, practical applications can also be found elsewhere in entirely different contexts.

In the eighteenth century, Nicolas Bernoulli posed a paradoxical problem now known as the St. Petersburg paradox.[16] It is based on a theoretical lottery game that leads to a random variable with infinite average. This game entails flipping a coin. If heads comes up in the first toss, the player wins one coin. If tails comes up, the player is allowed to flip the coin again, until heads comes up. The prize 2^N is determined by the number N of tails the player has drawn before obtaining a heads. The paradox is that the mean winning $(1 \times \frac{1}{2} + 2^2 \times \frac{1}{4} + 2^3 \times \frac{1}{8} + \cdots)$ is infinite, so that we cannot determine a fair ante to enter the game, and the banker should have infinite resources to cover the expected loss. Feller took up this problem in a classic book on probability theory,[4] where he showed that, when the game is repeated n times, the accumulated winning converges to $n \log_2 n$ as n increases.[5] This superlinear behavior implies that the distribution does not have a finite mean, since a finite mean would require the accumulated sum to vary linearly with n. Thus, the absence of a well-defined mean does not imply the absence of any rule. Instead, for a large number of games, the probability distribution of the overall winning should display the universal scaling property expected from Lévy's mathematical results.[2] This point hints at possible physical systems that exhibit a distribution without a finite mean. In this paper, we present a particular example of such a system: the power law distribution of earthquakes.

II. POWER-LAW DISTRIBUTION OF EARTHQUAKE ENERGIES

The Gutenberg–Richter law of earthquake size distribution is of fundamental importance not only in seismology but more generally in physics of self-similar phenomena.[17–19] The famous Gutenberg–Richter relation states that $\log P(M) = a - bM$ with $b \simeq 1$ for the cumulative number $P(M)$ of earthquakes with a magnitude larger than M.[20] Combined with the equation connecting the earthquake's magnitude M and its energy E,[17] $\log E = 1.5M+$ const., we have $P(E) \propto E^{-\beta_{\rm GR}}$ with $\beta_{\rm GR} = b/1.5$. Then, if $f(E)$ is the probability of an earthquake having an energy E, $P(E) = \int_E^\infty f(E)dE$ or $f(E) = -dP/dE$, and we have $f(E) \propto E^{-1-\beta_{\rm GR}}$. Although slightly different from the original formulation, this power-law relation is commonly called the Gutenberg–Richter law today. It was actually originally formulated by Wadati as $f(E) \propto E^{-w}$ with $w(= 1 + \beta_{\rm GR}) \simeq 5/3$.[21] The exponent $\beta_{\rm GR}$ has been found to be universal with a value close to 2/3 (Fig. 1).[1] The power-law distribution suggests that it originates from a critical branching process or from self-organized criticality.[18,22] In addition, we study the power-law distribution of the earthquakes' energy E instead of their magnitude scale M.

While the simple law provides remarkably good one-parameter fits of available data,[23] it poses theoretical problems caused by the divergence of the moments $\langle E^n \rangle$ ($n = 1, 2, 3, \ldots$), including the mean value $\langle E \rangle$.[1] To avoid this difficulty, additional parameters are introduced to cut off the heavy tail of the distribution for large E.[24,25] In fact, theoretically, the power law cannot be true without limit because the planet is finite. Observationally, however, no significant deviation from the power law has been measured, so that any energy cut-off would occur at an energy larger than that of the greatest known earthquake.[25–28] While practical studies for verifying the validity of the power law should remain important, it is not only equally important but of physical significance to investigate the theoretical consequences of having an infinite mean for this power-law distribution, which holds insofar as the Gutenberg–Richter law is obeyed. It should be remembered, however, that deviations from this law may be found in the future.

