Note

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Instantaneous Measurement of Elastic Constants by Analysis of the Tap Tone of Wood

Application to flexural vibration of beams*[1]

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木材の打音のFFT分析による弾性定数の瞬間測定

——梁の曲げ振動への適用 *[1]——

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両端自由はりの打撃音から，木材のヤング率 E とせん断弾性係数 G を同時に，瞬間的に求める方法を検討した。FFT スペクトルアナライザによる打音の周波数分析から得られる固有振動数とはりのせん断変形および回転慣性を考慮したチモシェンコはり理論により，E と G を分離して求めた。はりの支持方法，効率的なはりの打撃位置，また曲げ振動の高次の固有振動数を分析スペクトルから自動的に選別する計算プログラム，および FM 電波による打撃音の送信法についても検討した。簡便な打撃法と合わせ，FFT アナライザと 16 ビット・パーソナルコンピュータの結合によって，E および G を計測するための高速自動処理が可能となることが確かめられた。

An instantaneous measurement for obtaining a Young's modulus (E) and a shear modulus (G) simultaneously from the tap tone of flexural vibrations of wooden beams was investigated. The resonance frequencies were obtained by an instantaneous frequency analysis of the tap tone using a FFT (Fast Fourier Transformation) spectrum analysis. The Timoshenko theory of flexural vibration taking the occurrence of shear and rotatory inertia effects into account was applied in the calculation of E and G. The hold position of a beam, the effective tap position to excite its vibration, the computer program to identify automatically the resonance frequencies of higher vibration modes from the original spectrum, and the transfer method of a tap tone by FM radio waves were investigated.

It was proved that a rapid and automated procedure to measure E and G was possible by the connection of a FFT analyzer and a 16-bit microcomputer.

1. INTRODUCTION

An elastic constant is one of the practical and principal strength predictors in nondestructive evaluation of wooden beams. Development of a rapid and simple testing method would contribute to the quality evaluation of woods by vibration methods.

This paper aims the development of a system for instantaneous measurement of Young's modulus (E)

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and shear modulus (G) of wooden beams by a flexural vibration method. The designed system has the following characteristics: (1) detection of a vibration by the tap tone of a beam, (2) instantaneous FFT (Fast Fourier Transformation) spectrum analysis, (3) application of the Timoshenko theory of a flexural vibrating beam to a simultaneous measurement of E and G, and (4) system control by a microcomputer. By connecting these, an automated and instantaneous measurement of the elastic constants of wooden beams was attempted.

2. EXPERIMENT

2.1 Materials

The experiment was made on specimens of hinoki (*Chamaecyparis obtusa* S. et Z.), buna (*Fagus crenata* Bl.), keyaki (*Zelkowa serrata* Mak.), and shirakashi (*Quercus myrsinaefolia* Bl.). Specimens of spruce (*Picea* sp.), western redcedar (*Thuya plicata* Donn), Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco), and meranti (*Shorea* sp.) also were used. The grain direction of beams was parallel to their lengths. The dimensions and the densities are given in Table 1 for the Japanese species.

Table 1. Dimensions and densities of specimens.

Specimen	Length (cm)	Thickness (cm)	Width (cm)	Density (g/cm^3)
Hinoki	40.1	1.03	2.50	0.368
Buna	40.2	1.03	2.50	0.635
Keyaki	40.2	1.03	2.50	0.651
Shirakashi	37.2	1.03	2.50	0.672

2.2 Measuring system

The measuring apparatus is shown in Figure 1. Besides a microphone, this system was made up of a microphone amplifier, a FFT spectrum analyzer (CF-910; Ono Sokki Co.), and a 16-bit microcomputer (PC-9801; NEC). The FFT analyzer was coupled to the microcomputer by a GP-IB interface board.

The specimens were held vertically and lightly by the fingers at the nodal points of vibrations, while they were tapped with a small plastic hammer. The tap tone was detected at an end of the beam by the microphone. The inherent error of the resonance frequency measurement by the FFT analyzer was 6.25 Hz in this work.

