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Correction Factors of the Resonance Frequency for Tapering and Shear Deformation of a Log in Flexural Vibration*1

Nobuo SOBUE*2

原木の細りとせん断変形に対するたわみ振動の共振周波数の補正係数*1

祖父江信夫*2

ヤング率に基づく原木丸太の選別に，たわみ振動法を適用するための検討を行なった。

丸太のように長さ方向に断面形状が変化する材料の振動問題は，変断面はりの振動問題と呼ばれ，振動方程式の解は簡単には得ることができない。そこで，数値解法の一つである伝達マトリックス法を用いて振動方程式を解き，両端を単純支持された丸太のたわみ共振周波数におよぼす丸太の細りと丸太直径の影響について検討した。その結果から，丸太の細り β（7式）に対する共振周波数の低減係数 α（10式）として，Table 2，3の値を得た。

次にこの低減係数と丸太と平均直径が等しい一様な円柱はりの振動理論式（9式）を適用して，丸太の正しいヤング率を求める計算式（11式）を導いた。

また，丸太のヤング率におよぼすせん断変形の影響についても数値的に検討した。

The effects of the tapering of a log and the log diameter on the resonance frequency of the flexural vibration of a tapered log were studied by using a transfer-matrix method to apply the flexural-vibration technique to the sorting of logs based on Young's modulus.

A simple formula to evaluate the correct Young's modulus of tapered logs from the vibration theory of an homogeneous beam is given by introducing the correction factor for the decrease of resonance frequency due to the tapering of the log.

The effect of the occurrence of shear effect on the Young's modulus of a log also was studied.

Keywords: logs, flexural vibration, tapering of logs, Young's modulus, transfer-matrix method.

1. INTRODUCTION

Recently in Japan, wood is to be used in large scale wood-construction, and the reasoned use of wood has been required.

The present work is intended to extend the application of a vibration technique to the sorting and grading of logs in non-destructive tests. It deals with Young's modulus measurement of the flexural vibration of tapered logs with both ends simply supported.

The vibration of a log belongs to the category of the problem of bars with variable cross-sections in the vibration theory. Therefore, the vibration equation becomes a differential equation which generally is unsolvable by formal integration because functional coefficients are involved in the equation. Some numerical solutions of the vibration equation of tapered beams have been given by the Rayleigh-Ritz

*1 Received July 6, 1989.

*2 名古屋大学農学部 School of Agriculture, Nagoya University, Nagoya 464-01

method for some simple cases in engineering problems such as the flexural vibrations of airplane propellers, ships, and turbine blades.[1)]

In this paper, the effects of the tapering of a log and the log diameter on the decrease of the resonance frequency is discussed by using a transfer-matrix method because this method is easy to apply to these complicated vibration problems. The goal of this work is to obtain the correction factor for the calculation of Young's modulus of a log which enables the application of the simple homogeneous-beam vibration theory to tapered logs.

2. THEORY

2.1 Fundamentals of the transfer-matrix method[2)]

A beam in flexural vibration is considered as a linear combination of spring elements and mass elements as shown in Figure 1.

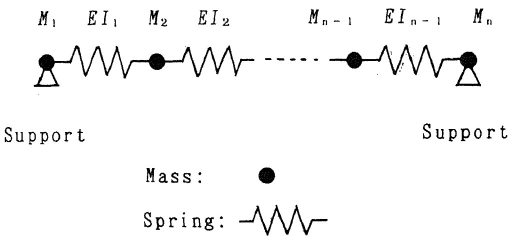

Fig. 1. Linear combination of elements.

A field matrix $[T_F]$ is defined as a matrix which relates the state of both edges of a spring element without mass considering the effect of deflection by a shearing force.

$$[T_F]=\begin{vmatrix} 1 & l & l^2/(2EI) & l^3/(6EI)-kl/(GA) \\ 1 & 1 & l/EI & l^2/(2EI) \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (1)$$

Where EI is the stiffness of a beam, l is the length, G is a shear modulus, A is the cross-sectional area, and k is a shear-deformation coefficient.

