

Limitations of a Relaxation Oscillator in Capacitance Measurements

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Abstract—Oscillation periods of a relaxation oscillator consisting of an op-amp-based integrator and a comparator are measured over a wide range of RC integration constants, and the results are compared with theoretical values derived in terms of the finite gain-bandwidth (GB) product and the slew rate of an op-amp and the response delay of a comparator. The comparison validates the theoretical derivation which gives the design criteria of a relaxation oscillator for capacitance measurements and sensor signal processing.

Index Terms—Capacitance measurement, comparator, operational amplifier, relaxation oscillator, signal processing.

I. INTRODUCTION

BECAUSE of its simple configuration, a relaxation oscillator consisting of an op-amp-based integrator and a comparator is widely applied to digital capacitance measurements and conventional interfaces of capacitive transducers [1]–[6]. Applications are based on the linear relation between the oscillation period and the integration time constant RC with C being the capacitance under measurement. Capacitances of micromachined capacitive transducers are usually in the range of a few picofarad. Does the linear relation hold true in such a small capacitance range? The work described in this paper has been motivated by this question.

An op-amp-based integrator is also a basic constituent of active filters. According to sensitivity analysis of typical Sallen and Key filters, the sensitivity of the pole frequency ω_p and the pole Q to the gain-bandwidth (GB) product of an op-amp increases with ω_p , and the product $\omega_p Q$ of a bandpass filter is limited to one hundredth of GB if the sensitivity smaller than 1% is required [7]. This implies that the pole frequency which would otherwise be determined by the external RC time constant is affected by nonidealities of an op-amp and such a limitation would also exist in a relaxation oscillator.

To answer the question, experimentally observed oscillation period is first presented in this paper. Analysis of the oscillation period which takes nonideal performances of an op-amp and a comparator into account is then given to explain the experimental results. Finally, limitation in capacitance measurements using a relaxation oscillator is described in terms of the nonidealities.

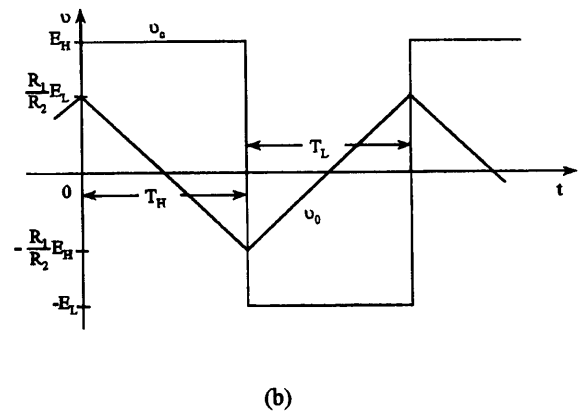
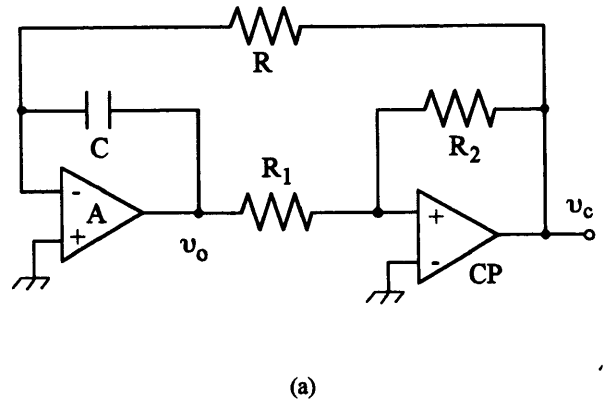


Fig. 1. (a) Circuit diagram and (b) waveforms of a relaxation oscillator.

II. RELAXATION OSCILLATOR

Fig. 1 shows a circuit diagram of a relaxation oscillator and waveforms observed at the outputs of the op-amp-based integrator and the comparator. Contrarily to the usual assumption, $t = 0$ in Fig. 1(b) does not correspond to power turn on but to an arbitrary transition in the output voltage. Assuming for the moment ideal op-amp A and comparator CP, we have the following equation for the integrator output v_o :

$$C \frac{dv_o}{dt} + \frac{v_c}{R} = 0 \quad (1)$$

where v_c is the output voltage of the comparator which assumes E_H until v_o reaches $-(R_1/R_2)E_H$. Solving (1) under the initial condition