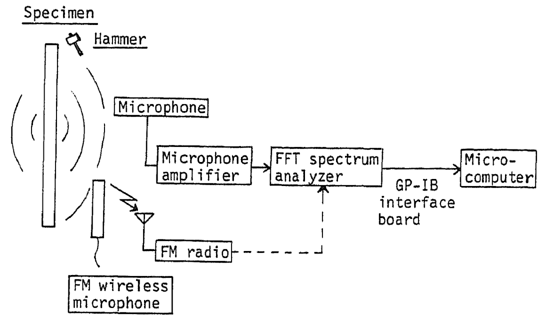

Fig. 1. Measuring apparatus.

2.3 Methods

2.3.1 Hearmon's method on the free-free vibration of beams

The full differential equation of a flexural vibration taking the occurrence of shear and rotatory inertia effects into account was proposed by Timoshenko.[1] According to Hearmon's solution on free-free vibration beams,[2]

$$E = \frac{4\pi l^2 \rho}{M_m{}^4 i^2} f_r{}^2 \cdot T, \text{ and}$$

$$T = 1 + \frac{i^2}{l^2} F_1(m) + \frac{i^2}{l_2} F_2(m) \left(1 - \frac{M_m{}^4 i^2}{F_2(m) l^2 T}\right) \cdot \frac{sE}{G} \quad (1)$$

where E = Young's modulus,
G = shear modulus,
i = radius of giration of cross-section,
ρ = density,
f_r = resonance frequency,
l = length of a beam,
s = shear deformation coefficient, and
$F_1(m)$, $F_2(m)$, and M_m are constants determined by vibration modes.

To obtain the values of E and G, provisional values of sE/G and T are given to the above equation; then the calculations are repeated until both values of E and sE/G converge on each constant using each improved value of sE/G and T.

The final values of E and G are obtained by applying the method of least squares to Equation 1.

The value of s for wooden beams has been discussed by Hearmon[2] and by Nakao and others[3] by experimental comparisons between flexural and torsional vibration tests. Hearmon obtained $s=1.06$ on average. In the recent work of Nakao and others, they obtained $s=1.18$ from vibration tests in which the support conditions of the beams were carefully considered. Thus, the $s=1.18$ obtained by Nakao was used in this work.

2.3.2 Computer program

Particular consideration was given to identify the resonance frequencies of the flexural vibrations from the original data in which the data of false peaks by noises were included, because the FFT analyzer can not discriminate the true peaks of flexural vibrations. The resonance frequencies therefore were searched successively from those of lower vibration modes by solving the Timoshenko equation at each step of the search of their higher vibration modes as follows:

First, the resonance frequency of fundamental vibration was searched for as the minimum value of the frequency data.

The resonance frequency of the 2nd vibration mode was searched from the result of the calculation of Equation 1 under the condition $5<sE/G<60$. Then the provisional value of sE/G for the 3rd vibration mode was calculated, and the estimated resonance frequency of the 3rd vibration mode (f_{r_3}') was obtained (Figure 2).

The search for the resonance frequency of the 3rd vibration mode was made as follows:

First, the frequencies which satisfied the condition $0.9f_{r_3}'<f_r<1.1f_{r_3}'$ were searched. Usually, a plural number of frequencies were found in the higher vibration mode. The resonance frequency was determined by what had the maximum intensity within the searched peaks. Then the estimated value of the 4th

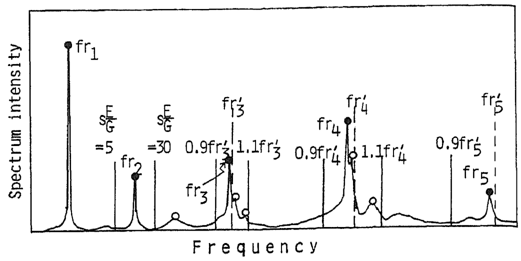

Fig. 2. Schematic diagram of the search process of resonance frequencies of higher-order vibration modes.

Notes: fr'_n is the estimated resonance frequency of the n-th vibration mode. Black dots are the true resonance frequencies of flexural vibrations. White dots are the false peaks by noises.

vibration mode was obtained in the same way as in the 3rd vibration mode.

The same procedure were repeated for the higher vibration modes until the 5th vibration mode.

