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Abstract

The purpose of this study is to develop students' competencies to solve social problems using mathematics by means of mathematics education in schools.

First, we defined the necessary competencies to solve social problems in the real world using mathematics as "the competencies to connect mathematics with society." It contains four domains: sense for quantities and shapes in society, competencies to solve social problems mathematically, competencies to communicate using mathematics in society, and competencies to use approximation. Each domain has two to four components. In addition, we defined awareness towards and the attitudes that support *the competencies to connect mathematics with society* as "awareness and attitudes for connecting mathematics with society."

Second, we developed a test that included items that assess each component of *the competencies to connect mathematics with society* and surveyed students from the 4th grade to the 11th grade in Japan to see how they measured in each of the components. The results showed that, for example, many students from the 7th grade to the 11th grade could not: make assumptions, create hypotheses, revise assumptions and hypotheses, interpret and communicate phenomena from mathematical expressions, interpret a sentence from everyday life that used mathematics, and use approximation. In addition, we saw little evidence of students applying mathematics to everyday life.

Third, we studied how classes in school should be to develop *the competencies to connect mathematics with society*. We practiced our findings in actual classrooms and verified the effectiveness.

Keywords: *competencies, connecting, real world, society, mathematical modelling*

Introduction

Our research group has been conducting practical research over the last 15 years with aims to create mathematics lessons that teach mathematics that is connected with our society. This is because Japanese students, compared to students in other countries, do not associate mathematics with their personal lives and the real world. They also think mathematics is not interesting (NIER, 2005). Although mathematics will be absolutely imperative in our society in the future, Japanese students have little to regard for it. Efforts to improve on this area have been made but in many cases issues such as teachers not understanding the value of real world problems in mathematics class and teachers having the misconception that the solution to this problem is to simply increase the number of real world problems covered in class have risen.

The purpose of this study is to develop students' competencies to solve social problems in the real world using mathematics by means of mathematics education in schools. Also, the term *society* used in this study is interchangeable with terms such as *real world, everyday life*, and *real life*.

The Competencies to Connect Mathematics with Society

The competencies to connect mathematics with society (CCMS) is based on "mathematical activity" (Shimada, 1977) and is the materialized and structured form of "the competencies and senses needed when addressing phenomena or problems in society" (Nagasaki, 2001a, 2001b)

Table 1 *Competencies to Connect Mathematics with Society*

A. Sense for quantities and shapes in society		
A01. Sense of length	A02. Sense of area	A03. Sense of volume
A04. Sense of weight	A05. Sense of angles	A06. Sense of time
A07. Sense of velocity	A08. Sense of shape	
B. Competencies to solve social problems mathematically		
B1. Translating social phenomena to mathematical form		
B11. Making assumptions	B12. Isolating variables	
B13. Controlling variables	B14. Forming hypotheses	
B2. Processing mathematically		
B21. Representing using tables, mathematical expressions, graphs, geometrical figures, etc.		
B22. Performing operations		
B3. Validating results in reality		
B31. Forecasting and speculating	B32. Revising	
C. Competencies to communicate using mathematics in society		
C01. Interpreting and communicating phenomena from mathematical expressions		
C02. Interpreting sentences from everyday life that use mathematics		
D. Competencies to use approximation		
D01. Formulating using approximation		
D02. Interpreting using approximation		

Competencies to solve social problems mathematically (B) involves translating a social problem into mathematical form, using mathematics within a model, and going through the process of validating and verifying the results within a real world setting.

Competencies to communicate using mathematics in society (C) is applied to the entire process of mathematical modeling and includes communicating the validity of results to others as well as critically understanding the ideas of others. In addition, it is the capacity to work within a group to solve problems.

Sense for quantities and shapes in society (A) and *competencies to use approximation* (D) are *competencies* that are applied with (B) and (C). The former is the sense related to the intuitive understanding of the meaning of numbers and figures in society. The latter is competencies related to approximating calculations based on objectives and understanding whether calculated results are acceptable in society.

