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# Unitary meson-exchange $\pi$NN models: NN and $\pi$d elastic scattering

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Within a unitary $\pi$NN theory a set of coupled integral equations defined in the NN$\oplus$N$\Delta$$\oplus$$\pi$NN subspace has been derived for examining the extent to which the NN and $\pi$d elastic scattering data can be *both* described by a class of meson-exchange $\pi$NN models. The considered models are constructued by using a previously developed subtraction procedure to extend the Paris, Bonn, Argonne-$V$14, and Reid NN potentials to include the coupling to the $\pi$NN production channel. The pion production through the $\Delta$ excitation is described by a $\pi N \leftrightarrow \Delta$ vertex and a $V_{NN\leftrightarrow N\Delta}$ transition potential, while the production through nonresonant $\pi$N interactions is described by a transition operator $F_{\pi NN\leftrightarrow NN}$. It is found that none of the constructed models can accurately reproduce all NN and $\pi$d elastic scattering data, in particularly the data of polarization observables. Our results indicate that either the conventional phenomenological parametrizations of baryon-baryon interaction at short distances are not valid in the intermediate energy region, or some genuine quark dynamics has already been revealed in the $\pi$NN data.

## I. INTRODUCTION

The starting point of conventional nuclear theory is a nucleon-nucleon (NN) potential which accurately describes the NN elastic scattering below the pion production threshold. An equally well-defined theory which can also describe nuclear reactions induced by intermediate energy probes must start with a $\pi$NN model Hamiltonian which can describe the following elementary processes (called the $\pi$NN processes):

$$\pi N \rightarrow \pi N \; (E_{\rm lab} \leq 300 \text{ MeV}) , \tag{1.1a}$$

$$NN \rightarrow NN \; (E_{\rm lab} \leq 1000 \text{ MeV}) , \tag{1.1b}$$

$$\rightarrow NN\pi , \tag{1.1c}$$

$$\pi d \rightarrow \pi d \; (E_{\rm lab} \leq 300 \text{ MeV}) , \tag{1.1d}$$

$$\rightarrow \pi NN , \tag{1.1e}$$

$$\rightarrow NN . \tag{1.1f}$$

In recent years there have been many attempts[1-11] to develop such a $\pi$NN theory. Each formulation of the problem has its own sophistication in handling the complications due to the presence of a three-body $\pi$NN reaction channel, and in defining the basic dynamics. But a realistic way of constructing a $\pi$NN model according to any one of the existing formulations should proceed as follows. First, it is necessary to carefully examine the extent to which all of the $\pi$NN processes listed above can be described by the well-studied meson-exchange mechanisms. The simplest procedure to carry out this study is to construct the $\pi$NN theory by extending the conventional meson-exchange model of nuclear force to include the delta ($\Delta$) and pion degrees of freedom. If this approach fails to bring complete success, one then takes the next step to consider possible genuine quark dynamics. For example, the dibaryonic excitation of a six-quark system, formulated in Ref. 9, as due to a $D \leftrightarrow BB$ vertex interaction, could be indispensable in understanding various strong energy dependencies of NN and $\pi$d cross sections.

From earlier coupled-channel NN calculations,[12-14] there are some indications that the meson-exchange model cannot provide a detailed description of all of the $\pi$NN processes. Specifically, the calculations of Ref. 13 had shown difficulties in describing various NN polarization observables, although a reasonably good fit to the Arndt phase shifts[15] had been obtained. However, these earlier calculations do not account for all of the pionic mechanisms within a unitary formulation of the $\pi$NN problem. They neglect the effects due to the coupling to the $\pi$d channel and the nonresonant $\pi$N interactions. Furthermore, the model dependence of the results has not been systematically studied. The question concerning the validity of the meson-exchange model in describing all $\pi$NN processes is therefore not fully answered with satisfaction. In this paper we report the progress we have made in trying to establish a quantitative answer to this important question. This is, of course, the basis for taking any step to explore the quark physics contained in the very rich experimental information on $\pi$NN reactions.

To have a proper assessment of this work, it is necessary to clearly define the scope of the present investigation. We start with the $\pi$NN formulation of Ref. 9, neglecting the dibaryonic excitation part of the proposed model Hamiltonian. In handling the crucial $\pi$NN production channel, we focus our attention on investigating the well-established one-pion-exchange model of the pion production mechanism. Accordingly, we neglect the excitation of the $\Delta\Delta$ state since, via $\Delta\rightarrow\pi$N decay, it can lead to intermediate states containing $2\pi$ or $\pi\Delta$ components which are known[13] to be unimportant in describing NN scattering below 1 GeV. With these simplifications, the $\pi$NN Hamiltonian takes the following form:

$$H = H_0 + H_{\text{int}} , \tag{1.2a}$$

$$H_{\text{int}} = H_1' + H_2' , \tag{1.2b}$$

where

$$H_1' = \sum_{i=1}^{2} [h_{\pi\text{N},\Delta}(i) + h_{\Delta,\pi\text{N}}(i)] + \tfrac{1}{2}\sum_{i\neq j} [V_{\text{NN},\text{NN}}(i,j) + V_{\text{NN},\text{N}\Delta}(i,j) + V_{\text{N}\Delta,\text{NN}}(i,j)] , \tag{1.2c}$$

$$H_2' = \sum_{i=1}^{2} v_{\pi\text{N}}(i) + \tfrac{1}{2}\sum_{i\neq j} [F_{\pi\text{NN},\text{NN}}(i,j) + F_{\text{NN},\pi\text{NN}}(i,j)] . \tag{1.2d}$$

In Fig. 1, we illustrate each mechanism of Eq. (1.2). $H_0$ is the free energy operator. $V_{\text{NN},\text{NN}}$ is a NN two-body potential. The $\Delta$ excitation and its associated pion production are described by a transition interaction $V_{\text{NN}\leftrightarrow\text{N}\Delta}$ and a vertex interaction $h_{\pi\text{N}\leftrightarrow\Delta}$. This is illustrated in Fig. 2(a) for a one-pion-exchange model of $V_{\text{NN}\leftrightarrow\text{N}\Delta}$. Note that the vertex interaction $h_{\pi\text{N}\leftrightarrow\Delta}$ generates a one-pion exchange N$\Delta\leftrightarrow\Delta$N interaction which is known to be an important ingredient in describing $\pi$d scattering. The N$\Delta\leftrightarrow$N$\Delta$ interaction induced by a $\Delta\leftrightarrow\pi\Delta$ vertex contains a $\pi$N$\Delta$ intermediate state and hence is also neglected, consistent with the neglect of the $\Delta\Delta$ contribution.

The term $H_2'$ [Eq. (1.2d)] describes the effects due to nonresonant $\pi$N interactions. $v_{\pi\text{N}}$ is a two-body $\pi$N potential and $F_{\text{NN}\leftrightarrow\pi\text{NN}}$ is a transition operator. We will consider a one-pion-exchange model of $F_{\pi\text{NN}\leftrightarrow\text{NN}}$, as illustrated in Fig. 2(b). The reasons for using this approach instead of explicitly including a $\pi$N$\leftrightarrow$N$'$ vertex to account for the $\pi$N nonresonant effects have been discussed in Ref. 9. Basically, it originates from the recognition of the well-known fact that a theory with a $\pi$N$\leftrightarrow$N$'$ vertex intrinsically contains a "many-body" problem in defining the *physical* nucleon, and hence the resulting unitary formulation of the $\pi$NN scattering processes listed in Eq. (1.1) is much more complicated than the straightforward scattering theory developed in Ref. 9. We do not claim that our formulation is better than others. It is just a different way of treating the pionic effects in developing a *tractable* unitary $\pi$NN model which can be used directly in *many-nucleon calculations.*

Starting from the model Hamiltonian defined by Eq. (1.2), it is straightforward to derive from the general formulation of Ref. 9 a set of coupled integral equations for the study of NN and $\pi$d elastic scattering. In this work

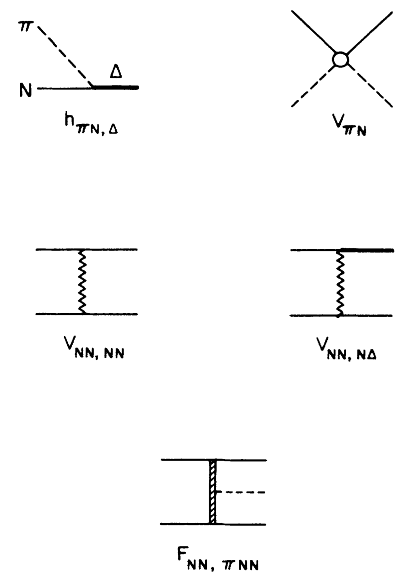


FIG. 1. Graphical representation of the model Hamiltonian Eq. (1.2).

we make a simplification that the nonresonant interaction $H_2'$ is treated perturbatively. This approximation, which greatly reduces the numerical complexities involved in the problem, is justified in the $\Delta$-excitation energy region in which extensive experimental data are available for a careful test of the $\pi$NN theory. In Sec. II we will explicitly present the resulting scattering equations and specify the dynamical input to our calculations.