$$v_o(t=0) = \frac{R_1}{R_2} E_L \quad (2)$$

we obtain the expression for T_H

$$T_H = CR \frac{R_1}{R_2} (1 + \gamma) \quad (3)$$

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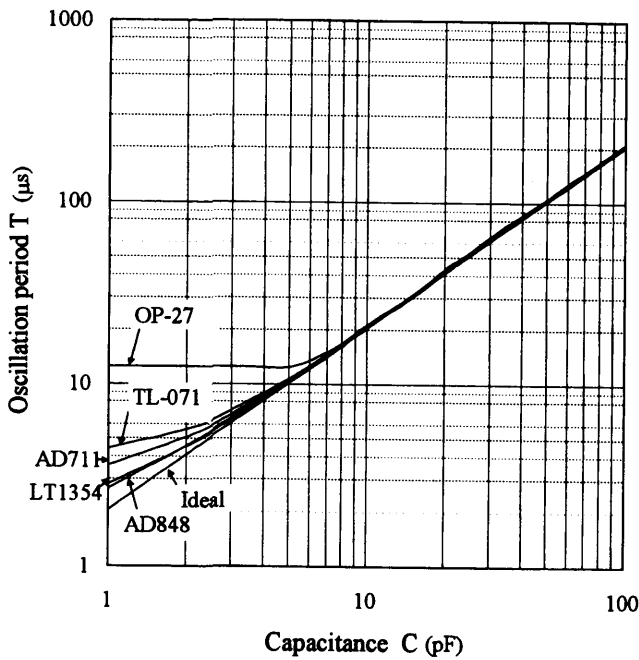


Fig. 2. Measured oscillation periods of relaxation oscillators.

where

$$\gamma = \frac{E_L}{E_H}. \quad (4)$$

Similarly, the period T_L during which the comparator assumes the negative constant voltage $-E_L$ is given by

$$T_L = CR \frac{R_1}{R_2} \left(1 + \frac{1}{\gamma}\right). \quad (5)$$

The oscillation period T is thus

$$T = T_H + T_L = CR \frac{R_1}{R_2} \left(2 + \gamma + \frac{1}{\gamma}\right). \quad (6)$$

Capacitance and resistance measurements using a relaxation oscillator are based on this linear relation between the oscillation period T and the time constant CR .

The measured relations are plotted in Fig. 2. Component values and circuit parameters are: $R = 1 \text{ M}\Omega$, $R_1 = 5.1 \text{ k}\Omega$, $R_2 = 10 \text{ k}\Omega$, $E_H = E_L = 13.5 \text{ V}$. Op-amps with different electrical performances were used for comparison. Taking a look at Fig. 2, one notices that the plots follow the ideal relation (6) in the capacitance range larger than 10 pF, but deviate from it in the smaller range and the deviation depends largely on the electrical performance of an op-amp.

III. NONIDEAL PERFORMANCE ANALYSIS

To explain the discrepancy mentioned in the previous section, we introduce nonidealities of an op-amp and a comparator into the performance analysis. In accordance with the experiment, $E_H = E_L = E$ is assumed hereafter.

Nonidealities of an op-amp are the finite GB, the slew rate, and the offset voltage. The finite GB can be described in terms of the frequency-dependent open loop gain $A(s)$

$$A(s) = \frac{A_o \omega_c}{s + \omega_c} \approx \frac{A_o}{s} \quad (7)$$

where

- A_o dc gain;
- ω_c dominant pole;
- $A_o \omega_c$ gain-bandwidth (GB) product.

The output of the integrator in Fig. 1 when driven by the unit step E_H is then given by

$$V_o(s) = -\frac{A_o \omega_c}{(C + C_i)R} \cdot \frac{1}{s + \frac{1 + A_o \omega_c}{(C + C_i)R}} \cdot \frac{E_H}{s^2} \quad (8)$$

where C_i is the input capacitance at the inverting input terminal of an op-amp. In deriving (8), the approximate expression of $A(s)$ given by (7) is used because the integrator operation at the frequency much higher than ω_c is analyzed. Transforming (8) into the time domain and taking the initial value (2) into account, we have

$$v_o(t) = \alpha E - \frac{E}{CR} \left\{ \frac{1}{1 + \frac{1}{A_o \omega_c CR}} t - \frac{1}{A_o \omega_c} \left(1 + \frac{C_i}{C}\right) \cdot \frac{1}{\left(1 + \frac{1}{A_o \omega_c CR}\right)^2} \cdot \left(1 - e^{-[(1 + A_o \omega_c CR)/(C + C_i)R]t}\right) \right\} \quad (9)$$

where

$$\alpha = \frac{R_1}{R_2}. \quad (10)$$

The first term inside the braces indicates that the finite GB product reduces the slope of the linear ramp voltage of the integrator. The second term inside the braces represents the offset voltage due to the finite GB product which invalidates the virtual-ground condition. The effect of each term inside the braces upon the oscillation period is evaluated using parameters of typical op-amps. The result shows that the effect of the last transient term is small enough to be neglected.

