A Cascade Phase-Sensitive Detector: II—Phase Filter Synthesis

MOHAMMAD MADIHIAN, STUDENT MEMBER, IEEE, TATSUO YAMAMOTO, MEMBER, IEEE, AND KENZO WATANABE, MEMBER, IEEE

***Abstract*—A synthesis procedure of the phase filter is described. The procedure consists of the Fourier-series approximation of the specified transfer function and the mechanization of each term in the series by means of the cascadable phase-sensitive detectors (CPSD's). The prototype filter implemented by four-stage CPSD confirms the procedure. The effect of input noise on the transfer function is analyzed to show that the signal-to-noise power ratio higher than 7.7 dB is necessary for the satisfactory operation of the fifth-order filter.**

Manuscript received January 7, 1981.

The authors are with the Research Institute of Electronics, Shizuoka University, Hamamatsu 432, Japan.

I. Introduction

A PHASE filter is defined as a network which passes a signal essentially unattenuated if its phase is in a specified range and suppresses it otherwise. It plays an important role in the signal processing in the phase domain, just as do conventional filters in the frequency domain. So far, however, no attention has been paid to it. This is due to the phase independence of the impedances presented by the passive networks and also to the absence of the active networks with the appropriate responses for phase filter synthesis.

0018-9456/81/1200-0247$00.75

The cascadable phase-sensitive detector (CPSD) reported recently [1] can provide various phase responses. Using this CPSD, the synthesis of a phase filter has been attempted. Following this introductory section, the synthesis procedure is described in Section II after a brief description of the CPSD. Unlike the frequency, the signal phase is disturbed by the white noise contained in the signal. It follows that the transfer characteristics of the phase filter depend on the input signal-to-noise power ratio. Its threshold value for the satisfactory operation is evaluated in Section III in terms of the phase distribution function. A prototype filter is described in Section IV to confirm the synthesis procedure.

II. Synthesis Procedure

A. Cascadable Phase-Sensitive Detector (CPSD)

The CPSD consisting of the homodyne system, the local oscillator, and the heterodyne system is shown in Fig. 1 [1]. The signal and reference carrier outputs were shown to take the same form as those of inputs, which made it possible to connect this CPSD in tandem. Under the linear law operation of each mixer, which is assured by the reference carrier with a large amplitude, the outputs of nth stage in the multistage connection were given by

$$v_{s(n)} = V_s \prod_{i=1}^{n} F_i \cos\{\theta + \sum_{j=1}^{i} (\Phi_j - \Psi_j)\} \cos \omega_n t \quad (1)$$

$$v_{r(n)} = G_n V_s \prod_{i=1}^{n-1} F_i \cos\{\theta + \sum_{j=1}^{i} (\Phi_j - \Psi_j)\} \cdot \cos\left[\omega_n t - \sum_{j=1}^{n} (\Phi_j - \Psi_j)\right] \quad (2)$$

where V_s and θ are the amplitude and the phase of an input signal, respectively, F_i is the gain of the signal channel, Φ_j and Ψ_j are the phase offset introduced in the reference and signal channels, respectively, in each stage, and G_n is the gain of the reference channel in nth stage.

B. Phase Filter

A basic idea of phase filter is depicted in Fig. 2. The input signal multiplied by the control voltage $H(\theta)$ which shapes the required transfer function produces the output. Hence

$$v_o = H(\theta) v_{\text{in}} \quad (3)$$

where $H(\theta)$ is assumed as 1 for any θ within the passband and 0 elsewhere. The synthesis of a phase filter then reduces to a way to produce the detected output proportional to $H(\theta)$.

Since $H(\theta)$ is a periodic function of θ with the period of 2π, all the phase filters are of bandpass type and the transfer function is symmetrical about the center phase θ_c of the passband. Expanding the transfer function $H(\theta)$ into the Fourier series, we have

$$H(\theta) = \sum_{n=0}^{N} A_n \cos n(\theta - \theta_c). \quad (4)$$

In (4), the series is truncated by the first N terms from the practical point of view. Referring to (1), the phase response

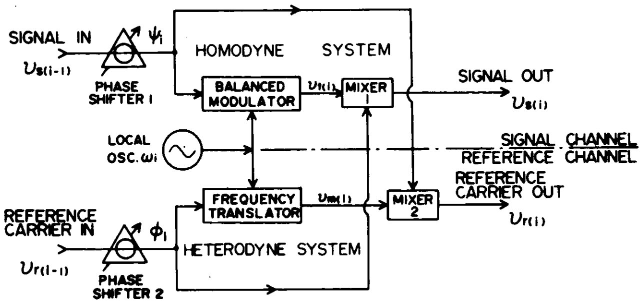

Fig. 1. Schematic diagram of the cascadable phase-sensitive detector (CPSD).