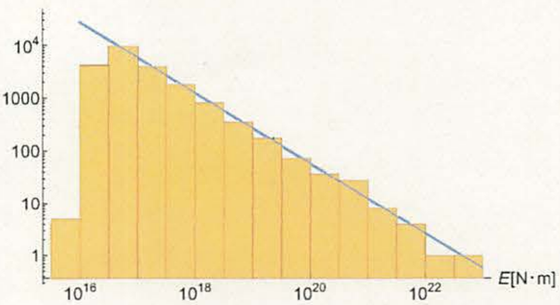

Fig. 1. The distribution of earthquakes' energy E (N m). Data are taken from the global centroid-moment-tensor catalog (2006–2017, depth < 70 km) (Refs. 29, 30, and 41). In this log–log histogram, the number of occurrence is proportional to $E^{-\beta_{\rm GR}}$, which is indicated by a solid line.

III. PROBABILITY DISTRIBUTION OF THE ACCUMULATED ENERGY

The earthquake energy E is treated as a random variable in time obeying the power-law probability distribution

$$f(E) = \frac{\beta_{\rm GR}}{a}\left(\frac{a}{E}\right)^{1+\beta_{\rm GR}}, \tag{1}$$

for $E > a$ and zero otherwise. A lower cut-off a is required for the normalization of probability ($\int_a^\infty f(E)dE = 1$). The parameters a and $\beta_{\rm GR}$ are assumed constant. We are interested in the t-dependence of the average

$$\langle E \rangle_t = \frac{1}{t}(E_1 + \cdots + E_t), \tag{2}$$

where E_k ($k = 1, 2, \ldots, t$) is a random variable. Mathematically, the average $\langle E \rangle_t$ should approach a Lévy distribution in the limit of $t \to \infty$.[2] In practice, however, the manner in which the convergence is achieved can only be investigated numerically.

For the purpose of illustration, first we present numerical results for $\langle E \rangle_t$ evaluated from $t = 100$ random numbers (E_k for $k = 1, 2, \ldots, 100$) that obey the distribution in Eq. (1). Figure 2 shows the distribution of $\langle E \rangle_t$ obtained from 100 000 numerical simulations. Since the probability distribution of $\langle E \rangle_t$ does not have a finite mean, the median provides a better indicator of the central tendency.[31] The nth quartile $E_{n/4}$ is defined as the energy separating the $n/4$ data of lowest energy from the highest $(4 - n)/4$ ($n = 1$, 2, 3). The median is the second quartile: $E_{2/4} = E_{1/2}$ (Fig. 2).

In our case, the sample size t corresponds to the time span of observation of earthquakes. As the number of time windows t increases, $E_{1/4}, E_{1/2}$, and $E_{3/4}$ rapidly become proportional to $t^{(1-\beta_{\rm GR})/\beta_{\rm GR}}$ (Fig. 3). Such a power-law asymptotical behavior implies self-similarity.[18,21] As a matter of fact, $\langle E \rangle_t$

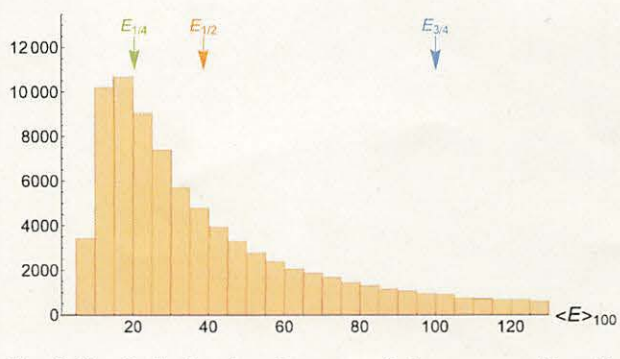

Fig. 2. The distribution, i.e., histogram, of the t-average $\langle E \rangle_t = (E_1 + \cdots + E_t)/t$ for $t = 100$. Each of the summand E_i varies randomly according to Eq. (1). (The lower cutoff is taken to be $a = 1$.) The nth quartile is indicated by an arrow labeled with $E_{n/4}$ ($n = 1$, 2, 3). The average $\langle E \rangle_t$ shows a long tail as its component E_i does. However, the former distribution does not depend on the specific details of the latter distribution. Note that the exponent $\beta_{\rm GR}$ governing the tail of the E_i distribution is of primary importance.