Similar analysis including the offset voltage E_{os} of op-amp shows that its effect on the integrator performance can be evaluated simply by shifting the input by the offset voltage. The slope SL of the triangular waveform at the integrator output is limited by the slew rate, SR , of an op-amp. Thus, the slope is given by

$$SL = \min \left\{ \frac{E \pm E_{os}}{CR \left(1 + \frac{1}{A_o \omega_c CR}\right)}, SR \right\} \quad (11)$$

where the positive sign corresponds to the negative and the minus sign to the positive ramp, respectively.

Fig. 3 shows the timing diagram of the integrator and comparator outputs when the comparator has the response delay. Referring to Fig. 3, one can obtain the expressions for T_H and T_L

$$T_H = \frac{(2\alpha + \gamma)E}{SL_{(-)}} + \frac{\tau_{HL} + \tau_{LH}}{2} \quad (12)$$

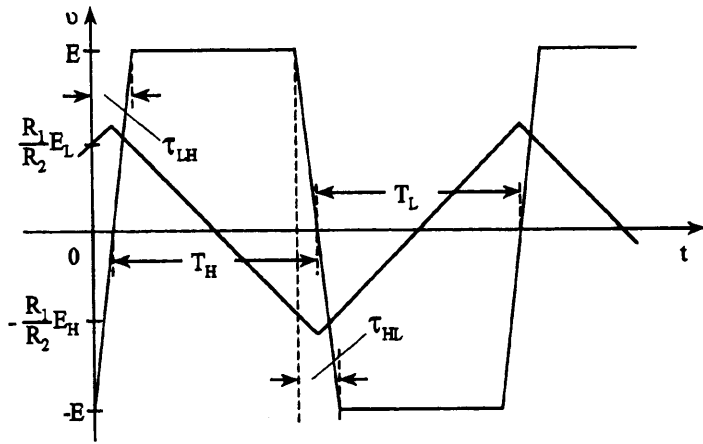


Fig. 3. Waveforms when a comparator has the response delay.

$$T_L = \frac{(2\alpha + \gamma)E}{SL_{(+)}} + \frac{\tau_{LH} + \tau_{HL}}{2} \quad (13)$$

where $SL_{(-)}$ and $SL_{(+)}$ are the slopes of the negative and positive ramps given by (11), τ_{HL} and τ_{LH} are the response delays of a comparator at the high-to-low and low-to-high transitions, respectively, and γ denotes the offset component due to the finite GB product in (9)

$$\gamma = \left(1 + \frac{C_i}{C}\right) \frac{A_o \omega_c CR}{(1 + A_o \omega_c CR)^2} \quad (14)$$

The delay time depends on the input overdrives, and hence on the slope of ramp voltage v_c , but the dependence is neglected and $\tau_{HL} = \tau_{LH} = \tau$ is assumed to the first order approximation. The oscillation period is then given by

$$\begin{aligned} T &= T_H + T_L \\ &= \max \left\{ (2\alpha + \gamma)CR \left(1 + \frac{1}{A_o \omega_c CR}\right) \cdot \left(\frac{1}{1 - \Delta} + \frac{1}{1 + \Delta}\right), \frac{4\alpha E}{SR} \right\} + 2\tau \\ &= \max \left\{ 2(2\alpha + \gamma)CR \left(1 + \frac{1}{A_o \omega_c CR}\right), \frac{4\alpha E}{SR} \right\} + 2\tau \end{aligned} \quad (15)$$

where

$$\Delta = \frac{E_{os}}{E} \quad (16)$$

It is noted in (15) that the effects of the offset voltage on T_H and T_L cancel, to first order, each other. This also holds true of the offset voltage of a comparator.