In Sec. III we discuss our numerical strategy in calculating the NN and $\pi$d elastic scattering amplitudes. It will be seen that the full calculation can be decomposed into two parts. The first part is to solve a Faddeev-Alt-Grass-Sandhas (AGS) equation for generating the multiple scattering part of the $\pi$d amplitude and a N$\Delta\rightarrow$N$\Delta$ amplitude which is needed in constructing one of the four driving terms of a two-body Lippmann-Schwinger equation. The solution of the Lippmann-Schwinger equation will then be used to generate the NN elastic scattering amplitude and a new N$\Delta\leftrightarrow$N$\Delta$ transition amplitude, which now contains the NN scattering information, to calculate the effect of pion absorption in $\pi$d scattering.

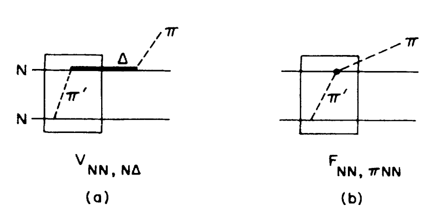


FIG. 2. Graphical representations of the one-pion-exchange mechanisms of the transition operators $V_{\text{NN},\text{N}\Delta}$ and $F_{\pi\text{NN},\text{NN}}$.

In the first part of Sec. IV, we present our NN results. We will analyze in detail the subtraction procedure utilized in defining $V_{\mathrm{NN,NN}}$ of Eq. (1.2). It will be clear that the main model dependences of our $\pi$NN study are in the parametrizations of the transition interactions $V_{\mathrm{NN}\leftrightarrow\mathrm{N}\Delta}$ and $F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}$, and in the choice of the starting low energy potential. We then explore whether the fit to NN data can be achieved by making use of these freedoms of the model. The NN potentials used in this search are the Paris,[16] Bonn (1975),[17] Argonne $V14$,[18] and Reid[19] potentials.

The second part of Sec. IV presents our $\pi$d results. We discuss the pion absorption effect in $\pi$d scattering, and its close relationship with the corresponding NN prediction in a unitary approach. We also compare our results with data in order to further establish the validity of the meson-exchange $\pi$NN model.

Section V is devoted to discussing possible sources of the problems encountered in our study and the necessary further improvements of the model.

## II. THE NN AND $\pi$d SCATTERING EQUATIONS

The derivation of NN and $\pi$d scattering equations from the model Hamiltonian Eq. (1.2) can be carried out straightforwardly following the general formulation in Ref. 9. To simplify the calculation, the nonresonant interaction $H_2'$ is treated in perturbation. Then the equation for calculating the NN amplitude can be cast into a one-channel Lippmann-Schwinger form,

$$T_{\mathrm{NN,NN}}(E)=\tilde{V}_{\mathrm{NN,NN}}(E)+\tilde{V}_{\mathrm{NN,NN}}(E)\frac{P_{\mathrm{NN}}}{E-H_0+i\epsilon}T_{\mathrm{NN,NN}}(E)\;, \tag{2.1}$$

where

$$\tilde{V}_{\mathrm{NN,NN}}(E)=V_{\mathrm{NN,NN}}+U^{(1)}_{\mathrm{NN,NN}}(E)+U^{(2)}_{\mathrm{NN,NN}}(E)+U^{(3)}_{\mathrm{NN,NN}}(E)\;, \tag{2.2}$$

with

$$U^{(1)}_{\mathrm{NN,NN}}(E)=V_{\mathrm{NN,N}\Delta}\frac{P_{\mathrm{N}\Delta}}{E-H_0-\Sigma_\Delta(E)}V_{\mathrm{N}\Delta,\mathrm{NN}}\;, \tag{2.3a}$$

$$U^{(2)}_{\mathrm{NN,NN}}(E)=V_{\mathrm{NN,N}\Delta}\frac{P_{\mathrm{N}\Delta}}{E-H_0-\Sigma_\Delta(E)}X_{\mathrm{N}\Delta,\mathrm{N}\Delta}(E)\times\frac{P_{\mathrm{N}\Delta}}{E-H_0-\Sigma_\Delta(E)}V_{\mathrm{N}\Delta,\mathrm{NN}}\;, \tag{2.3b}$$

$$U^{(3)}_{\mathrm{NN,NN}}(E)=F_{\mathrm{NN},\pi\mathrm{NN}}\sum_{i=1}^{2}\frac{P_{\pi\mathrm{NN}}}{E-H_0-v_{\pi\mathrm{N}}(i)+i\epsilon}\times F_{\pi\mathrm{NN,NN}}\;. \tag{2.3c}$$

Here $P_{\mathrm{NN}}$, $P_{\mathrm{N}\Delta}$, and $P_{\pi\mathrm{NN}}$ are, respectively, the projection operators for the NN, N$\Delta$, and $\pi$NN intermediate states. The $\Delta$ excitation is contained in $U^{(1)}_{\mathrm{NN,NN}}$ and $U^{(2)}_{\mathrm{NN,NN}}$. The $\Delta$ self-energy is defined by

$$\Sigma_\Delta(E)=\sum_{i=1}^{2}h_{\Delta,\pi\mathrm{N}}(i)\frac{P_{\pi\mathrm{NN}}}{E-H_0+i\epsilon}h_{\pi\mathrm{N},\Delta}(i)\;. \tag{2.4}$$

The transition potential $V_{\mathrm{NN}\leftrightarrow\mathrm{N}\Delta}$ is parametrized as a static one-pion exchange with a dipole form factor $(\Lambda_\pi^2-\mu^2)/(\mathbf{q}^2+\Lambda_\pi^2)$ at each meson-baryon-baryon vertex. Its form has been given explicitly in Ref. 23. The operator $X_{\mathrm{N}\Delta,\mathrm{N}\Delta}$ in Eq. (2.3b) describes the coupling to the $\pi$d channel. It will be defined later.

To evaluate $U^{(3)}_{\mathrm{NN,NN}}$ [Eq. (2.3c)] we assume that the transition operator $F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}$ can be parametrized according to the static one-pion exchange followed by a $\pi$N two-body interaction $v_{\pi\mathrm{N}}$ of Eq. (1.2d) [Fig. 2(b)]. We then have (suppressing isospin indices)

$$U^{(3)}_{\mathrm{NN,NN}}(E)=\frac{1}{(2\pi)^3}\left[\frac{f}{\mu}\right]^2\left\langle \mathrm{NN}\left|\sum_{i\neq j}^{2}\sigma_i\cdot\mathbf{q}\frac{1}{\mathbf{q}^2+\mu^2}F_{\pi\mathrm{NN}}(\mathbf{q}^2)g(E,j)F_{\pi\mathrm{NN}}(\mathbf{q}'^2)\frac{1}{\mathbf{q}'^2+\mu^2}\sigma_i\cdot\mathbf{q}'\right|\mathrm{NN}\right\rangle\;, \tag{2.5}$$

where $\mathbf{q}$ is the momentum of the exchanged pion, $\mu$ is the pion mass, $f^2=4\pi$ (0.08) is the usual coupling constant, and

$$g(E,j)=v_{\pi\mathrm{N}}(j)\frac{P_{\pi\mathrm{NN}}}{E-H_0-v_{\pi\mathrm{N}}(j)+i\epsilon}v_{\pi\mathrm{N}}(j)\equiv t[E-E_{\mathrm{N}}(p_i)]-v_{\pi\mathrm{N}}(j),\quad i\neq j\;. \tag{2.6}$$

Here we have defined a $\pi$N scattering operator,

$$t(\omega)=v_{\pi\mathrm{N}}+v_{\pi\mathrm{N}}\frac{P_{\pi\mathrm{N}}}{\omega-H_0-v_{\pi\mathrm{N}}+i\epsilon}v_{\pi\mathrm{N}}\;. \tag{2.7}$$

The $\pi$NN form factor is taken to be the usual dipole form,

$$F_{\pi\mathrm{NN}}(\mathbf{q}^2)=\frac{\beta_\pi^2-\mu^2}{\mathbf{q}^2+\beta_\pi^2}\;. \tag{2.8}$$

The NN$\rightarrow$NN interaction $V_{\mathrm{NN,NN}}$ in Eq. (2.2) is defined by the subtraction procedure developed in Ref. 13. For a given choice of the starting low-energy NN potential $V_0$, it is defined by

$$V_{\mathrm{NN,NN}}=V_0-\sum_{i=1}^{3}U^{(i)}_{\mathrm{NN,NN}}(E_s)\;. \tag{2.9}$$

The subtraction energy $E_s$ is chosen such that the low energy NN phase shifts and various NN cross sections in the intermediate energy region can be best reproduced when Eq. (2.1) is used in the calculation. Qualitatively, the subtraction procedure is to remove the effects due to the excitations of intermediate N$\Delta$ and $\pi$NN states, which are contained in the phenomenological part of the starting low energy NN potential $V_0$, so that there is no double counting.