In addition, awareness and positive attitudes are necessary when dealing with social problems in mathematics, which further fosters developed awareness and attitudes.

Table 2 *Awareness and Attitudes for Connecting Mathematics with Society*

E01. Awareness towards mathematics
E02. Awareness towards expressive methods in mathematics
E03. Awareness towards mathematical processes
E04. Awareness towards cooperative learning in mathematics
E05. Applicative attitudes in mathematics
E06. Developmental attitudes in mathematics

Researching the Competencies to Connect Mathematics with Society

We developed a test that included questions that assess each component of the CCMS and surveyed students from the 4th grade to the 11th grade in Japan to see how they measured in each of the components.

The test items were created for each component of the CCMS (table 1), assessed in a preliminary study, and selected.

Table 3 shows the set structure and table 4 shows the number of respondents. Appendix 1 includes sample test items. In addition, for each component of the CCMS, we set 60% as the passing level to signify that the group had acquired the competencies. Table 5 shows the average percentage of items answered correctly per grade.

The results showed that, for example, many students from the 7th grade to the 11th grade could not: make assumptions, create hypotheses, revise assumptions and hypotheses, interpret or communicate phenomena from mathematical expressions, interpret a sentence from everyday life that used mathematics, and use approximation.

Moreover, table 6 shows the percentage of positive answers for items regarding the *awareness and attitudes for connecting mathematics with society* (E). The percentage of positive answers for items regarding applicative attitudes in mathematics in everyday life (E05) was less than 50% for grades 5 and above.

Table 3 *Set Structure*

Set	Items	Awareness/attitude Items
I	20	14
II	24	14
III, IV	24	14
V, VI	35	14

Table 4 *Number of Respondents per Grade*

Grade	Set	Schools	Classes	Students
4	I	9	19	568
5	II	9	18	608
6	III	9	9	270
	IV	9	9	295
7	V	8	9	325
	VI	8	8	268
8	V	8	8	261
	VI	9	9	315
9	V	8	8	280
	VI	8	8	282
10	V	8	10	365
	VI	9	12	420
11	V	7	7	259
	VI	7	7	258

Developing the Competencies to Connect Mathematics with Society

We studied how classes in school should be to develop the CCMS. We do not believe it can be taught simply by covering real world problems in class. As seen from the results in section 3, there were few students who had acquired all of the competencies from all components and many students who could not solve the given social problems on their own. Therefore, you need to think about the following points when planning your lessons.

1) Have a clear idea as to which component of the CCMS you want to foster and a clear goal as to what level you want to achieve.
2) Think about the guiding questions and supporting materials (questions, experiments, observations, explorations, teaching aids, materials, etc.) needed to achieve these goals. This includes methods such as working in small groups or as a class and communicating mathematically to solve the problem.

Then, we conducted 50 case studies at elementary, junior and senior high schools and qualitatively demonstrated that the CCMS could be improved for students. Appendix 2 includes a sample lesson in a elementary school (Makino, 2001).

In addition, as part of the lessons to develop the CCMS, Nishimura, K. (2001) devised lessons that focused on creating new mathematical concepts that were yet to be learned rather than applying concepts already learned when processing problems

Table 5 *Average Percentage of items Answered Correctly per Grade for each Domain of the Competencies to Connect Mathematics with Society*