3. RESULTS AND DISCUSSIONS

3.1 Effect of the hold position of a beam on the resonance frequencies

The support position of a beam affects the resonance frequency a little in low vibration modes.[2,5]

Table 2 compares the resonance frequencies when a beam is held at the nodal points of five vibration modes. The values in parentheses can be considered as the standard resonance frequencies of the vibration modes because their nodal points coincide with their hold positions on a beam. The difference of fre-

Table 2. Effect of the hold position of a keyaki beam on the resonance frequency.

Hold position is nodal point of N-th vibration mode	Measured resonance frequency (Hz)				
	1st	2nd	3rd	4th	5th
1st	(293.75)	793.75	1525.00	2456.25	3562.50
2nd	293.75	(793.75)	1531.25	2462.50	3562.50
3rd	293.75	800.00	(1531.25)	2456.25	3562.50
4th	293.75	793.75	1525.00	(2462.50)	3562.50
5th	293.75	793.75	1525.00	2456.25	(3562.50)
$\Delta f/f_r$ (%)	0	0	0.41	0.25	0

Note: $\Delta f/f_r$ means the relative error of the resonance frequency of each vibration mode when a beam is held at the nodal point of fundamental vibration against the standard value of the resonance frequency marked with parentheses.

quencies between the standard values and the measured values was within 6.25 Hz. This is the inherent error of the frequency measurement of the FFT analyzer used. Accordingly, there was no significant effect of the hold position of a beam on the measured resonance frequency.

3.2 Effect of the tap position on power spectra

Figure 3 shows the relationship between the spectrum intensity of each vibration mode and the tap positions in keyaki when the beam was held at the nodal point of fundamental vibration. It shows that the spectrum intensities are strong when a beam is tapped on a loop of its vibration mode, and that the spectrum intensities diminish or disappear when it is tapped at a nodal point.

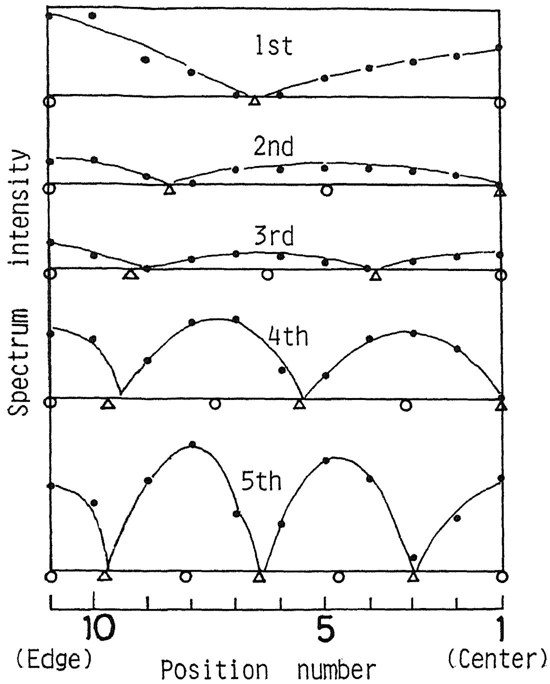

Fig. 3. Relationship between spectrum intensity of each vibration mode and tap position for keyaki.
Notes: △, nodal point; ○, loop.

Considering these facts, the following two manners of tapping were investigated to obtain a clear power spectrum of flexural vibration. One was a tap at an end of a beam because all vibration modes of free-free beam have their loops at the end of a beam. However, the spectra intensities of 2nd and 3rd vibration modes were weak, and in the case of mistaps, the intensity of the 2nd vibration mode was confused with noises. This misreading of the data could be avoided by a check of the spectrum in the monitor TV or by an averaging procedure of multiple taps for the spectrum.