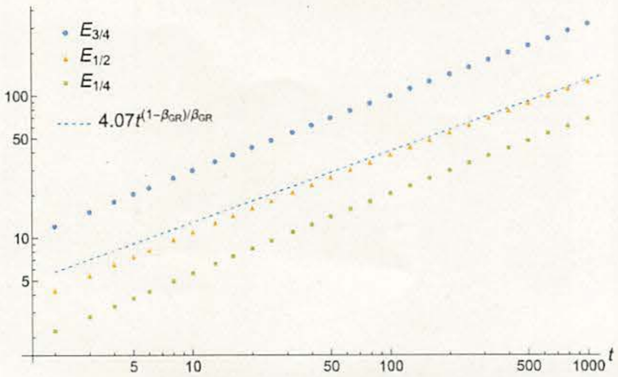

Fig. 3. Quartiles $E_{1/4}, E_{1/2}$ and $E_{3/4}$ of the $\langle E\rangle_t$ distribution are plotted against t ($\beta_{\rm GR}=2/3$ and $a=1$). The almost linear dependence in the log–log plot indicates that the quartiles vary according to a power law of the number of samples t. This behavior is contrasted with the usual behavior of distributions converging towards a constant value (mean). A dashed line shows the theoretical limit $E_{1/2} \simeq 4.07t^{(1-\beta_{\rm GR})/\beta_{\rm GR}}$ as $t \to +\infty$.

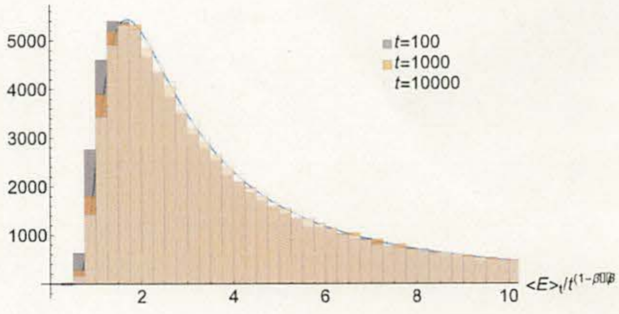

Fig. 4. Distributions, i.e., histograms, of the scaled variable $\langle E\rangle_t/t^{(1-\beta_{\rm GR})/\beta_{\rm GR}}$ for $t=100$, 1000, and 10 000 ($\beta_{\rm GR}=2/3$ and $a=1$). Each of these distributions is numerically obtained from 100 000 simulations. For comparison, the solid curve corresponds to a Lévy distribution (see the text).

scaled by $t^{(1-\beta_{\rm GR})/\beta_{\rm GR}}$ converges to a universal distribution (Fig. 4). By contrast, in the case of a distribution with a finite mean, the typical value $E_{1/2}$ and the width $E_{3/4}-E_{1/4}$ of the distribution converge, as t increases, to expected value $\bar{\mu}$ and zero, respectively, so that the average E_t converges to $\bar{\mu}$ with certainty. The certainty and constancy are both not achieved in the present case. In another case, certainty holds true while constancy is violated.[31] As mentioned before, the limit distribution of $\langle E\rangle_t$ is Lévy's distribution. In the present case, the scaled average $\langle E\rangle_t/t^{(1-\beta_{\rm GR})/\beta_{\rm GR}}$ obeys the distribution $S(x;\beta_{\rm GR},1,\gamma,0)$, so that the median of $\langle E\rangle_t$ varies as $4.07at^{(1-\beta_{\rm GR})/\beta_{\rm GR}}$ with $\beta_{\rm GR}=2/3$ in the $t\to+\infty$ (see the Appendix for the meaning of the Lévy distribution variable). This theoretical result is plotted with a dashed line in Fig. 3.