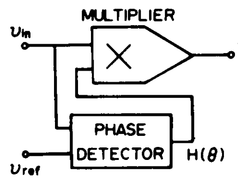

Fig. 2. A block diagram of the phase filter.

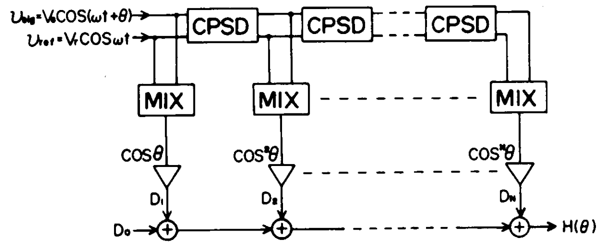

Fig. 3. Transversal-type phase filter.

now available is of the form of not $\cos n\theta$ but $\cos^n \theta$. Hence, expanding $H(\theta)$ in terms of $\cos^n(\theta - \theta_c)$, we have

$$H(\theta) = \sum_{n=0}^{N} D_n \cos^n(\theta - \theta_c) \quad (5)$$

where D_n is the expansion coefficient to be determined. Expressing $\cos^N\theta$ in terms of $\cos n\theta$ as

$$\cos^N\theta = \sum_{n=0}^{N} B_{Nn} \cos n\theta \quad (6)$$

substituting (6) into (5), and comparing the resultant coefficients with those of (4), we have

$$[D_n] \cdot [B_{Nn}] = [A_n] \quad (7)$$

where $[A_n]$ and $[D_n]$ are raw vectors and $[B_{Nn}]$ is a lower triangular matrix the entries of which are easily calculated by the trigonometric formulas. If the specification for the passband and the number of the stages to be used are given, $[A_n]$ and $[B_{Nn}]$ can be determined by (4) and (6), respectively, and hence $[D_n]$ by (7). The center phase θ_c corresponds to $\Psi_1 - \Phi_1$ in (1). It can assume any specified value without affecting D_n, so $\Psi_j - \Phi_j$, and hence θ_c is assumed to be zero in the following discussion.

The mechanization of $H(\theta)$ by means of (5) is straightforward and is shown in Fig. 3. The mixers here operate as phase detectors providing the outputs proportional to $\cos^i\theta$ ($i = 1, 2, \cdots, N$). Scaling each output by the appropriate D_i and summing up provide $H(\theta)$.

III. Effect of Noise on Transfer Function

The phase response described above assumes the noise-free signal. In the presence of noise, the signal phase is disturbed and the transfer function of the phase filter is correspondingly degraded. The degradation is evaluated in the following, under the assumption that the input signal contaminated by the noise is applied to the phase filter after an IF amplification followed by the amplitude limiting.

The input signal to the IF amplifier can be written as

$$\begin{aligned} v(t) &= V_s \cos(\omega t + \theta) + n \cos(\omega t + \alpha_n) \\ &= r_s \cos(\omega t + \theta - \theta_s) \end{aligned} \tag{8}$$

where

$$r_s = \sqrt{V_s^2 + n^2 + 2V_s n \cos(\alpha_n - \theta)} \tag{9}$$

and

$$\theta_s = \tan^{-1} \frac{n \sin(\alpha_n - \theta)}{V_s + n \cos(\alpha_n - \theta)}. \tag{10}$$