The effect of the $\pi$d channel on NN scattering is contained in the amplitude $X_{N\Delta,N\Delta}$ of Eq. (2.3b). Assuming that the NN interaction in the presence of a spectator pion can be taken as a separable form, $X_{N\Delta,N\Delta}$ can be obtained by solving the following coupled Faddeev-AGS equation in the subspace $N\Delta \oplus \pi d$:

$$X_{N\Delta,N\Delta}(E)=Z_{N\Delta,N\Delta}(E)+Z_{N\Delta,N\Delta}(E)G_{N\Delta}(E)X_{N\Delta,N\Delta}(E)+Z_{N\Delta,\pi d}(E)G_{\pi d}(E)X_{\pi d,N\Delta}(E)\ , \tag{2.10a}$$

$$X_{N\Delta,\pi d}(E)=Z_{N\Delta,\pi d}(E)+Z_{N\Delta,\pi d}(E)G_{\pi d}(E)X_{\pi d,\pi d}(E)+Z_{N\Delta,N\Delta}(E)G_{N\Delta}(E)X_{N\Delta,\pi d}(E)\ , \tag{2.10b}$$

$$X_{\pi d,N\Delta}(E)=Z_{\pi d,N\Delta}(E)+Z_{\pi d,N\Delta}(E)G_{N\Delta}(E)X_{N\Delta,N\Delta}(E)\ , \tag{2.10c}$$

$$X_{\pi d,\pi d}(E)=Z_{\pi d,N\Delta}(E)G_{N\Delta}(E)X_{N\Delta,\pi d}(E)\ , \tag{2.10d}$$

where

$$Z_{N\Delta,N\Delta}(E)=\left\langle N\Delta \left| h_{\Delta,\pi N}(1)\frac{P_{\pi NN}}{E-H_0+i\epsilon}h_{\pi N,\Delta}(2)\right| \Delta N\right\rangle \tag{2.11}$$

is the one-pion-exchange term and

$$Z_{N\Delta,\pi d}(E)=\left\langle N\Delta \left| h_{\Delta,\pi N}\frac{P_{\pi NN}}{E-H_0+i\epsilon}g_{NN,d}\right| \pi d\right\rangle\ , \tag{2.12a}$$

$$Z_{\pi d,N\Delta}(E)=\left\langle \pi d \left| g_{d,NN}\frac{P_{\pi NN}}{E-H_0+i\epsilon}h_{\pi N,\Delta}\right| N\Delta\right\rangle \tag{2.12b}$$

is the one-nucleon-exchange term. $g_{d,NN}$ is the form factor in the separable representation of the NN potential *in the presence of a spectator pion.* The propagators in Eq. (2.10) are

$$G_{N\Delta}(E)=\frac{P_{N\Delta}}{E-H_0-\Sigma_\Delta(E)}\ , \tag{2.13a}$$

$$G_{\pi d}(E)=\frac{1}{E-H_0-S_d(E)}\ , \tag{2.13b}$$

where $\Sigma_\Delta(E)$ is given in Eq. (2.4) and

$$S_d(E)=\left\langle \pi d \left| g_{d,NN}\frac{P_{\pi NN}}{E-H_0+i\epsilon}g_{NN,d}\right| \pi d\right\rangle\ . \tag{2.14}$$

In the same approximation, the $\pi$d scattering amplitude can be cast into the following form:

$$T_{\pi d,\pi d}(E)=X_{\pi d,\pi d}(E)+T^{\rm abs}_{\pi d,\pi d}(E)+T^{\rm NR}_{\pi d,\pi d}(E)\ . \tag{2.15}$$

The first term is the multiple scattering amplitude due to the $\pi N \leftrightarrow \Delta$ mechanism. It is the solution of Eq. (2.10d). The pion absorption effect is calculated from

$$T^{\rm abs}_{\pi d,\pi d}=X_{\pi d,N\Delta}(E)G_{N\Delta}(E)T_{N\Delta,N\Delta}(E)\times G_{N\Delta}(E)X_{N\Delta,\pi d}(E)\ , \tag{2.16a}$$

with

$$T_{N\Delta,N\Delta}(E)=V_{N\Delta,NN}\left[\frac{1}{E-H_0+i\epsilon}+\frac{1}{E-H_0+i\epsilon}T_{NN,NN}(E)\frac{1}{E-H_0+i\epsilon}\right]V_{NN,N\Delta}\ . \tag{2.16b}$$

The third term on the right-hand side of Eq. (2.15) contains the effect due to the nonresonant $\pi$N interaction,

$$T^{\rm NR}_{\pi d,\pi d}(E)=\left\langle \chi^{(-)}_{\pi d,E}\left| \sum_{i=1}^{2} t_{\pi N}[E-E_N(\mathbf{p}_i)]\right|\chi^{(+)}_{\pi d,E}\right\rangle , \tag{2.17}$$

where $|\chi^{(\pm)}_{\pi d,E}\rangle$ is the $\pi$d scattering wave function generated from the solution of the Faddeev-AGS Eq. (2.10). $t_{\pi N}$ is the nonresonant $\pi$N $t$ matrix defined by Eq. (2.7) and is calculated in the presence of a spectator nucleon.

## III. NUMERICAL PROCEDURES

All of the numerical calculations are carried out in the partial-wave representations of the NN, N$\Delta$, and $\pi$-"d" channels, where d denotes an eigenstate of the NN subsystem in the $\pi$NN channel. For a given total angular momentum $J$ and parity $P$, the momentum-space matrix elements of all $\pi$NN scattering equations given above are reduced to a set of coupled one-dimensional integral equations. To solve these equations, we need to carry out partial-wave decompositions of their driving terms. This task is straightforward but tedious, as can be seen in Ref. 20 for the one-particle-exchange driving terms, and many existing publications[21] for baryon-baryon interactions. We omit this part of the presentation and concentrate on the methods of solving scattering equations. For a concise presentation, all of the orbital-spin-isospin quantum numbers needed to specify the resulting one-dimensional integral equations will be suppressed. Only the channel labels NN, N$\Delta$, and $\pi$d will be kept.

The first step of the calculation is to solve the Faddeev-AGS equations defined by Eq. (2.10). In the partial-wave representation it can be written as

$$X_{\alpha\beta}(p,p_0,E)=Z_{\alpha\beta}(p,p_0,E)+\sum_{\gamma={\rm N}\Delta,\pi d}\int_0^\infty p'^2dp'Z_{\alpha\gamma}(p,p',E)G_\gamma(p',E)X_{\gamma\beta}(p',p_0,E) , \tag{3.1}$$

with $\alpha,\beta={\rm N}\Delta,\pi$d. $p_0$ is the $\pi$d on-shell momentum

$$E=(m_d^2+p_0^2)^{1/2}+(\mu^2+p_0^2)^{1/2} .$$

Note that in solving Eq. (3.1) the nonresonant $\pi$N interaction is not included. Its effect is included in $U^{(3)}_{\rm NN}$ [Eq. (2.3c)] for NN scattering and $T^{\rm NR}_{\pi d,\pi d}$ [Eq. (2.17)] for $\pi$d scattering.

Because of the logarithmic singularities of the driving term $Z_{\alpha\beta}$, Eq. (3.1) is solved by using the standard method of contour rotation.[22] The procedure is to use the property that all of the singularities of Eq. (3.1) are in the upper-half of the complex $p$ plane and hence we can write the integration of Eq. (3.1) as an integration over a complex axis

$$p_c=p\exp(-i\theta) .$$

For $\theta>0$ and $p\geq 0$, $p_c$ is in the lower half of the complex momentum plane and hence the driving terms $Z$'s defined on the $p_c$ axis do not have singularities. By choosing appropriate mesh points for the integration over $p_c$, Eq. (3.1) can be cast into a finite complex matrix equation and can be solved easily to obtain the matrix element $X_{\pi d,\pi d}(p_0,p_0,E)$ for calculating the multiple scattering part of the $\pi$d scattering amplitude in Eq. (2.15), the half-off-shell matrix elements $X_{\pi d,{\rm N}\Delta}(p_0,p_c,E)$ and $X_{{\rm N}\Delta,\pi d}(p_c,p_0,E)$ for calculating the pion absorption effect Eq. (2.16) in $\pi$d scattering, and the fully-off-shell matrix element $X_{{\rm N}\Delta,{\rm N}\Delta}(p_c,p_c',E)$ for evaluating the NN driving term $U^{(2)}_{\rm NN,NN}$ of Eq. (2.3b). We now move to discuss these two calculations.