	Average Percentage of Questions Answered Correctly per Grade (%)							
Grade	4	5	6	7	8	9	10	11
A. Sense for quantities and shapes in society	**55.9**	**43.1**	**64.5**	**71.9**	**70.2**	**77.8**	**74.8**	**80.1**
B. Competencies to solve social problems mathematically	**55.9**	**45.4**	**54.6**	**52.6**	**55.0**	**59.9**	**60.1**	**66.6**
B1. Translating social phenomena to mathematical form	37.9	57.5	50.6	54.9	53.5	57.7	58.4	65.6
B11. Making assumptions	25.9	45.2	36.4	46.6	43.8	49.1	48.6	59.9
B12. Isolating variables	46.0	72.5	73.1	80.4	80.0	80.1	80.8	83.0
B13. Controlling variables	45.8	52.0	50.0	55.5	56.8	59.6	62.1	67.0
B14. Forming hypotheses	---	---	---	23.5	22.9	33.0	34.0	43.8
B2. Processing mathematically	78.5	44.0	64.8	56.0	60.7	69.1	68.6	74.3
B21. Representing using tables, mathematical expressions, graphs, geometrical figures, etc.	64.6*	42.4	40.3	42.2	44.9	56.9	55.7	63.4
B22. Performing operations	83.1	47.2*	79.5	85.6	90.2	93.6	93.1	96.3
B3. Validating results in reality	---	31.2	44.5	46.0	50.3	51.9	52.6	59.1
B31. Forecasting and speculating	---	34.6	49.8	58.3	61.7	61.2	62.2	69.1
B32. Revising	---	20.9	22.2	26.6	30.5	36.2	34.3	42.5
C. Competencies to communicate using mathematics in society	**---**	**32.7**	**49.3**	**47.1**	**52.4**	**58.2**	**60.3**	**62.1**
C01. Interpreting and communicating phenomena from mathematical expressions	---	39.1	47.7	45.9	51.1	57.5	59.9	63.3
C02. Interpreting sentences from everyday life that use mathematics	---	19.9*	53.3	52.0	57.5	61.2	61.8	57.4
D. Competencies to use approximation	**42.9**	**48.5**	**52.6**	**46.7**	**59.0**	**57.3**	**56.7**	**56.1**
D01. Formulating using approximation	66.2	70.2	76.9	76.1	87.0	88.3	88.1	91.1
D02. Interpreting using approximation	19.5	26.8	25.9	22.5	25.3	26.1	20.5	21.2

*Extreme value compared to other grades because only one question was posed for the component.

Table 6 *Average Percentage of Positive Answers for Questions regarding the Awareness and Attitudes for Connecting Mathematics with Society per Grade (%)*

Grade	4	5	6	7	8	9	10	11
E01. Awareness towards mathematics	78.2	64.0	65.4	53.8	50.3	47.4	45.1	42.7
E02. Awareness towards expressive methods in mathematics	84.9	77.4	81.9	69.5	68.7	69.4	63.9	69.3
E03. Awareness towards mathematical processes	53.4	40.2	41.2	35.1	30.9	38.3	27.0	32.4
E04. Awareness towards cooperative learning in mathematics	62.6	48.4	50.8	41.3	28.2	34.2	19.6	21.8
E05. Applicative attitudes in mathematics	54.5	43.7	42.5	29.0	23.8	27.4	19.6	22.0
E06. Developmental attitudes in mathematics	45.1	37.9	36.5	25.4	24.2	28.8	18.3	22.7

mathematically (Type 1). Shimada, & Nishimura. (2006) devised lessons that put emphasis on developing the competencies *making assumptions* (B11) and *revising*

(B32), which were competencies we found many students did not have, as outlined in section 3 (Type 2, 3). We then conducted lessons and proved their effectiveness.

Type1: Lessons where, during the process of working out the solution, new mathematical concepts that have yet to be learned are created and used to work out the solution.

Type2: Lessons with emphasis placed on making assumptions and transforming social problems into mathematical form.

Type3: Lessons with emphasis placed on thinking about assumptions that are not originally apparent and verifying hypotheses made after solving problems already transformed into mathematical form by others.

Furthermore, Shimada (2009) expanded on these findings to focus on the relationship between decision making and the CCMS, and is researching the structure of lessons that respect students' "values." The following lesson for the 6th grade is one such example.

Our class will be going to Disneyland on our next field trip. There are 3 rides:
Space Mountain, Big Thunder Mountain, Micro Adventure
As a group, decide the order your group will ride them.