Obviously, r_s and θ_s are random variables with the joint probability density function [2]

$$p(r_s, \theta_s) = (r_s/2\pi N_i) \exp[-(r_s^2 + V_s^2 - 2r_s V_s \cos\theta_s)/2N_i], \quad r_s \geqslant 0, \quad \pi \geqslant \theta_s \geqslant -\pi \tag{11}$$

where N_i denotes the noise power. Since the amplitude variation in r_s is limited by the limiter, the input to the phase filter is a constant-amplitude signal with the following density function:

$$\begin{aligned} p(\theta_s) = \int_0^\infty p(r_s, \theta_s) dr_s &= (1/2\pi) \exp(-V_s^2/2N_i) \\ &+ (V_s \cos\theta_s/\sqrt{8\pi N_i}) \exp(-V_s^2 \sin^2\theta_s/2N_i) \\ &+ (V_s \cos\theta_s/\sqrt{2\pi N_i}) \Phi(V_s \cos\theta_s/\sqrt{N_i}) \\ &\cdot \exp(-V_s^2 \sin^2\theta_s/2N_i), \quad \pi \geqslant \theta_s \geqslant -\pi \end{aligned} \tag{12}$$

where

$$\Phi(k) = (1/\sqrt{2\pi}) \int_0^k \exp(-u^2/2)\, du.$$

The input signal power is $V_s^2/2$ and hence $V_s^2/2N_i$ represents the signal-to-noise power ratio (SNR). For a small SNR, θ_s approaches the noise phase α_n and $p(\theta_s) \simeq 1/2\pi$. For an SNR larger than 3 dB, $p(\theta_s)$ can be approximated by

$$p(\theta_s) = (V_s/\sqrt{2\pi N_i}) \cos\theta_s \exp(-V_s^2 \sin^2\theta_s/2N_i), \quad |\theta_s| < \pi/2. \tag{13}$$

The outputs of the nth CPSD in the presence of noise are expressed by (1) and (2) replaced θ by $\theta - \theta_s$. The mean value of the response is then given by

$$\overline{V_{s(n)}} = V_s \int_{-\pi/2}^{\pi/2} \prod_{i=1}^{n} F_i \cos(\theta - \theta_s) p(\theta_s)\, d\theta_s. \tag{14}$$

Fig. 4(a) and (b) plots the phase responses for $n = 5$ and 9, respectively, with SNR as a parameter. As SNR degrades, the peak level of the response decreases and its phase selectivity becomes lost. Also, it is apparent by comparing parts (a) and (b) of Fig. 4 that the degradation is more serious for the latter stage. Defining the threshold value for SNR as that which causes 10-percent reduction of the response peak, we have found that it is approximately given by

$$[\mathrm{SNR}]_{\mathrm{thr}} = 4.1\sqrt{n} + 0.6. \tag{15}$$

The transfer function of the phase filter in the presence of noise is given by

$$\overline{H(\theta)} = \sum_{n=0}^{N} D_n \overline{\cos^n\theta} \tag{16}$$

where

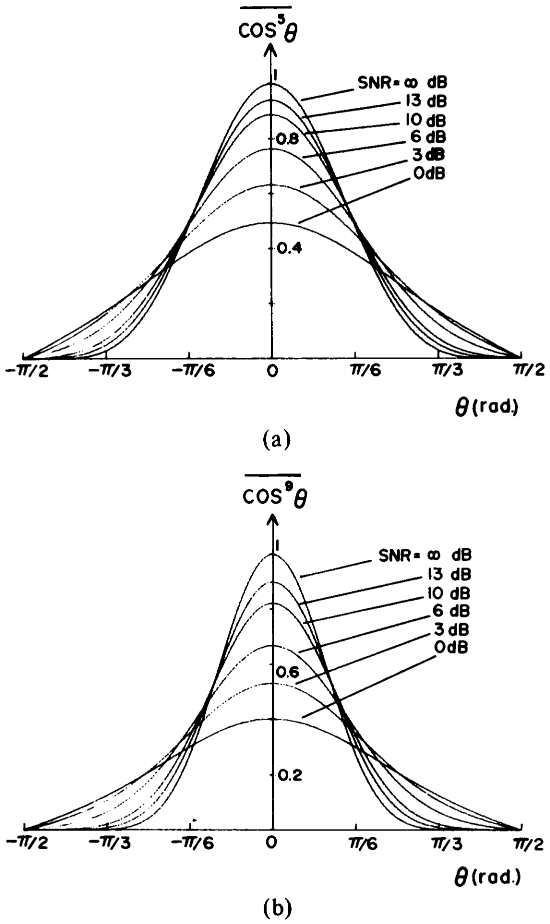

Fig. 4. Mutilated phase responses. (a) $n = 5$. (b) $n = 9$.