In the partial-wave representation the Lippmann-Schwinger Eq. (2.1) can be written as

$$T^{\alpha}_{l'l}(p',p_0,E)=\tilde V^{\alpha}_{l'l}(p',p_0,E)+\sum_{l''}\int_0^\infty \frac{\tilde V^{\alpha}_{l'l''}(p',p,E)T^{\alpha}_{l'',l}(p,p_0,E)}{E-2(m^2+p^2)^{1/2}+i\epsilon}p^2dp , \tag{3.2}$$

where $\alpha$ denotes the usual NN eigenchannel quantum numbers $JST$ and $l$ is the relative orbital angular momentum. The driving term $\tilde V^{\alpha}_{ll'}$ is the partial-wave projection of the following momentum-space matrix elements:

$$\tilde V_{\rm NN,NN}(\mathbf{p}',\mathbf{p}_0,E)=V_{\rm NN,NN}(\mathbf{p}',\mathbf{p}_0)+\sum_{i=1}^{3}U^{(i)}_{\rm NN,NN}(\mathbf{p}',\mathbf{p}_0,E) . \tag{3.3}$$

In terms of the chosen integration variable $\mathbf{p}$ on the real axis and $\mathbf{p}_c$ on the complex axis, the $\Delta$ excitation parts of the effective NN potential can be written explicitly as

$$U^{(1)}_{\rm NN,NN}(\mathbf{p}',\mathbf{p}_0,E)=\int d\mathbf{p}V_{\rm NN,N\Delta}(\mathbf{p}',\mathbf{p})G_{\rm N\Delta}(\mathbf{p},E)V_{\rm N\Delta,NN}(\mathbf{p},\mathbf{p}_0,E) , \tag{3.4a}$$

$$U^{(2)}_{\rm NN,NN}(\mathbf{p}',\mathbf{p}_0,E)=\int d\mathbf{p}_cd\mathbf{p}_c'V_{\rm NN,N\Delta}(\mathbf{p}',\mathbf{p}_c')G_{\rm N\Delta}(\mathbf{p}_c',E)X_{\rm N\Delta,N\Delta}(\mathbf{p}_c',\mathbf{p}_c,E)G_{\rm N\Delta}(\mathbf{p}_c,E)V_{\rm N\Delta,NN}(\mathbf{p}_c,\mathbf{p}_0) . \tag{3.4b}$$

We follow Ref. 7 to calculate the matrix element of the N$\Delta$ propagator in the nonrelativistic baryon approximation

$$G_\Delta(E,p_\Delta)=\frac{1}{E-m-m_\Delta-p_\Delta^2/2\mu_{N\Delta}-\Sigma_\Delta(E,p_\Delta)}\ , \tag{3.5}$$

where $m_\Delta$ is the $\Delta$ bare mass and $\mu_{N\Delta}=mm_\Delta/(m+m_\Delta)$. The $\Delta$ self-energy evaluated in the presence of a spectator nucleon is

$$\Sigma_\Delta(E,p_\Delta)=\int\frac{|h(q)|^2q^2dq}{E-m-p_\Delta^2/2m-p_\Delta^2/2[m+E_\pi(q)]-E_\pi(q)-E_N(q)+i\epsilon}\ . \tag{3.6}$$

The form factor $h(q)$ and the bare mass $m_\Delta$ of the $\Delta$ are determined by fitting the $\pi$N $P_{33}$ phase shifts. In our fit we have $m_\Delta=1280$ MeV and

$$h(q)=0.98\frac{1}{\sqrt{2(m+\mu)}}\frac{q}{\mu}\left[\frac{\Lambda_\Delta^2}{\Lambda_\Delta^2+q^2}\right]^2\ , \tag{3.7}$$

with $\Lambda_\Delta=358$ MeV/$c$. We have also tried other parametrizations of $h(q)$. But we find that the $\pi$NN results are not very sensitive to this change, as far as $m_\Delta$ and the parameters of $h(q)$ are fixed by the same fit to the $P_{33}$ phase shift. All results presented in this paper are based on Eq. (3.7).

The matrix element of the nonresonant interaction defined in Eq. (2.5) in the $T=1$ NN channel can be written explicitly as

$$\begin{aligned}U^{(3)}_{NN,NN}(\mathbf{p}',\mathbf{p},E)=\int d\mathbf{k}\left[\frac{1}{2\pi}\right]^3\left[\frac{f}{\mu}\right]^2(\mathbf{q}'\cdot\boldsymbol{\sigma}_2)F_{\pi NN}(\mathbf{q}'^2)\frac{1}{\mathbf{q}'^2+\mu^2}\\ \times\{\tfrac{1}{3}[T^S_{11}(q',q,\omega)+8T^S_{31}(q',q,\omega)]+T^P_{13}(q',q,\omega)\mathbf{q}'\cdot\mathbf{q}\\ +\tfrac{1}{3}[8T^P_{31}(q',q,\omega)-T^P_{13}(q',q,\omega)](\boldsymbol{\sigma}_1\cdot\mathbf{q}')(\boldsymbol{\sigma}_1\cdot\mathbf{q})\}\frac{1}{\mathbf{q}^2+\mu^2}(\mathbf{q}\cdot\boldsymbol{\sigma}_2)F_{\pi NN}(\mathbf{q}^2)\ ,\end{aligned} \tag{3.8}$$

where $\omega=E-E_N(\mathbf{k})$, $\mathbf{q}=\mathbf{p}-\mathbf{k}$, and $\mathbf{q}'=\mathbf{p}'-\mathbf{k}$. $T^\alpha_{2I,2J}$ is the $\pi$N amplitude in the channel of isospin $I$ and total angular momentum $J$, with $\alpha=S$ and $P$ denoting the $l=0$ and 1 $\pi$N partial wave, respectively. The coefficient of each term in Eq. (3.8) is obtained from calculating the matrix element of the isospin factor in the considered $T=1$ NN channel.

Note that only the $\pi$N interaction in the $S_{11}$, $S_{31}$, $P_{13}$, and $P_{31}$ channels are included in Eq. (3.8). The $\pi$N $P_{11}$ channel is dropped here since the contribution from its nucleon pole term is a two-pion exchange interaction which can be generated from the one-pion-exchange part of the $V_{NN,NN}$ in solving the scattering Eq. (3.2). Including the $P_{11}$ channel in calculating $U^{(3)}_{NN,NN}$ obviously will cause a double counting problem. This treatment of $P_{11}$ could be inaccurate in the energy near the pion production threshold. But it is not an important issue in the present study, since our focus is on the $\Delta$ excitation region. We use the $\pi$N amplitudes of Ref. 24 in our calculation of Eq. (3.8).

Equation (3.2) can be solved by the standard matrix method,[21] since the $\pi$NN singularities have been integrated out and the driving term $\tilde{V}^\alpha_{ll'}$ is finite on the real momentum axis. Once the NN amplitude is obtained, we can then calculate the pion absorption effect on $\pi$d scattering according to Eq. (2.16),

$$T^{\rm abs}_{\pi d,\pi d}(p_0,p_0,E)=\int p_c^2dp_cp_c'^2dp_c'X_{\pi d,N\Delta}(p_0,p_c,E)G_{N\Delta}(p_c,E)T_{N\Delta,N\Delta}(p_c,p_c',E)G_{N\Delta}(p_c',E)X_{N\Delta,\pi d}(p_c'p_0,E)\ , \tag{3.9a}$$

where

$$\begin{aligned}T_{N\Delta,N\Delta}(p_c,p_c',E)=\int p^2dp\ V_{N\Delta,NN}(p_c,p)\frac{1}{E-2E_N(p)+i\epsilon}V_{NN,N\Delta}(p,p_c')\\ +\int p^2dp\,p'^2dp'\frac{V_{N\Delta,NN}(p_c,p)}{E-2E_N(p)+i\epsilon}T_{NN,NN}(p,p',E)\frac{V_{NN,N\Delta}(p',p_c')}{E-2E_N(p')+i\epsilon}\ .\end{aligned} \tag{3.9b}$$

Here $E_N$ is the nucleon energy and $T_{NN,NN}$ is the NN amplitude calculated from the partial-wave solution $T^\alpha_{l'l}$ of Eq. (3.2). Again, in order to evaluate Eq. (3.9a), we must evaluate the $N\Delta\rightarrow N\Delta$ matrix element on the complex mesh points $p_c$.

The last step is to add the effect of $\pi$N nonresonant interaction to the $\pi$d amplitude, according to Eq. (2.17). We use the standard on-shell approximation for the $\pi$d wave function in each partial wave,

$$\chi^{(+)}_{\pi d,E}(p)=\chi^{(-)*}_{\pi d,E}(p)\rightarrow\frac{1}{p_0^2}\delta(p-p_0)S^{1/2}_{\pi d}\ , \tag{3.10}$$

where $S_{\pi d}$ is the $S$ matrix calculated from the Faddeev-AGS solution [Eq. (3.1)]. We then have

$$T^{\rm NR}_{\pi d,\pi d}(p_0,p_0,E)=\sum_\alpha\int p^2dp\ S^{1/2}_{\pi d}X_{\pi d,N\alpha}(p_0,p)G_\alpha(p,E)X_{N\alpha,\pi d}(p,p_0)S^{1/2}_{\pi d}\ , \tag{3.11}$$

where $\alpha$ specifies the quasiparticle channel in a separable representation of the $\pi$N $t$ matrix and $G_\alpha$ is the corresponding quasiparticle propagator.

The total $\pi$d amplitude then takes the form

$$T_{\pi d,\pi d}(p_0,p_0,E)=X_{\pi d,\pi d}(p_0,p_0,E)+T^{\rm abs}_{\pi d,\pi d}(p_0,p_0,E)+T^{\rm NR}_{\pi d,\pi d}(p_0,p_0,E)\ . \qquad (3.12)$$

This completes our discussion of our numerical procedures. The calculations of NN and $\pi$d elastic scattering observables are well documented.

## IV. NUMERICAL RESULTS

The main finding of Ref. 13 is that the meson theory of nuclear forces can describe the main features of NN scattering data up to 1 GeV, but cannot reproduce the strong energy dependencies of polarization cross sections $\Delta\sigma_T^{\rm tot}$ and $\Delta\sigma_L^{\rm tot}$ near 800 MeV. In the first part of this section, we want to use the formalisms presented in Secs. II and III to reassess this conclusion by investigating several questions concerning the model dependence of the results obtained in Ref. 13, and examining the extent to which the fit to the data can be improved by including the effects due to nonresonant $\pi$N interactions. We then present our results of $\pi$d elastic scattering.