The gain-bandwidth products $A_o \omega_c$, slew rates SR , and input capacitances C_i of op-amps and the response delays of comparators used in the experiment are listed in Table I. The oscillation periods calculated by (15) using these parameter values are compared with the measured periods. Fig. 4 shows the error defined by

$$\delta = \frac{T_{\text{calculated}} - T_{\text{measured}}}{T_{\text{calculated}}} \times 100(\%) \quad (17)$$

In the capacitance range smaller than 7 pF, the oscillation period of the OP-27-based oscillator is solely determined by the

TABLE I
GAIN-BANDWIDTH PRODUCTS, SLEW RATES, AND INPUT CAPACITANCES OF OP-AMPS AND RESPONSE DELAYS OF COMPARATORS. *: MANUFACTURERS' SPECIFICATION

	OP-27	TL-071	AD711	LT1354	AD848
$A_o \omega_c / 2\pi$ (MHz)	6	4	5	12	175*
SR (V/ μ s)	2.5	11	15	400*	300*
C_i (pF)	—	—	11.0*	3.0*	1.5*
τ_{LH} (μ s)	0.53	0.31	0.32	0.30	0.30
(μ s)	0.53	0.31	0.34	0.33	0.33

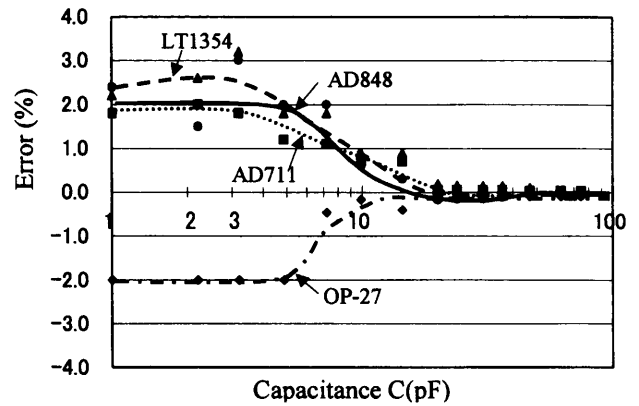


Fig. 4. Errors between the calculated and measured oscillation periods.

slew rate, and the error is independent of the capacitance. Errors for the other three op-amps are almost the same, though their GB products and input capacitances are different. This verifies the theoretical derivation given by (15) and indicates that the residual errors are largely due to the uncertainties of the delay time which depends on the input overdrives. Difficulty of an accurate characterization of a small capacitance is another source of the residual error.

IV. DESIGN CRITERIA

The deviation of the oscillation period from the linear relation is the error in capacitance measurement

$$\begin{aligned} \varepsilon &= \frac{\Delta C}{C} \\ &= \frac{\Delta T}{T} \\ &= \frac{1}{A_o \omega_c CR} \left\{ 1 + \frac{1}{2\alpha} \left(1 + \frac{C_i}{C}\right) \right\} + \frac{\tau}{2\alpha CR} \end{aligned} \quad (18)$$

The derivation of (18) assumes

$$SR > \frac{E}{CR} \quad (19)$$

For the error to be less than $\varepsilon\%$, an op-amp and a comparator with the following specifications are required:

$$A_o \omega_c > \frac{100}{\varepsilon CR} \left\{ 1 + \frac{1}{2\alpha} \left(1 + \frac{C_i}{C}\right) \right\} \quad (20)$$

$$\tau < \frac{2\alpha \varepsilon CR}{100} \quad (21)$$

Specifically, if 1% accuracy is required in capacitance measurement of 1 pF with R , E , and α being 1 M Ω , 10 V, and 0.5, respectively, then an op-amp with $A_o\omega_c$ and SR larger than 72 MHz and 10 V/ μ s, respectively, and C_i smaller than 2 pF should be used to build a relaxation oscillator. A high-speed comparator with τ smaller than 10 ns is also required.

V. CONCLUSIONS

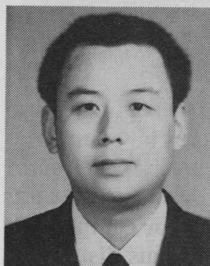
The effect of nonideal performances of an op-amp and a comparator upon the oscillation period of a relaxation oscillator was analyzed. The derived expression explains the nonlinear relation between the oscillation period and the integration time constant, and gives design criteria of a relaxation oscillator for capacitance measurement and sensor signal processing.

A relaxation oscillator described in this paper is also a basic building block of a voltage-to-frequency converter, charge-balancing and dual-slope analog-to-digital converters, and an oversampling $\Delta\Sigma$ modulator. The criteria derived here will also be useful for designing these data converters.

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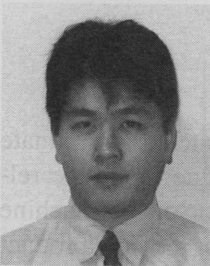
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