（課程博士・様式７）

学 位 論 文 要 旨

専　　攻：　情報科学専攻　　　　　　　氏　名：葛西　陽介

論文題目：　Dowker spaces with normal products from combinatorial principles

論文要旨：

The class of normal spaces is one of the most important subjects in topology, as theories on metric spaces represent. Not only about metric spaces, theories on the normality in products also form a significant field, especially in set theoretical topology. Recall that products of normal spaces are not generally normal as for example ordinal spaces ω_1 and $\omega_1 + 1$ are normal but their product $\omega_1 \times (\omega_1 + 1)$ is not normal (Dieudonné, 1939). Historically, many researchers have attempted to find conditions for the normality in products of normal spaces. In a stream of such investigations, the *Dowker's problem* have arisen. This asks the equivalence of normality and *binormality.* A binormal space is a space for which the product with the unit interval $[0, 1]$ is normal. Since binormality has been frequently required in many theorems of homotopy theory, many researchers hoped that Dowker's problem is affirmative. A normal space which provides a counterexample for the problem is called a *Dowker space.* In 1951 C. H. Dowker has proved that for normal spaces, binormality coincides to countable paracompactness. Therefore the Dowker's problem turns to ask whether normal spaces are always countably paracompact. In fact, Dowker also gave a useful equivalent condition for coutable paracomactness in the same paper.

As many counterexamples in topology, extraordinary techniques have been required to construct Dowker spaces. In 1955, M. E. Rudin has constructed the first example of Dowker space using an ω_1-Suslin tree. But in that era the consistency of the existence of a Suslin tree was not known (this is proved by S. Tennenbaum in 1968). Hence the next subject was that to find a Dowker space which is constructed only using ZFC. Such space is called a *ZFC Dowker space.* The existence of a ZFC Dowker space had become a big problem in topology. About 20 years later since Dowker's problem arised, Rudin has constructed the first ZFC Dowker space. Thus finally Dowker's problem has negatively solved. However, it remains a quite important question: Is there a "small" ZFC Dowker space? By small we mean not only the cardinalities of spaces but also for topological properties that is, the cardinality of basis, local basis , density and etc. The existence of a ZFC Dowker space itself has already been solved. But to find ZFC Dowker spaces with such small properties still now stand as extreme problems. This is called *the small Dowker space problem.* By its definition, the possible minimum cardinality of Dowker spaces is $\aleph_1$ and the following are the best current solutions of small ZFC Dowker spaces: (1) Of size $2^{\aleph_0}$ given by Z. T. Balogh in 1996; (2) Of size $\aleph_{\omega+1}$ given by M. Kojman and S. Shelah in 1998. Note that the space of (2) is obtained as a subspace of Rudin's first ZFC Dowker space, and the sizes of both of the entire space and its neighborhood basis have size $\aleph_\omega^{\aleph_0}$. It is still unknown the existence of a ZFC Dowker space which has any of first countability, local compactness or separability.

On the other hand, by assuming certain set theoretical hypothesis, many and various small Dowker spaces have been constructed. Even then, only a couple examples of Dowker spaces with normal products are known. In 2010, P. Szeptycki presented a Dowker space with normal square, and in fact so is Rudin's first ZFC Dowker space (this is proved by K. P. Hart in 1981). Our motivation is to give more examples of Dowker spaces with normal products. Finally we have proved the following results: (1) Rudin's (truly) first Dowker spaces from Suslin trees have Dowker products; (2) There is a collectionwise normal Dowker space with Dowker square. Hence we newly presented two more examples of Dowker spaces with normal products. Both normality in products are proved using two properties: (a) λ-*additivity*, that is every union of less than λ many closed subsets is closed and hence every subspace of size $\leq \lambda$ is normal; (b) Each space doesn't have any pairs of disjoint dominating closed subsets. However in fact those spaces must have size at least $\aleph_2$ for the normal products. Actually, Szeptycki's space and Rudin's ZFC Dowker space have size $\aleph_2$ and $\aleph_\omega^{\aleph_0}$ respectively. We conclude the thesis by explaining a necessary condition on Dowker spaces to have normal products. Related to this, we state a problem that asks the existence of a Dowker space of size $\aleph_1$ which satisfies the necessary condition.