To compare the above results with empirical data, shallow earthquakes (i.e., of depth <70 km) in the global CMT (Global Centroid-Moment-Tensor Project) catalog covering the period 1977–2017 were analyzed.[29,30,41] The total number of events is 40 390. Dividing the period of 41 years (14 984 days) into consecutive t days, we evaluated the average $\langle E\rangle_t$ in Eq. (2) by regarding E_k as the energy released in day k. Figure 5 is the t-dependence of the median of this average. As can be seen, the median and therefore $\langle E\rangle_t$, increases as t grows larger. Although it may seem counterintuitive, Fig. 5 shows that the rate of energy release depends on the width of the considered time window. This is an immediate consequence of the fact that there is no upper cut-off for the power-law distribution. When the average is ill-defined, owing to the long tail of the power-law distribution, it should strongly depend on the presence or absence of outliers on the tail, i.e., extremely large events of extremely rare occurrence. It, thus, varies with the width of the observation window, because wider time frames are more likely to contain outliers.

IV. DISCUSSION

It is often stated that the power-law distribution of earthquakes indicates the absence of a characteristic scale. An upper bound scale is introduced only if the finite size of the system, the Earth's crust, is taken into consideration. However, the expression of the median of the probability distribution provides a characteristic scale that is almost independent of the upper bound scale. The median of the probability distribution in Eq. (1) is $E_{1/2}=2^{1/\beta_{\rm GR}}a \simeq 2.8a$.

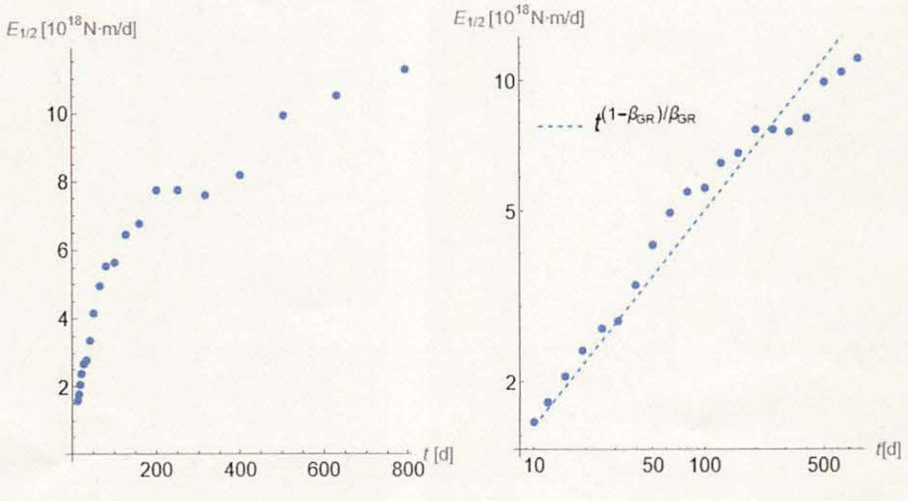

Fig. 5. The median $E_{1/2}$ of the t-day average $\langle E\rangle_t$ of earthquake energy is plotted against t (Refs. 29, 30, and 41). A steady increase in the median $E_{1/2}$, and therefore of $\langle E\rangle_t$, shows the absence of an upper cutoff in the power-law tail of the earthquake size distribution. The left panel is a normal plot. The right panel is a log–log plot of the same data, where a dashed line shows $E_{1/2} \propto t^{(1-\beta_{\rm GR})/\beta_{\rm GR}}$. It should be remarked that the lower cutoff is provided by practical incompleteness of the data catalogue (Refs. 29, 30, 32, and 41).

Similarly as for the upper limit, the attempts to find the lower limit a have not yielded reliable results. Indeed, the lower limit can be extremely small: molecular dynamics simulations[33] and laboratory experiments[34] suggest that the self-similarity extends to the molecular scale. The present results should open a new way for investigating elusive boundary scales at both ends.