Students translate this into mathematical form with personal "values," such as taking the shortest route, minimizing time losses, and having fun, in mind. They think about the order of the rides by using mathematics. When deciding the order of the rides within the group, the students learn about the diverse values of others and experience conflict between each other.

As with this lesson, decisions are often made for social problems by making assumptions based on values and forming hypotheses. There is also a need to define what values each thought is based on when assessing the decision made. In other words, the CCMS plays an important role when making decisions based on mathematical ground. At the same time, it is just as important to take this concept of values into consideration when developing the CCMS.

Furthermore, Nishimura (2010) focused on "the interrogative cycle" (Wild & Pfannkuch, 1999). Wild & Pfannkuch centered on statistical thinking but Nishimura believed "the interrogative cycle" could be applied to solving social problems as well. He then defined "the critical cycle" as a dimension to consistently think critically and reflectively throughout the entire problem-solving process, where phases such as "generate," "seek," "interpret," "criticize," and "judge" function continuously. Next, he analyzed the problem-solving processes of students and identified how "the critical cycle" works. It involves backtracking through the phases of the mathematical modelling process, advancing to the next modelling cycle, and improving the students' preexisting competencies in the components of the CCMS. Consequently, teachers can develop the CCMS further by preparing guiding questions and supporting materials that work on these functions. Take the Disneyland lesson for example. In order to elicit "criticizing", have two groups present different ways to go around the park and have the class think about which is better. By doing so, the assumptions that need to be made and the variables that need to be considered become exposed, which in turn raises the students' competencies to "make assumptions" and "isolate variables."

Conclusion

In this study, we structured the competencies to solve social problems using mathematics and defined them as the CCMS. Then, we clarified that students had have acquired the aspects for some components while they had not acquired the aspects for others. In order to develop the CCMS, there is a need focus on the specific component, set goals that match the students' preexisting competencies in these components, and conduct lessons with appropriate guiding questions and supporting materials in mind. It is effective to think of these guiding questions and supporting materials from "the critical cycle" perspective. As for the lesson composition, we outlined 3 types of lessons aside from lessons where social problems are presented and students use mathematical concepts already learned. In addition, we stated the importance of considering the relationship between the CCMS and students' "values."

As it has passed ten years since the survey was conducted, we will investigate the present state of students on the CCMS and compare the result with the former one.

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Appendix1

Sample Items and Percentage of Items Answered Correctly

Grade	D
4	53.2%
5	---
6	63.7%
7	72.3%
8	71.3%
9	80.0%
10	83.8%
11	83.8%

Example 1) Sense of length (A01)

Which of the following, A to E, has a length (height) closest to 10 meters?

A. Height of a blackboard in a classroom
B. Height of a slide at a park
C. Length of a car
D. Height of a three-storey school building
E. Length of a platform at Tokyo station

Example 2) Making assumptions (B11)

Ayako runs 300 meters per second for 20 minutes. She thought she could find the distance she ran by calculating 20 x 300. In order for Ayako's calculation to work, what do you need to consider? Choose the best answer from A to E below.

A. Ayako ran without resting
B. Ayako ran at the same speed
C. Ayako did not ride a bicycle
D. Ayako's strides were all the same length
E. Ayako ran without taking any detours

Grade	B
4	43.7%
5	55.3%
6	59.3%
7	63.4%
8	66.3%
9	67.9%
10	65.5%
11	65.3%

Example 3) Revising (B32)

Toru is thinking of rolling a spinning top, like the one shown in figure A, down a channel made by two angled rulers lined together, as shown in figure B.

When a ball is rolled down this slope, the relationship between the time (x sec) and distance rolled (y cm) was as follows.

x	0	1	2	3	4	5
y	0	2	8	18	32	50

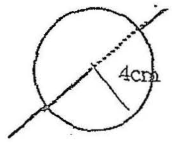


Figure A

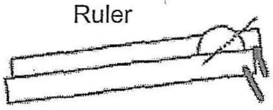


Figure B

The relationship expressed as a formula was $y=2x^2$ and so Toru thought it would be the same for the top. However, wher actually rolling the top down the slope, things looked different Toru decided to conduct a proper experiment. The distance rolled, y cm, after x seconds was measured in the experimen and the results were as follows.

x	0	1	2	3	4	5
y	0	1	3	5	7	9

To calculate the distance rolled by an actual top, what formula should we consider? Choose the best answer from A to E below.