A point matrix $[T_P]$ is defined as a matrix which relates the state of both edges of a mass element.

$$[T_P]=\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \underline{M}\omega^2 & 0 & 0 & 1 \end{vmatrix} \quad (2)$$

ere $\underline{M}$ is mass, and ω is a circular frequency.

he transfer matrix of an entire beam $[N]$ is given by multiplying the field matrix and the point matrix in turn.

$$\begin{vmatrix} y_n \\ \theta_n \\ M_n \\ Q_n \end{vmatrix}=[T_{Pn}][T_{Fn}][T_{Pn-1}][T_{Fn-1}]\cdots[T_{P1}]\begin{vmatrix} y_1 \\ \theta_1 \\ M_1 \\ Q_1 \end{vmatrix} \quad (3)$$

$$=[N]\begin{vmatrix} y_1 \\ \theta_1 \\ M_1 \\ Q_1 \end{vmatrix} \quad (4)$$

where $[N]$, y, θ, M, and Q are the (4×4) matrix, deformation, slope of a beam, moment, and shear force, respectively. The subscripts 1 and n are the position numbers of both ends of the beam.

The boundary condition of a simply supported beam is:

$$y_n=y_1=0 \text{ and } M_n=M_1=0 \quad (5)$$

The resonance frequency is solved for a boundary condition by substituting Equation 5 into Equation 4. Then, the non-trivial solution of the resonance frequency is given by the following equation:

$$\begin{vmatrix} N_{12} & N_{14} \\ N_{32} & N_{34} \end{vmatrix}=0 \quad (6)$$

2.2 Numerical calculation

2.2.1 Accuracy of the solution

The accuracy of the solution depends on the number of elements considered. In this work, the number of field matrices were 60. The accuracy was examined as a homogeneous cylindrical beam. The difference of the resonance frequency between the transfer-matrix method and the analytical solution by the elementary vibration theory was less than 0.002%.

2.2.2 Parameters for the calculation

In this study, the three conditions of the calculation were examined which correspond to sugi (*Cryptomeria japonica* D. Don), hinoki (*Chamaecyparis obtusa* Endl.), and Douglas-fir ((*Pseudotsuga menziesii* (Mirb.) Franco)). Two length of logs, 3 m and 4 m, were employed considering the actual application to commercial logs.

The values of Young's modulus E, shear modulus G, and density ρ used for each species[3,4)] are shown in Table 1. The shear deformation coefficients k used were 6/5 and 4/3.[5)]

The tapering of log β was considered to be between 0 to 3 cm/m, where the tapering of the log was defined as the decrease of the log diameter per unit of log length:

Table 1. Elastic and physical constants of models.

Assumed species	Young's moduli (GPa)	Shear moduli (GPa)	Densities (g/cm^3)
Sugi	7.0	0.47	0.40
Hinoki	9.0	0.84	0.45
Douglas-fir	11.0	0.79	0.50

$$\beta=\frac{d_b-d_t}{l} \quad (\text{cm/m}) \tag{7}$$

where d_b and d_t are the log diameters at the bottom and the top, respectively, and l is the log length.

The mean log diameter $\bar{d}$ is defined as;

$$\bar{d}=\frac{d_b+d_t}{2} \tag{8}$$

2.3 Definition of the correction factor and its application to Young's modulus determination of logs

The simulation results then are used for practical application to evaluate the Young's modulus of a log.

In the transfer-matrix method, a Young's modulus of a log can be calculated directly by solving Equation 6. However, it is time consuming and is not a proper way to apply in factories and fields for the sorting of logs.

Then, the following correction factor of the resonance frequency α is introduced: Young's modulus E is given for a beam with a homogeneous cross-section by:

$$E=\frac{4\pi^2 A l^4 \rho f r_h^2}{m^4 I} \tag{9}$$

where A: areas of the cross-section,

l: length of a beam,

ρ: density,

fr_h: resonance frequency,

I: moment of inertia,

m: a constant given for each vibration mode (m$=\pi$ for the fundamental vibration mode of a simply supported beam).

Then, I consider the way to apply the simple theory of vibration of a homogeneous beam to a tapered log.

The resonance frequency of a log fr calculated by the transfer-matrix method is related with that of a homogeneous cross-section beam with the mean log diameter of the same log by introducing the following correction factor α:

$$fr=\alpha\, fr_h, \tag{10}$$

where α is the functions of the cross-section and the tapering of a log.