More generally, the example of earthquakes' sizes shows that we should not *a priori* assume that a distribution has a finite mean (expected) value. This implicit assumption comes from our everyday experiences. In most cases, the significance of expected value is warranted by the law of large numbers, which allows us to ignore the distinction between the measured result of a given sample (the average) and the theoretical property of a probability distribution (the expected value). However, an infinite or undefined expected value, when it occurs, does not necessarily cause a practical difficulty. It does not necessarily mean that each realization has an infinite or indefinite result. In fact, there are abundant interesting statistics in which the mean value is not a valid indicator of the typical outcome.[1,31] In this respect, it should be remembered that the power-law distribution, also known as the Pareto distribution in economy and sociology, was originally discovered in studying the distribution of wealth in a society, i.e., a large portion of wealth is held by a small fraction of the population.[35] Power-law distributions occur in a diverse range of natural phenomena, including the sizes of earthquakes, moon craters,[36] solar flares,[37] and rainfall depths.[38] The recent surge of interest in scale-free networks shed a new light on the power-law distributions in the number of citations of scientific papers,[39] the number of hits on web pages,[40] and other network-related phenomena.

V. CONCLUSION

While we took earthquakes as an illustrative example, similar applications may be found in many other power-law phenomena. For distributions with an undefined mean, the median provides a well-defined measure of a typical outcome. Accordingly, it is a good practice to monitor whether and how median varies as the sample size grows. In so doing, we find pedagogical value especially in taking a critical view on a non-trivial assumption that the average of sample data should approach to a definite value as sample size increases indefinitely. The present study shows that the violation of the law of large number does not mean the absence of any rule, while its empirical verification may require to collect a large set of data. It is generally a formidable task to show unequivocally that the underlying distribution obeys a power law, because it requires a significant amount of data ranging over several orders of magnitude. Still, it is straightforward just to find an indication of the absence of a well-defined mean. Thus, it is instructive to remind that the long tail of the underlying distribution is predicted from the manner in which the median varies.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available within the article and its supplementary material.[41]

APPENDIX: LÉVY DISTRIBUTION

The Lévy distribution, or also known as the stable distribution, $S(x;\alpha,\beta,\gamma,\mu)$ is defined by

$$S(x;\alpha,\beta,\gamma,\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi(t)e^{-ixt}dx, \tag{A1}$$

where

$$\phi(t) = \exp\left(i\mu t - \gamma^{\alpha}|t|^{\alpha}(1 - i\beta\,\mathrm{sgn}(t)w(\alpha,t))\right), \tag{A2}$$

$$w(\alpha,t) = \tan(\pi\alpha/2), \tag{A3}$$

and $w(\alpha,t) = \tan(\pi\alpha/2)$ for $\alpha \neq 1$ and $w(\alpha,t) = -2/\pi \log|\gamma t|$ for $\alpha = 1$.[3] The parameters α and β are real constants satisfying $0 < \alpha < 2$ and $-1 \leq \beta \leq 1$, while $\gamma > 0$ is a scale parameter. In the case of $\alpha = 2$ and $\beta = 0$, the distribution reduces to a Gaussian distribution with variance $2\gamma^2$ and mean μ.

The generalized central limit theorem states that the superposition $(\sum_{i=1}^{n} X_i)$ of independent, identically distributed random variables (X_i) converges to $S(\alpha,\beta,\gamma,0)$. According to a theorem by Gnedenko and Kolmogorov,[3] for a random variable X_i obeying a probability distribution,

$$f(x) \simeq \begin{cases} c_+ x^{-(\alpha+1)} & \text{for} \quad x \to \infty, \\ c_-|x|^{-(\alpha+1)} & \text{for} \quad x \to -\infty, \end{cases} \tag{A4}$$