### A. NN elastic scattering

To make contact with the NN study of Ref. 13, let us first consider only the one-pion-exchange $\Delta$-excitation mechanism[25] (i.e., setting $U^{(3)}_{\rm NN,NN}=0$) and choose $V_0$ of Eq. (2.9) to be the Paris potential.[16] Our NN calculation then only has two parameters, $E_s$ for the subtraction in defining $V_{\rm NN,NN}$ by Eq. (2.9), and the cutoff parameter $\Lambda_\pi$ for the dipole form factors of the $V_{\rm NN\leftrightarrow N\Delta}$. Since an acceptable $\pi$NN model for nuclear studies must be as good as the conventional nuclear theory in describing the low energy nuclear properties, we require that the parameters $E_s$ and $\Lambda_\pi$ must be chosen such that the original Paris phase shifts below 200 MeV must be reproduced within 5%. It is found that $E_s \le 50$ MeV is acceptable, as shown in Fig. 3. The cutoff parameter $\Lambda_\pi$ is limited in the range $\Lambda_\pi < 750$ MeV/$c$. In Fig. 4 we see that the $^1S_0$ and $^3P_0$ phase shifts calculated with $\Lambda_\pi = 900$ and 1200 MeV are clearly not acceptable. For higher partial waves, the low-energy phase shifts can be reproduced to a much higher accuracy than that seen in Figs. 3 and 4.

The optimum value of the cutoff parameter $\Lambda_\pi$ is determined by considering various NN total cross sections in the intermediate energy region. In Fig. 5, we again see that $\Lambda_\pi > 750$ MeV/$c$ is not acceptable for describing the total reaction cross section $\sigma_R$. Our final choice of $\Lambda_\pi$ is made by requiring the best fit to the NN total cross sections. In this way we arrive at the value $\Lambda_\pi = 650$ MeV, since it gives the best description of the famous polarization data[26] as shown in Fig. 6. Accordingly, the phase shifts calculated with $\Lambda_\pi = 650$ MeV are in good agreement with the Arndt phase shifts, as reported in Refs. 13 and 25. The discrepancies shown in Fig. 6 show the limitation of the $\pi$NN model with only $\Delta$ excitation included.

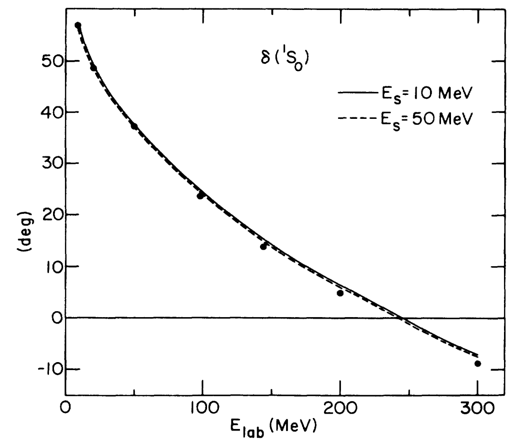


FIG. 3. The $^1S_0$ phase shifts calculated with $E_s = 10$ and 50 MeV are compared with the Paris phases (dots). $E_s$ is the subtraction energy of Eq. (2.9).

The results discussed so far are based on the assumption that the NN$\to$N$\Delta$ transition is due to one-pion exchange only. The conventional model also includes a $\rho$ exchange. It is therefore interesting to see whether the fit to the NN data can be improved if we also consider this shorter range $\Delta$ excitation mechanism. It is well known that the $\rho$ exchange tends to cancel the $\pi$ exchange. Therefore, when the $\rho$ exchange is included, the cutoff $\Lambda_\pi$ for $\pi$ exchange has to be increased accordingly in order to retain the best reproduction of the low ener-

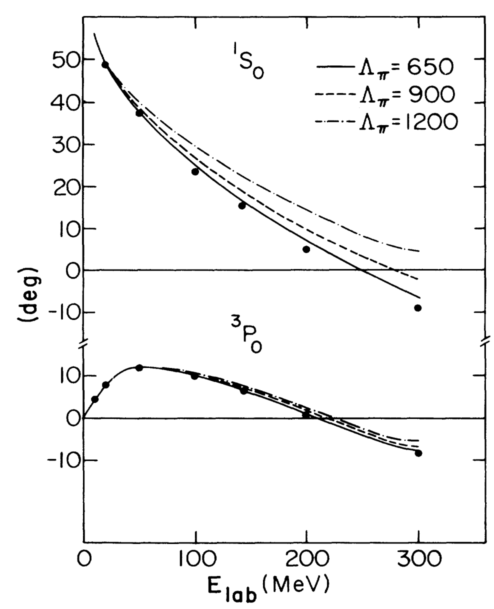


FIG. 4. Dependence of the calculated NN phase shifts on the cutoff parameter $\Lambda_\pi$ of the one-pion-exchange NN$\leftrightarrow$N$\Delta$ potential.

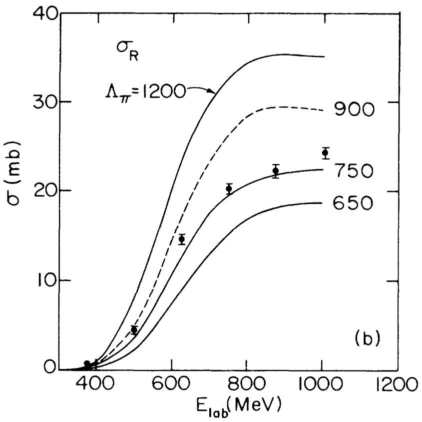


FIG. 5. Dependence of the calculated NN total reaction cross section on the cutoff parameter $\Lambda_\pi$ of the one-pion-exchange NN$\leftrightarrow$N$\Delta$ potential. The data are taken from the compilation of Ref. 26.

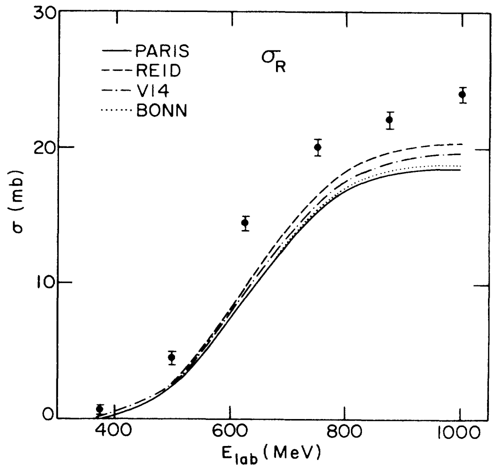


FIG. 7. Dependence of the calculated total NN reaction cross section on the starting low energy NN potential. The calculations are done with $\Lambda_\pi=650$ MeV/$c$ for $V_{\mathrm{NN}\leftrightarrow\mathrm{N}\Delta}$.

gy phase shifts and the total cross sections up to 1 GeV. We have explored this possibility and have not been able to find a set of $\Lambda_\pi$ and $\Lambda_\rho$ which can give a better fit to the polarization total cross sections shown in Fig. 6. In all of our subsequent calculations we will therefore only keep the $\pi$ exchange in defining the NN$\rightarrow$N$\Delta$ interaction.

We now turn to investigating the major model dependence in our approach. So far we use the Paris potential as the starting low energy potential $V_0$ to define the NN$\rightarrow$NN interaction by the subtraction defined in Eq. (2.9). It is natural to ask whether our NN results can be improved if other existing low energy NN potentials are used in our calculation. The chosen low energy NN potentials must be as good as the Paris potential in describing low energy NN phase shifts and various important nuclear properties (such as the nuclear binding in three-nucleon systems and nuclear matter). In addition, they must all have the well-established one-pion-exchange tail. The potentials which satisfy these criteria and are used in this investigation are the Bonn[17] (1975), ANL $V14$,[18] and Reid[19] potentials. Because of their different treatments of the intermediate range interaction, their short-range parametrizations are radically different from each other and also from the Paris potential. This difference in defining short-range dynamics will certainly be seen clearly at higher energies.

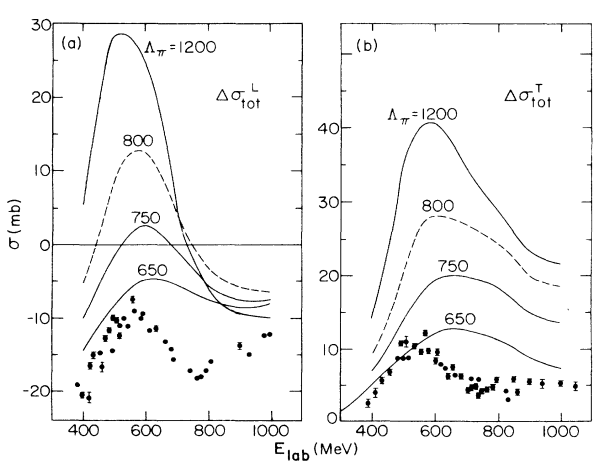


FIG. 6. Same as Fig. 5, but for the total cross sections with spin orientations of the projectiles and the target nucleon specified. We follow the notations of Ref. 26.