Accordingly, Young's modulus of a log is given by

$$E=\frac{4\pi^2 l^4 \rho}{m^4}\frac{A}{I}\frac{fr^2}{\alpha^2} \tag{11}$$

where A and I are the cross-section and moment of inertia, respectively, of the homogeneous beam with the mean log diameter.

3. RESULTS AND DISCUSSIONS

3.1 Effect of the tapering of a log and the log diameter on the correction factor

Tables 2 and 3 show the effects of the taperings of logs and the mean log diameters on the correction factors of the resonance frequency. The correction factor was not affected by the consideration of shear deformation in a log; namely, not only by the E-to-G ratio but also by the shear-deformation constant. Thus, the correction factor obtained is applicable to all species considered. Then, the corrected Young's modulus, that is not a pure Young's modulus because the effect of the shear deformation in a log is involved, can be calculated from Equation 11 by using the correction factor and the resonance frequency obtained by experiments.

The correction factor decreased with decreases of the mean log diameter and with increases of the

Table 2. Correction factors of resonance frequency α for logs of 3 m lengths.

Mean log diameters (cm)	Taperings of log β (cm/100 cm) 0.5	1.0	1.5	2.0	2.5	3.0
10	0.997	0.988	0.973	0.951	0.923	0.887
15	0.999	0.995	0.988	0.979	0.966	0.951
20	0.999	0.997	0.993	0.988	0.981	0.973
25	1.000	0.998	0.996	0.992	0.988	0.983
30	1.000	0.999	0.997	0.995	0.992	0.988
35	1.000	0.999	0.998	0.996	0.994	0.991
40	1.000	0.999	0.998	0.997	0.995	0.993

Table 3. Correction factors of resonance frequency α for logs of 4 m lengths.

Mean log diameters (cm)	Taperings of log β (cm/100 cm) 0.5	1.0	1.5	2.0	2.5	3.0
10	0.995	0.979	0.951	0.912	0.859	0.791
15	0.998	0.991	0.979	0.962	0.940	0.912
20	0.999	0.995	0.988	0.979	0.966	0.951
25	0.999	0.997	0.992	0.986	0.978	0.969
30	0.999	0.998	0.995	0.990	0.985	0.978
35	1.000	0.998	0.996	0.993	0.989	0.984
40	1.000	0.999	0.997	0.994	0.992	0.988

tapering of the log. The result showed that the decrease of the resonance frequency by the tapering of a log was not so much in the limit of the practical use because the tapering of a log generally is smaller than about 2 cm/m.[6] Assuming that the allowable error in the calculation of Young's modulus using the homogeneous-beam theory is 2.5%, the limit of the decrease of the resonance frequency becomes within 1.25% because a Young's modulus is proportional to the square of the resonance frequency ; namely, the threshold level of α where the homogeneous-beam theory is applicable is 0.9875. In a case when α is less than the threshold level, the Young's modulus should be calculated by using Equation 11 and the correction factors given in Tables 2 or 3. The intermediate values in the Tables can be obtained by interpolating the adjacent two values.

3.2 Effect of the shear deformation on Young's modulus

The resonance frequency of flexural vibration is influenced by the occurrence of shear effect in the beam. The magnitude of the decrease of the resonance frequency is affected by the slenderness ratio (mean log-diameter/log-length) and the E-to-G ratio of a log as shown in Table 4 in the case of logs of 3 m lengths.

The apparent Young's modulus decreases with increases of log diameters. The calculation of the pure Young's modulus in flexural vibration may not have an important meaning from the viewpoint of the actual sorting or grading of logs. However, the effect of shear deformation in flexural vibration should be considered when we compare the apparent Young's modulus measured by the flexural vibration with that measured by the longitudinal vibration because the latter is not affected by the shear effect.

Table 4. Decreases of the apparent Young's modulus due to the effects of shear deformations in logs.

Mean log diameters (cm)	Decreases of apparent Young's modulus (%)		
	Sugi	Hinoki	Douglas-fir
10	1.2	0.8	1.2
15	2.8	1.8	2.6
20	4.8	3.6	4.6
25	7.6	5.4	7.0
30	10.8	7.8	10.0
35	14.4	10.6	13.6
40	18.8	13.6	17.6

Note : calculated for logs of 3 m lengths.

3.3 Application of the numerical correction

The numerical correction was applied to eight green logs of hinoki (*Chamaecyparis obtusa* Endl.) whose lengths and mean diameters, excluding bark thickness, were 303 to 309 cm and 15 to 22 cm, respectively.