where c_+ and c_- are positive constants, we obtain

$$Y_n = \frac{\sum_{i=1}^{n} X_i}{n^{1/\alpha}} = \langle X\rangle_n / n^{(1-\alpha)/\alpha} \to S(\alpha,\beta,\gamma,0), \tag{A5}$$

for $0 < \alpha < 1$, where the expected value $\langle X\rangle_n = \sum_{i=1}^{n} X_i/n$ diverges as $n \to \infty$. The parameters β and γ are given by

$$\beta = \frac{c_- - c_-}{c_+ - c_-}, \tag{A6}$$

and

$$\gamma = \left(\frac{\pi(c_+ + c_-)}{2\alpha \sin(\pi\alpha/2)\Gamma(\alpha)}\right)^{1/\alpha}, \tag{A7}$$

where $\Gamma(z) = \int_0^{\infty} x^{z-1}e^{-x}dx$ is the gamma function.[3]

In the present case of Eq. (1), we have $c_+ = \beta_{\mathrm{GR}} a^{\beta_{\mathrm{GR}}}$, $\alpha = \beta_{\mathrm{GR}} = 2/3$ and $c_- = 0$, so that $\beta = 1$, $\gamma = (\pi/2/\sin(\pi\beta_{\mathrm{GR}}/2)/\Gamma(\beta_{\mathrm{GR}}))^{1/\beta_{\mathrm{GR}}} a$ and $\langle X\rangle_n / n^{(1-\beta_{\mathrm{GR}})/\beta_{\mathrm{GR}}} \sim S(\beta_{\mathrm{GR}}, 1, \gamma, 0)$, which has a median of $4.074\,32a$.

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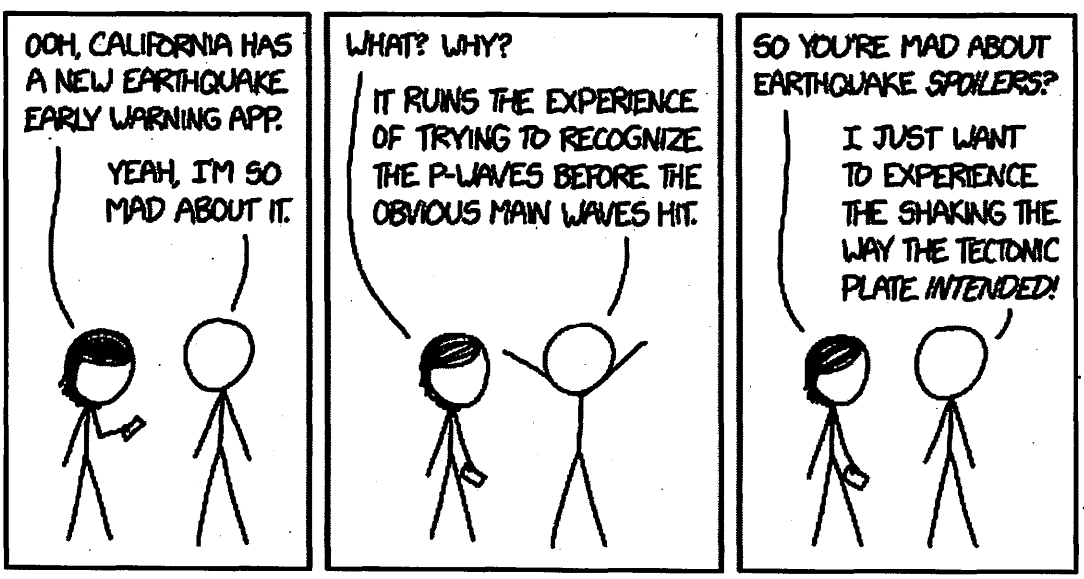
[41]See supplementary material at https://www.scitation.org/doi/suppl/10.1119/10.0010261 for the data for time and energy of earthquakes used in this study.



I was fired by the National Weather Service five minutes after they hired me for going into their code base and renaming all the tornado warnings to "tornado spoiler alerts." (Source: https://xkcd.com/2219/)