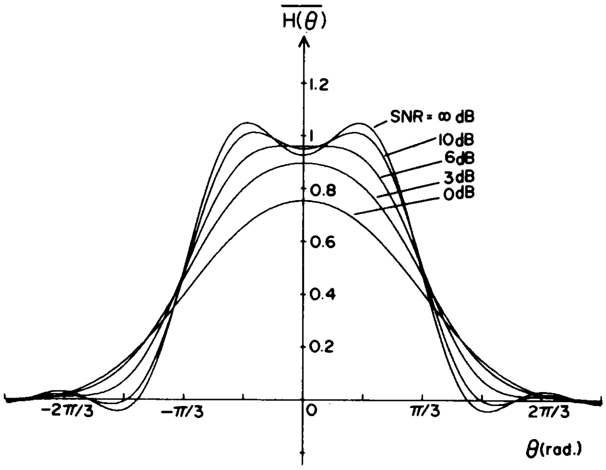

Fig. 5. Mutilated transfer function of the phase filter with 120° phase bandwidth.

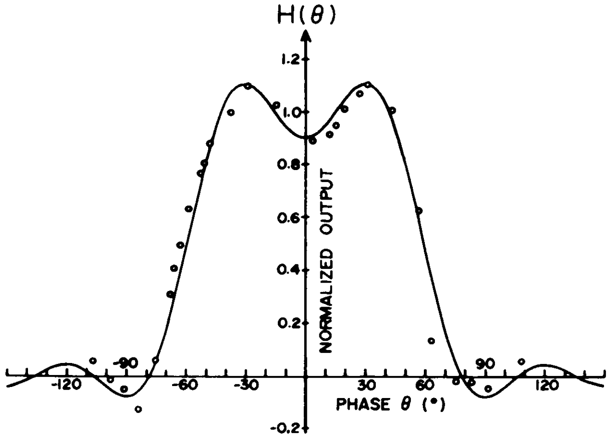

Fig. 6. Measured transfer function of the phase filter with 120° phase bandwidth.

$$\overline{\cos^n \theta} = \int_{-\pi/2}^{\pi/2} \cos^n(\theta - \theta_s) p(\theta_s)\, d\theta_s.$$

Fig. 5 represents $\overline{H(\theta)}$ of the fifth-order filter of 120° bandwidth with SNR as a parameter. As SNR decreases, the passband characteristic becomes flat while the cutoff characteristic becomes obscure. Since the transfer function is a weighted sum of the phase responses, it is reasonable to define the threshold SNR for a phase filter as that of N stages which provides the maximum weight. In the fifth-order filter in Fig. 5, D_3 is maximum. The threshold SNR for this filter is hence found to be 7.7 dB, a plausible value approved by inspecting Fig. 5.

IV. Prototype System

A fifth-order phase filter with the passband of 120° has been constructed. $[D_n]$ is found to be

$$[-0.08, -0.01, 1.66, 2.18, -1.12, -1.77].$$

The signal frequency is set at 10.7 MHz. The local frequencies in the cascaded PSD are 3.2, 2, 1, and 0.455 MHz, respectively. The circuit configurations of CPSD's are all the same as those described in [1]. The balanced demodulators in IC form are used for the mixers in Fig. 3. Scaling and summing are accomplished by one op-amp.

The signal amplitude is held fixed at 0.1 V, much larger than the noise level. The measured transfer function is plotted in Fig. 6 along with the theoretical curve. The data are in good agreement with the theoretical values, confirming the synthesis procedure.

V. Conclusions

The synthesis procedure of the phase filter has been described. Emphasis was placed upon how to provide a control voltage which forms the transfer function of the filter. The effect of noise on the transfer function has also been analyzed to show that SNR higher than 7.7 dB is necessary for the satisfactory operation of the fifth-order filter.

The phase filter described here can be made compatible to any specification by changing only the coefficient $[D_n]$. The demerit is that the steeper cutoff characteristics require quite a high-order filter and also higher SNR. Though primitive, it will find many applications in the phase domain.

Acknowledgment

The authors wish to thank K. Kondo for his help in the construction and the measurement of the prototype phase filter.

References

[1] K. Watanabe, M. Madihian, and T. Yamamoto, "A cascade phase sensitive detector-phase response," *IEEE Trans. Instrum. Meas.*, vol. IM-29, pp. 3–6, Mar. 1980.

[2] P. F. Panter, *Modulation, Noise, and Spectral Analysis.* New York: McGraw-Hill, 1965, ch. 4.