The Young's moduli of logs with bark were measured by the both-ends simply-supported flexural-vibration method, and after having been sawn into lumber whose cross-section was 10.9 cm × 10.9 cm the Young's moduli were again measured. The vibration was detected by a piezo-electric sensor located at the center of the specimen, and the resonance frequency was determined by using a Fast Fourier Transformation spectrum analyzer.

Figure 2 shows the relationships between the Young's moduli of logs and those of lumber before applying the numerical corrections. The Young's moduli of logs were smaller by about 8.5% than those of the lumber.

Table 5 shows the results of the application of the numerical corrections. The slenderness ratios of logs and lumber were about 1/18 and 1/28, respectively. Therefore, the effect of the shear deformation cannot be ignored in comparing their Young's moduli. The correction for shear deformation was made by using the mean diameter calculated from the top- and bottom-diameter of each log and the depth of the lumber, and the correction factors in Table 4 were

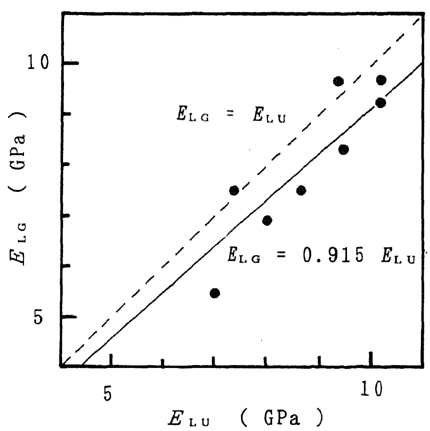

Fig. 2. Relationships between Young's moduli of logs E_{LG} and those of lumber E_{LU}.

Table 5. Results of corrections.

Log no.	E_{LG} (GPa)	E_{LU} (GPa)	E_{LG}/E_{LU} (1)	(2)	(3)	β (cm/m)
1	5.508	6.973	0.790	0.821	0.837	1.70
2	6.925	7.991	0.867	0.891	0.908	1.42
3	7.490	8.629	0.868	0.895	0.905	1.20
4	7.516	7.381	1.018	1.045	1.055	1.17
5	8.314	9.440	0.881	0.906	0.919	1.30
6	9.254	10.16	0.911	0.933	0.940	0.87
7	9.682	9.360	1.034	1.056	1.068	1.12
8	9.716	10.21	0.952	0.973	0.979	1.75
Min.	5.508	6.973	0.790	0.821	0.837	0.87
Max.	9.716	10.21	1.034	1.056	1.068	1.70
Ave.	8.051	8.768	0.915	0.940	0.951	1.32

Legend: E_{LG}: Young's modulus of a log, E_{LU}: Young's modulus of lumber, (1) : no correction, (2) : after correction for shear deformation, (3) : after corrections for shear deformation and tapering of log, β : tapering of log.

used. By these corrections, the differences of Young's moduli between the logs and the lumber decreased by 2.5% on average.

The effect of the correction for the tapering of a log was about 1% on average. This effect was smaller than that of the shear deformation.

The total amount of the corrections for the shear effect and for the tapering was 3.5% on average. However, the deviation of the data to which both corrections were applied was larger than the amount of the correction for tapering. This fact occurred probably because the physical properties vary in the radial direction of a log, and because the cross-section of log was out of roundness.

Consequently, the effect of the tapering of a log was negligible in the practical measurement of Young's modulus of the log; namely, we can treat a log as a homogeneous cylindrical beam whose diameter is equivalent to the mean diameter of a log for the purpose of sorting and grading logs.

4. CONCLUSIONS

The effects of the tapering of a log, the log diameter, and the shear deformation on the resonance frequency in flexural vibration of tapered logs were studied by using the transfer-matrix method.

The obtained correction factor of the resonance frequency which is the function of the tapering of a log and the log diameter enabled the application of the simple vibration theory of flexural vibration of a beam with a homogeneous cross-section to tapered logs (Equation 11). The effect of shear deformation was greater than that of the tapering of a log in usual cases.

The results obtained gave the theoretical base for the application of the simple flexural-vibration technique to tapered logs.

A limited experiment proved that the correction for tapering of a log was negligible in the practical sorting and grading of logs except for extremely tapered logs.

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