The more reliable manner was the following "two-times tapping". As shown in Figure 3, the center of a beam coincides with the loops of the odd-numbered vibration modes, and the position between position numbers 3 and 4 nearly coincides with the loops of the even-numbered vibration modes. Figure 4 (a) shows the result of a tap at the center of a beam; odd-numbered modes are dominant. Figure 4 (b) shows the spectrum when a beam was tapped at the position between the position numbers 3 and 4; strong peaks of even-numbered modes are seen. Figure 4 (c) shows the sum of the two spectra. This operation gave a clear spectrum of flexural vibration. Here, a beam was held at its center when it was tapped for even-numbered vibration modes because

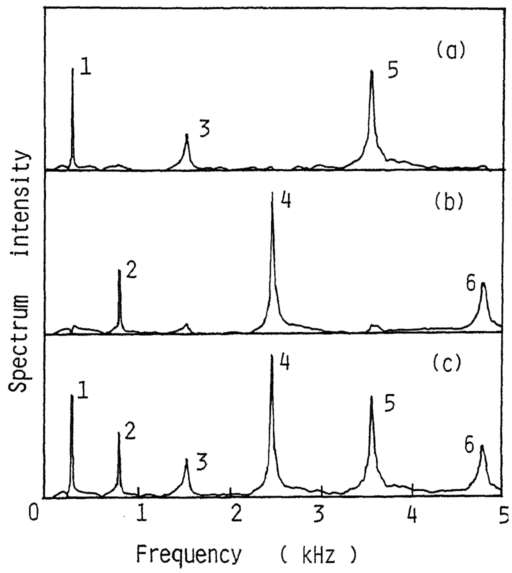

Fig. 4. Spectra patterns of the two-times tapping procedure process for keyaki.
(a) Tap position is the center of the beam.
(b) Tap position is centered between tap positions 3 and 4 in Figure 3.
(c) Sum of spectra (a) and (b).

the center coincides with the nodal points of all even-numbered vibration modes. A "two-times tapping" therefore was made in the following tests.

3.3 Calculation of E and G

Figures 5 and 6 show the results of the calculations

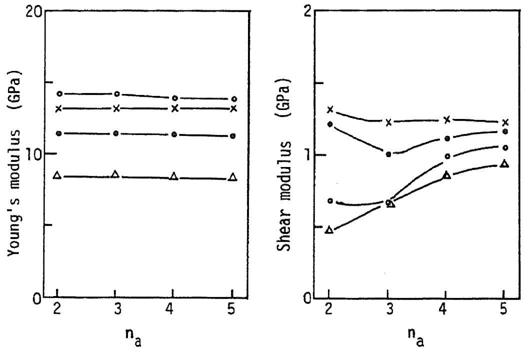

Fig. 5. Relationship between Young's modulus or the shear modulus and the resulting number of the averaging in the least squares procedure (n_a) in the case of a flatwise tap.
Legend: △, hinoki; ○, buna; ×, keyaki; ●, shirakashi.

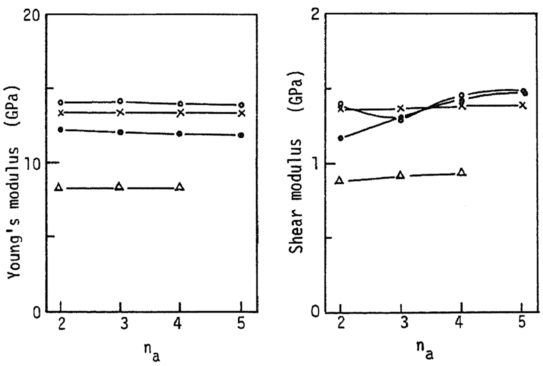

Fig. 6. Relationship between Young's modulus or the shear modulus and the resulting number of the averaging in the least squares procedure (n_a) in the case of an edgewise tap.
Note: For legend refer to Figure 5.

of E and G in the cases of a flatwise tap (a beam was tapped on a broad face) and of an edgewise tap (a beam was tapped on a narrow face), respectively. Their abscissae are the resulting numbers of the averaging in the least squares procedure (n_a) in Hearmon's method.

The Young's modulus was not affected by n_a. On the other hand, the shear modulus was affected very much in some cases by n_a. However, the value had a tendency to converge to a constant (G_f) with an increase of n_a.