For each one of the considered low energy NN potentials, we repeat the same procedure discussed above to see whether the NN data can be fitted. To fit both the low energy phase shifts and the total reaction cross section, we find that all potentials favor $E_s<50$ MeV and $\Lambda_\pi<750$ MeV/$c$. For the same parameters $E_s=10$ MeV and $\Lambda_\pi=650$ MeV/$c$ their phase shifts are comparable to that of the Paris potential. Their corresponding predictions of total reaction cross section $\sigma_R$ are compared in Fig. 7. Clearly, the total reaction cross section $\sigma_R$ is insensitive to the choice of the starting low energy NN potential. However, their predictions of the polarization total cross sections are very different. Figure 8 shows that with $E_s=10$ MeV and $\Lambda_\pi=650$ MeV/$c$ none of the constructed models can reproduce the strong oscillation of the polarization data. We have carried out extensive calculations in the region $E_s<50$ MeV and $\Lambda_\pi<750$ MeV/$c$ for all of the considered potentials and have also investigated possible improvements by using a $\pi+\rho$ model of $V_{\mathrm{NN,N}\Delta}$. No improvement has been found.

The above-mentioned studies suggest that the $\Delta$-excitation model cannot give a detailed description of all $\pi$NN processes even in the $\Delta$-excitation energy region. The best we can obtain is the model based on the Paris potential with the parameters $E_s=10$ MeV and $\Lambda_\pi=650$ MeV. This model is essentially the model first developed in Ref. 13. To see clearly the starting point of our subsequent investigations, we show in Fig. 9 the NN phase shifts (solid curves) calculated from this $\Delta$ excitation model. The higher partial wave phase shifts are similar to what have been presented in Ref. 13 and therefore are neglected here because they are irrelevant to our studies.

Within our formulation, the next possibility of im-

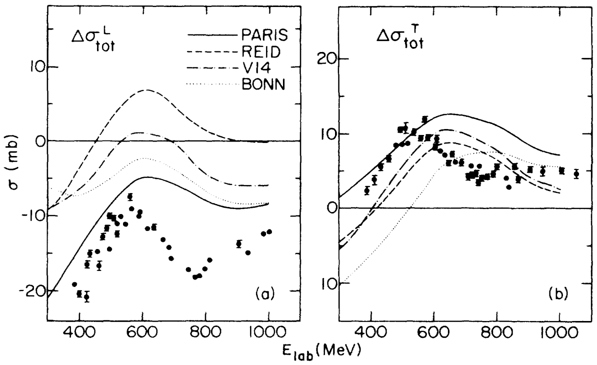


FIG. 8. Same as Fig. 7, but for the total polarization cross sections as defined in Fig. 6.

proving the meson-exchange $\pi$NN model is to consider the nonresonant interaction $H_2'$ of Eq. (1.2). In NN scattering, this additional pionic effect is described by $U^{(3)}_{\mathrm{NN,NN}}$ [Eq. (2.5)]. We will still focus our attention in the $\Delta$ resonant energy region in which our perturbative treatment of nonresonant $\pi$N interactions is suitable. The question we want to answer is the following: Can the discrepancies, shown in Fig. 9, between the data and the predictions by the $\Delta$-excitation model (solid curves) be removed by including $U^{(3)}_{\mathrm{NN,NN}}$ in our NN calculation?

Within the one-pion-exchange model of the transition interaction $F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}$, the strength of $U^{(3)}_{\mathrm{NN,NN}}$ is determined by the range parameter $\beta_\pi$ of the form factor $F_{\pi\mathrm{NN}}(q)$, Eq. (2.8). We consider two different methods in determining $\beta_\pi$. The first one is to assume that the $\pi$N$\leftrightarrow$N form factor [Fig. 2(b)] for the transition operator $F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}$ is identical to that of the $V_{\mathrm{NN}\leftrightarrow\mathrm{N}\Delta}$. Then the parameter $\beta_\pi$ is taken to be 650 MeV/$c$ as determined above. The results calculated from this choice of the parameter are the dashed curves in Fig. 9. It is seen that the nonresonant $\pi$N interactions only significantly influence the NN phase shifts in the $^1S_0$ and $^3P_2$ channels. In the $^1S_0$ channel, the inelasticity is increased by a factor of about 2, in better agreement with the data. Note that it is very difficult to improve this inelasticity in the $\Delta$-excitation model since the $^1S_0$ NN channel only couples with the $L=2$ $^5D_0$ N$\Delta$ channel. The effect of nonresonant $\pi$N interaction on polarization total cross sections is found to only increase the theoretical values by about 1 mb at all energies, and hence does not yield the needed strong energy dependence near 800 MeV seen in Fig. 6.

The second way of determining the strength of the nonresonant transition interaction is to adjust the cutoff $\beta_\pi$ for the transition operator $F_{\pi\mathrm{NN}\leftrightarrow\mathrm{NN}}$ to fit a piece of NN data which is most sensitive to the nonresonant $\pi$N interaction. As shown in Fig. 9(a), the $^1S_0$ inelasticity is most influenced by the nonresonant $\pi$N interaction. We, therefore, first adjust $\beta_\pi$ to fit this data and then see whether the overall fit to NN data can be improved. The resulting cutoff is found to be about $\beta_\pi=1200$ MeV. The improvements are illustrated in Table I. It is seen that the largest effect due to the change $\beta_\pi=650$

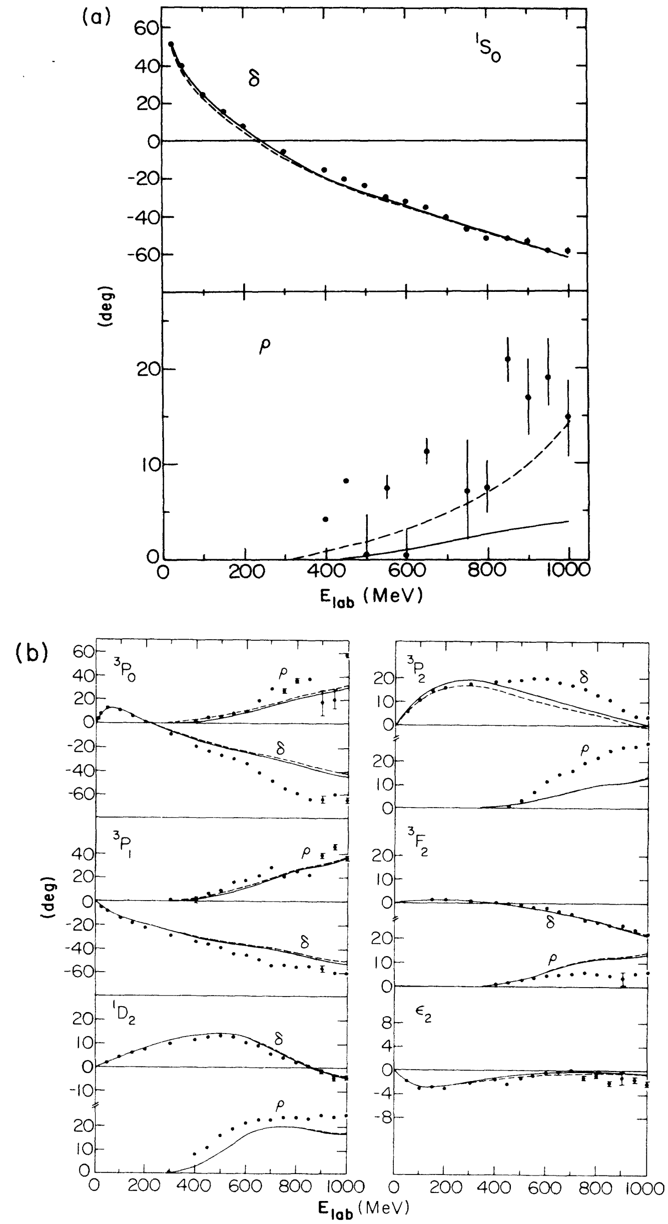


FIG. 9. The calculated $T=1$ NN phase shifts are compared with Arndt's data (Ref. 15). The solid curves only contain the $\Delta$ excitation, while the dashed curves also contain the effect due to the nonresonant term $U^{(3)}_{\mathrm{NN,NN}}$. The differences are negligible for $l>2$ partial waves. The results of $l>2$ partial waves have been published in Ref. 13 and hence are omitted here.

MeV/$c\rightarrow 1200$ MeV/$c$ is to change the $^1S_0$ inelasticity from 7.61 to 10.12, which is much closer to the data. However, the other partial waves are relatively insensitive to the cutoff $\beta_\pi$ and hence no improvement in fitting the polarization cross sections is obtained.

In summary, the main effect of nonresonant $\pi$N interactions is to increase the inelasticity of the $^1S_0$ channel to a value comparable to the data. Its effects in all other partial waves (except $^3P_2$) are small compared

TABLE I. Dependence of phase shifts on the cutoff parameter $\beta_\pi$ [Eq. (2.8)] of the form factors of $U^{(3)}_{\mathrm{NN,NN}}$ at $E_{\mathrm{lab}}=800$ MeV. $\delta$ and $\rho$ are in degrees.

| | $^1S_0$ | | $^3P_0$ | | $^3P_1$ | | $^1D_2$ | | $^3F_3$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_\pi$ | $\delta$ | $\rho$ | $\delta$ | $\rho$ | $\delta$ | $\rho$ | $\delta$ | $\rho$ | $\delta$ | $\rho$ |
| a | −48.61 | 3.21 | −35.83 | 19.82 | −44.89 | 25.40 | 2.75 | 20.04 | −7.02 | 19.30 |
| 650 | −48.17 | 7.61 | −33.24 | 21.51 | −43.32 | 26.11 | 2.84 | 20.23 | −6.80 | 19.41 |
| 1200 | −48.78 | 10.12 | −30.81 | 22.77 | −42.17 | 26.68 | 2.75 | 20.30 | −6.79 | 19.43 |

[a] Calculation with $U^{(3)}_{\mathrm{NN,NN}}=0$.

with the $\Delta$ resonant effect, and therefore cannot explain the strong energy dependencies of the polarization total cross sections near 800 MeV.

