Discontinuous homomorphisms on C(X) and the forcing axioms for EPC

（課程博士・様式７）

学 位 論 文 要 旨

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論文要旨：

The existence of a discontinuous homomorphism on the Banach algebra $C(X)$ of all complex-valued continuous functions on a compact Hausdorff space X is a long-standing problem, which is in the context of normed algebras and of automatic continuity. In 1949, Kaplansky showed that any algebra norm on $C(X)$ is greater than or equivalent to the supremum norm. Bade and Curtis showed that the existence of an algebra norm on $C(X)$ is equivalent to the existence of a discontinuous (algebra) homomorphism on $C(X)$. Dales and Esterle independently showed that the continuum hypothesis implies the existence of a discontinuous homomorphism. However, Woodin showed that there exists a model in which every homomorphism on $C(X)$ is continuous and Martin's axiom holds. Thus the existence of a discontinuous homomorphism is independent from ZFC. Furthermore, in 1993, Woodin showed that the existence of a discontinuous homomorphism is consistent even if assuming the negation of the continuum hypothesis. More specifically, Woodin constructed a model using the ω_2-length finite support iteration of Cohen forcing and showed that an ultrapower of the real number field has the property β_1, which implies the existence of a discontinuous homomorphism. At that time, Woodin asked whether Martin's axiom is consistent with the existence of a discontinuous homomorphism or not. In other words, the independency of the existence of a discontinuous homomorphism on $C(X)$ from Martin's axiom and negation of the continuum hypothesis. This question is the main theme of this paper.

In this paper, two properties of forcing notions are introduced; one is called EPC and the other is called ProjCes(E). The main result is that the forcing axiom for EPC and ProjCes(E) is consistent with the negation of the continuum hypothesis and with the existence of a β_1 ultrapower of the real number field, and hence, with the existence of a discontinuous homomorphism, simultaneously. This is a generalization of Woodin's model construction and the first partial solution to Woodin's question.

Both properties of EPC and ProjCes(E) are quite stronger than the countable chain

condition respectively. Thus the next problem is the existence of non-trivial forcing notions that satisfy both EPC and ProjCes(E) other than Cohen forcing. The uniformization of ladder system coloring is an important forcing in the context of the independency of the Whitehead conjecture of group theory. By an argument using a countable elementary submodel, it can be shown that the ladder system coloring uniformization is EPC. Moreover, the uniformization of a ladder system coloring on a stationary set E is ProjCes(E). Hence, both EPC and ProjCes(E) are non-trivial properties of forcing notions. As a corollary of this result, a new consistency result is given; the existence of a discontinuous homomorphism on $C(X)$, the failure of the Whitehead conjecture, and the negation of the continuum hypothesis. Furthermore, the forcing axiom for σ-centered does not imply the one for EPC.

The main result is given by replacing the countability of Cohen forcing by EPC in Woodin's argument. It is expected that EPC preserves set theoretical objects preserved by Cohen forcing. Indeed, the author showed that Luzin sets of size $\aleph_1$ and the mad family added by Hechler's forcing notion are preserved by any EPC forcing. Furthermore, any EPC forcing of size $\leq \aleph_1$ preserves any non-meager set and any tight mad family. As a consequence, there is a model of forcing axiom for EPC in which the Cichoń-Blass diagram is same as the one in the Cohen model with $\mathfrak{c} = \aleph_2$. Moreover, it can be shown that the forcing axiom for EPC does not imply the one for σ-centered, and hence, the two forcing axioms are mutually independent.

For a more detailed description of the position of EPC among other properties of forcing notions, we use Kunen's forcing notions which fills a gap. A pregap defines a Kunen forcing, and every pregap has its own type. The author completely characterizes EPC among Kunen forcings using the type of the pregap; a Kunen forcing is EPC if and only if each side of the type of the pregap is not ω_1.