The constant G_f was compared with the shear modulus (G_t) which was obtained directly by the free-free torsional vibration test. The G_t was calculated from the anisotropic theory of torsional vibration as Nakao and others[3] made. In spite of the change of the shear modulus with n_a, the value of G_f agreed very much with G_t as shown in Figure 7. The

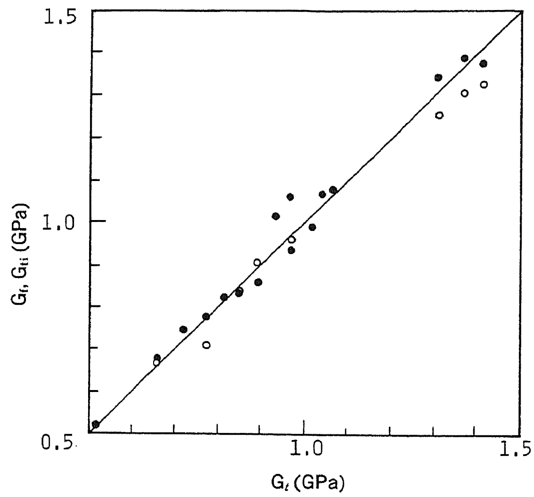

Fig. 7. Comparisons of the shear modulus obtained from flexural vibration (G_f) and that obtained from the isotropic theory of torsion (G_{ti}) with that obtained from the anisotropic theory of torsion[3] (G_t).
Note: The above values were measured using the same specimens and four additional specimens of spruce, western redcedar, Douglas-fir, and meranti prepared for this purpose.
Legend: ●, G_f; ○, G_{ti}.

ratio G_f/G_t was 1.013. This agreement is not an unexpected coincidence because the value $s=1.18$ obtained by Nakao and others is the experimental value which was calculated by the same experimental comparison as in this work. Furthermore, this value agrees very much with the theoretically estimated $s(s=1.1\text{-}1.2)$,[2,4] but the value $s=1.18$ should be considered as an experimental coefficient. The value of s which was calculated inversely from the results of this work is 1.17. This agrees very much with that obtained by Nakao and others.[3]

To calculate the shear modulus of such anisotropic material as wood, it is necessary to know the ratio of the two shear muduli concerning the torsion, for example G_{55}/G_{44}. This procedure is troublesome or, sometimes, difficult. Therefore, usually the shear moduli are calculated from the isotropic theory of torsion.

The shear modulus (G_{ti}) which was calculated from the isotropic theory also is compared with G_t in Figure 7. The G_{ti} is smaller than the G_t by about 3%. Comparing G_{ti}/G_t with G_f/G_t, they are 0.974 and 1.013, respectively, and their standard deviations are 0.0349 and 0.0381, respectively. Accordingly, there is not much difference in accuracy between the calculations by the isotropic theory of torsion and those by the theory of flexural vibration.

It has been noted that s depends on the grain direction of wood. Therefore, the value $s=1.17$ obtained in this work should be applied within the limit that the grain direction of a beam is parallel to its length. However, this would be the most probable case in nondestructive tests on wooden beams.

The operation time of this system is an important factor from the viewpoint of instantaneous measurements. The reading time of tap-tone data by the FFT analyzer is 160 mS, and the FFT analysis time and the transfer time of the data from the FFT analyzer to the computer are both about 200 mS. Therefore, the rate-determining stage of this system is that of the calculation of the elastic constants. This calculation time was two seconds when the averaging procedure was made up to the 5th vibration mode. This speed would become faster by improvements in the calculation program.

An improved method of the tap-tone transfer by FM radio waves also was investigated using a FM wireless microphone and the usual FM radio. A transfer of about 20 m was obtained in a building. This way is convenient not only in lumber yards but also in laboratories.

4. CONCLUSIONS

A new system for measuring the Young's modulus and the shear modulus by the flexural vibration of a beam was designed. An automated and instantaneous measurement of these constants was obtained by connecting a tap-tone measurement, a FFT spectrum analysis, the Timoshenko theory of flexural vibration of beams, and the system control of a microcomputer.

The equipment for measuring the vibration of beams were simplified very much compared with that of steady-state vibration methods by applying a tap to a beam for exciting a vibration and the detection of the tap tone with a microphone.

The shear modulus obtained by this system agreed very much with that obtained by the torsional vibration method of anisotropic bodies. The measurement of the shear modulus by this system is recommended as a convenient means when the application of torsional vibration is difficult.

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