This completes our NN study in this work. We will discuss necessary future works in Sec. V.

### B. $\pi d$ elastic scattering

In this subsection we present our $\pi d$ results. To compare with the well-established Faddeev-AGS $\pi d$ calculation, we first consider our calculation in the limit that the pion absorption effect is neglected; namely, setting the absorption amplitude $T^{\mathrm{abs}}_{\pi d,\pi d}$ in Eq. (2.15) to zero. In this multiple scattering limit, our approach differs from the exact Faddeev-AGS $\pi d$ calculation[3–5,27–30] only in

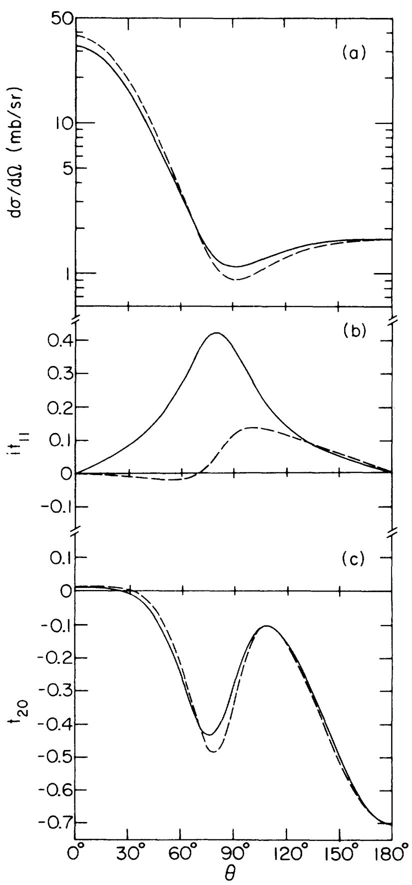

FIG. 10. The effect of the nonresonant amplitude $T^{\mathrm{NR}}_{\pi d,\pi d}$ [Eq. (2.15)] on the $\pi d$ differential cross section (a), vector (b) and tensor (c) analyzing power at $T_\pi=140$ MeV.

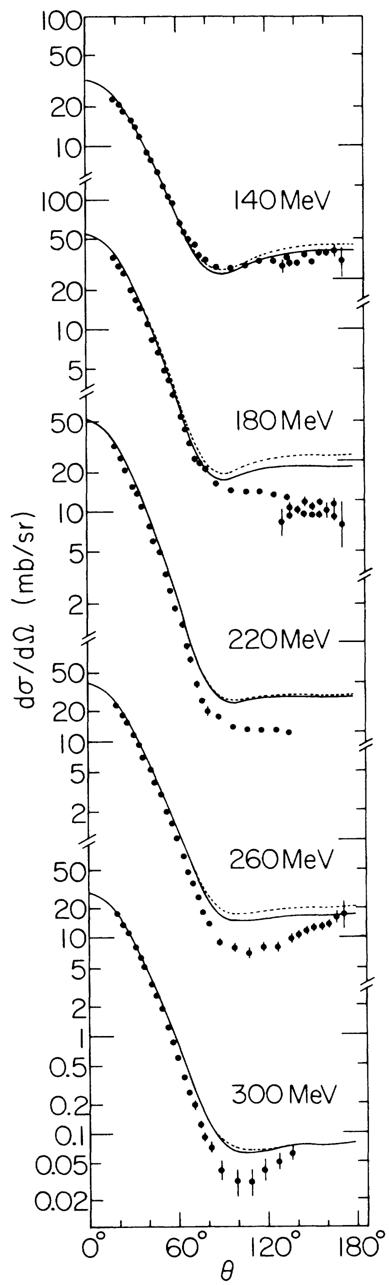

FIG. 11. The $\pi d$ elastic differential cross sections calculated without (solid) and with (dashed) the pion absorption effect are compared with the experimental data (Ref. 26).

our perturbative treatment of the nonresonant $\pi$N interaction. To have a sensible comparison with the data, it is necessary to justify this approximation which drastically simplifies the numerical task. This is done by examining whether the effect of the nonresonant amplitude described by $T^{\mathrm{NR}}_{\pi d,\pi d}$ of Eq. (2.15) is similar to what has been found in the exact Faddeev-AGS $\pi$d calculation, including all $s$ and $p$ $\pi$N amplitudes. In Fig. 10 we show our results calculated with (solid curves) and without (dashed curves) including the nonresonant amplitude $T^{\mathrm{NR}}_{\pi d,\pi d}$ of Eq. (2.15). The effects shown in Fig. 10, in particular the dramatic change in the vector analyzing power $it_{11}$, agree with that of the calculation by Giraud *et al.*[27] (see their Fig. 6). On this ground, we consider that our perturbative treatment is adequate and can be used as a good starting point for investigating the pion absorption effect.

We now show the effect of pion absorption and compare our results with the data. To see the main feature of our approach, let us recall that the pion absorption term $T^{\mathrm{abs}}_{\pi d,\pi d}$ of Eq. (2.15) is directly calculated from the full NN amplitude $T_{\mathrm{NN,NN}}$ through Eq. (2.16). We also recall that the full NN amplitude $T_{\mathrm{NN,NN}}$ is in turn influenced by the coupling to the $\pi$d channel through the calculation of the NN driving term $U^{(2)}_{\mathrm{NN,NN}}$ of Eq. (2.3b) from the Faddeev-AGS amplitude $X_{\mathrm{N\Delta,N\Delta}}$. This mutual coupling is, of course, the consequence of our unitary formulation of the $\pi$NN problem. Note that the strength of this coupling is mainly determined by the range parameter $\Lambda_\pi$ of the form factor of the transition interaction $V_{\mathrm{NN}\leftrightarrow\mathrm{N\Delta}}$. Its value $\Lambda_\pi = 650$ MeV has been determined in the NN study. Therefore, there is no adjustable parameter in our calculation of the pion absorption effect in $\pi$d scattering. The direct constraint from the fit to NN data is the characteristic of a unitary approach.

We compare in Figs. 11–13 the $\pi$d data with our results. The solid curves are the multiple scattering results obtained by setting the absorption amplitude $T^{\mathrm{abs}}_{\pi d,\pi d}$ of Eq. (2.10) to zero. It is seen that only the differential cross sections at large angles are significantly changed by

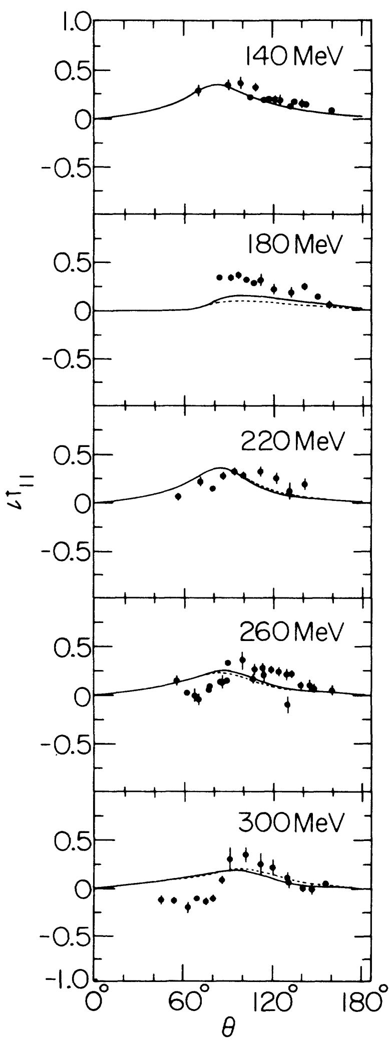


FIG. 12. Same as Fig. 11, but for the vector analyzing power.

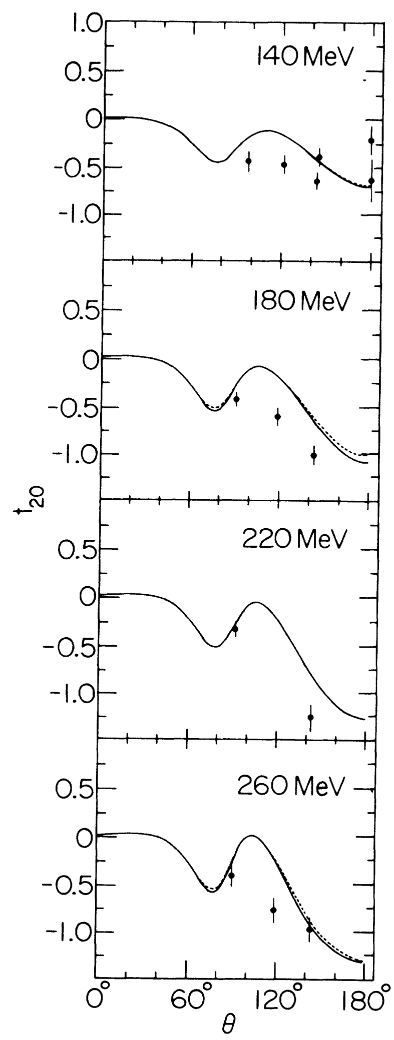


FIG. 13. Same as Fig. 11, but for the tensor analyzing power.

the $\pi$-absorption effect (dashed curves). Its effects on tensor and vector analyzing powers are negligible. Agreements of our results with the data are comparable to all previous calculations. Clearly, much work remains to be done to remove the discrepancies between the theory and the data.