A. The distance rolled becomes longer and approaches $y=2x^2$
B. The distance rolled becomes constant and approaches $y=2x-1$
C. The distance rolled becomes constant and approaches $y=x$
D. The speed changes part way and then becomes constant the rest of the way while the distance approaches $y=2x$
E. It is irregular because it is not a ball

Grade	B
4	---
5	---
6	---
7	27.1%
8	34.1%
9	37.1%
10	36.7%
11	40.9%

Example 4) Interpreting and communicating phenomena from math expressions (C01)

The table below shows the world's population and the population increase for each year. What is this table trying to tell us? Choose the best answer from A to E below.

Year	World Population (million people)	Yearly population increase (million people)
1990	5278	87
1991	5361	83
1992	5444	83
1993	5526	82
1994	5606	81
1995	5687	81
1996	5766	79

Grade	D
4	---
5	53.9%
6	65.4%
7	62.3%
8	72.4%
9	73.0%
10	75.7%
11	79.5%

A. The world population and the population increase goes up
B. Even if the world population increases, the population increase does not change

C. The world population and the population increase does not change very much
D. Even if the population increase goes down, the world population continues to increase
E. Because the population increase is going down, the world population will decrease.

Appendix2

Sample lesson "Investigating Traffic Lights" (Makino,2001)

Problem outline: Consider whether the amount of time for a green light, "green time," for a traffic light in our community is long enough for wheelchair users and elderly people to cross safely.

Goals related to the CCMS: Making assumptions for walking speeds when crossing a crosswalk and the relationship between the street width and amount of "green time". (B11)

Isolating variables such as types of pedestrians, amount of traffic, time of day, time needed to cross the street, walking speeds, street width, etc. (B12)

Controlling these variables and focusing on the street width and the time needed to cross the street. (B13)

Forming hypotheses for the relationship between the amount of "green time" and the pedestrian's walking speed. (B14)

Graphing observed results and recognizing tendencies. (B21)

Approximating the length of the crosswalk and "green time". (D02)

Guiding questions and supporting materials: Present information on traffic light types and their roles.

Ask "what needs to be considered when determining the amount of time for the red, yellow, and green lights on a traffic light?" to bring out variables and discuss as a class.

In small groups, plan an investigation on "green time," have them collect actual data, and present their findings.

Have students summarize their findings and thoughts in a notebook, and exchange ideas with each other.

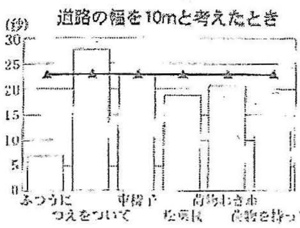
Actual lesson: The class was able to isolate crosswalk length and walking speed as variables. (B12)

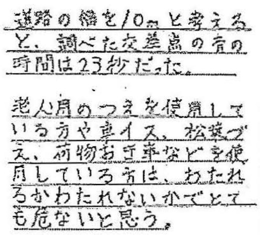
Measuring the length of a crosswalk is dangerous and so the students counted paces. They measured the distance of 10 paces 5 times and used the average as the distance of one pace. (B21, D02)

To determine the speed, students measured the time it took to advance 10 meters across the schoolyard using crutches and wheelchairs. (B13)

Students thought about whether actual "green times" were safe enough for people to cross. (B31, C01)

Example of student's note





I thought of the street width as being 10 meters and when I measured the green time, it was 23 seconds.
I think it is dangerous for people such as those who use canes made for elderly people, people in wheel chairs, people using crutches, and people who push carts for carrying baggage as they may or may not be able to cross the street.