## V. DISCUSSION

In this work we have investigated the extent to which the NN and $\pi$d elastic scattering can be described by a meson-exchange $\pi$NN model of the form of Eqs. (1.2). Although the main features of the data have been reproduced, it is clear that none of the constructed models can be considered to be a complete success. In this section we would like to discuss possible sources of the problem and necessary future works.

First, it is likely that the pion production mechanism cannot be completely described by the conventional one-pion-exchange parametrization as shown in Fig. 2. It is possible that the pion can also be produced by some short-range mechanisms which cannot be phenomenologically accounted for by adjusting the range parameters of the pionic form factors of the transition operators $V_{NN\leftrightarrow N\Delta}$ and $F_{\pi NN\leftrightarrow NN}$. For example, we can imagine within the existing quark-pion models[31] that the pion can be produced in the region where two nucleons have lost their own identities and a six-quark "dibaryon" state is formed. If this dibaryon state does not have a large overlap with the N$\Delta$ wave function, then this process cannot be represented by a NN$\rightarrow$N$\Delta$ transition followed by a $\Delta\rightarrow\pi$N decay. Similar arguments can also be put forward to question the adequacy of the one-pion-exchange model of the nonresonant pion production operator $F_{NN\leftrightarrow NN\pi}$. A microscopic construction of such a quark model of pion production could be difficult in practice. A more realistic way of developing a *working* $\pi$NN model is to extend the well-defined one-pion-exchange models for $V_{NN\leftrightarrow N\Delta}$ and $F_{NN\leftrightarrow \pi NN}$ to include some phenomenological terms so that the NN data, such as polarization data $\Delta\sigma_L^{\text{tot}}, \Delta\sigma_T^{\text{tot}}$, can be fitted. Since the results obtained from the present model are already very close to the data, the search for these phenomenological terms is probably not too difficult in practice.

The second possible source of the problem could be due to the intrinsic deficiencies of the starting low energy NN potential. A common feature of the considered potentials (Paris, Bonn, Argonne $V14$, and Reid) is that their short-range part is determined phenomenologically by fitting the NN data below the pion production threshold. In using the subtraction procedure Eq. (2.9) to define the NN interaction in the presence of pion and $\Delta$ degrees of freedom, we essentially assume that the same phenomenological short-range core is also valid at higher energies. Although this turns out to be rather successful in preserving the good description of the low energy NN phase shifts, it is difficult to decide whether this is a good assumption. For example, within the quark picture, a nonlocal form could be more realistic in representing the short-range baryon-baryon interactions, because of the composite structure of the baryons. A local form of potential could be sufficient for describing low energy phenomena, but could be a bad representation of some essential nonlocal quark effects which are important in the high energy region. Perhaps the problem will be solved if we start our construction of the $\pi$NN model with a low energy potential which has incorporated the nonlocal nature of the baryon-baryon interaction, deduced from a well tested quantum chromodynamics (QCD)- based quark model of the nuclear force. Such a desirable NN model has yet to be constructed.

To close we want to point out that the predictions of the $\pi$NN model based on the Paris potential reproduce the main features of all NN and $\pi$d scattering data. Although further improvements are needed, it can be used as a good starting point for investigating the other $\pi$NN processes listed in Eq. (1.1), the intermediate energy nuclear reactions, and also the questions concerning the presence of $\Delta$ or $\pi$ in nuclei.

## ACKNOWLEDGMENTS

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[1] M. Stingl and A. T. Stelbovics, J. Phys. G **4**, 1371 (1978); **4**, 1389 (1978).

[2] Y. Avishai and T. Mizutani, Nucl. Phys. **A326**, 359 (1979); **A338**, 377 (1980); Phys. Rev. C **27**, 312 (1983).

[3] I. R. Afnan and B. Blankleider, Phys. Rev. C **22**, 1638 (1980); B. Blankleider and I. R. Afnan, *ibid.* **24**, 1572 (1981); I. R. Afnan and R. J. McLeod, *ibid.* **31**, 1821 (1985).

[4] A. W. Thomas and A. S. Rinat, Phys. Rev. C **20**, 216 (1979); A. S. Rinat, Y. Starkand, and E. Hammel, Nucl. Phys. **A364**, 486 (1981); A. S. Rinat and Y. Starkand, *ibid.* **A397**, 381 (1983).

[5] T. Mizutani, C. Fayard, G. H. Lamot, and R. S. Nahabetian, Phys. Lett. **107B**, 177 (1982); C. Fayard, G. H. Lamot, and T. Mizutani, Phys. Rev. Lett. **45**, 524 (1980).

[6] W. M. Kloet and R. R. Silbar, Nucl. Phys. **A338**, 281 (1980); **A338**, 317 (1980); **A364**, 346 (1981); Phys. Rev. Lett. **45**, 970 (1980).

[7] M. Betz and T.-S. H. Lee, Phys. Rev. C **23**, 375 (1981).

[8] M. Araki, Y. Koike, and T. Ueda, Nucl. Phys. **A369**, 346 (1981); **A389**, 605 (1982); T. Ueda, Phys. Lett. **141B**, 157 (1984).

[9] T.-S. H. Lee and A. Matsuyama, Phys. Rev. C **32**, 516 (1985).

[10] I. R. Afnan and B. Blankleider, Phys. Rev. C **32**, 2006 (1985).

[11] H. Pöpping and P. Sauer, in Proceedings of the Symposium on $\Delta$-Nucleus Dynamics, Argonne National Laboratory Report ANL-PHY-83-1, 1983, p. 355; H. Pöpping, P. Sauer, and Zhang Xi-Zhen, University of Hanover Report ITP-UH, 1987.

[12]E. L. Lomon, Phys. Rev. D **26**, 576 (1982).
[13]T.-S. H. Lee, Phys. Rev. Lett. **50**, 157 (1983); Phys. Rev. C **29**, 195 (1984); T.-S. H. Lee and A. Matsuyama, *ibid.* **32**, 1986 (1985).
[14]E. E. Van Faassen and J. A. Tjon, Phys. Rev. C **28**, 2354 (1983); **30**, 285 (1984); J. A. Tjon and E. E. Van Faassen, Phys. Lett. **120B**, 39 (1983).
[15]R. A. Arndt, L. D. Roper, R. A. Bryan, R. B. Clark, B. J. VerWest, and P. Signell, Phys. Rev. C **28**, 97 (1983).
[16]M. Lacombe, B. Loiseau, J. N. Richard, R. Vinh Mau, J. Côté, P. Pires, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[17]K. Holinde and R. Machleidt, Nucl. Phys. **A247**, 495 (1975).
[18]R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[19]R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).
[20]I. Afnan and A. W. Thomar, in *Modern Three-Hadron Physics*, edited by A. W. Thomas (Springer-Verlag, Heidelberg, 1977).
[21]The numerical method for solving two-body Lippmann-Schwinger equations in momentum space can be found in many existing publications, for example, M. Haftel and F. Tabakin, Nucl. Phys. **A158**, 1 (1970).
[22]J. H. Hetherington and L. H. Schick, Phys. Rev. **137B**, 935 (1965).
[23]G.-H. Nieplaus, M. Gari, and B. Sommer, Phys. Rev. C **20**, 1096 (1979).
[24]K. Schwarz, H. F. K. Zingl, and L. Mathelitsch, Phys. Lett. **83B**, 297 (1979).
[25]In the study of Ref. 13, $\rho$ exchange is also included, but its effect is very small, since the chosen cutoff $\Lambda_\rho = 650$ MeV/$c$ is very close to the $\rho$ mass 760 MeV.
[26]A recent compilation of NN and $\pi$d data is given in a review article by M. P. Locher, M. E. Sainio, and A. Svarc, Adv. Nucl. Phys. **17**, 47 (1986), and references therein.
[27]N. Giraud, Y. Avishai, C. Fayard, and G. H. Lamot, Phys. Rev. C **19**, 465 (1979); N. Giraud, C. Fayard, G. H. Lamot, *ibid.* **21**, 1959 (1980).
[28]A. S. Rinat, Y. Starkand, E. Hammel, and A. W. Thomas, Phys. Lett. **80B**, 166 (1979); C. Fayard, G. H. Lamot, and T. Mizutani, Phys. Rev. Lett. **45**, 524 (1980).
[29]H. Garcilazo, Phys. Rev. Lett. **45**, 780 (1980); Nucl. Phys. **A360**, 411 (1981).
[30]A. Matsuyama and K. Yazaki, Nucl. Phys. **A364**, 477 (1981).
[31]A. W. Thomas, Adv. Nucl. Phys. **13**, 1 (1984).