Chaos Synchronization and Chaotic Signal Masking in Semiconductor Lasers With Optical Feedback

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Invited Paper

***Abstract*—Theoretical and experimental investigations of chaos synchronization and its application to chaotic data transmissions in semiconductor lasers with optical feedback are presented. Two schemes of chaos synchronization—complete and generalized synchronization—are discussed in the delay differential systems. The conditions for chaos synchronization in the systems and the robustness for the parameter mismatches are studied. The possibility of secure communications based on the chaos masking technique in semiconductor lasers with optical feedback is also discussed, and message transmission of a 1.5-GHz sinusoidal signal is demonstrated. The method of bandwidth enhancement of chaotic carriers is proposed for broad-band chaos communications.**

***Index Terms*—Chaos, chaos communications, chaos synchronization, optical feedback, semiconductor lasers.**

I. Introduction

SINCE THE FIRST prediction of chaos synchronization by Pecora and Carroll [1], [2], synchronization of chaotic oscillations between two nonlinear systems has been reported in various fields of engineering. In the proposed method of chaos synchronization, two similar nonlinear systems—transmitter and receiver systems—that show chaotic dynamics are prepared. The receiver system is divided into two subsystems (driving and response systems) and the receiver consists of one of the subsystems (response system). Without a signal transmission from the transmitter to the receiver, the outputs from the two systems never show the same waveform since chaos is sensitive to the initial condition of a system. But when a chaotic output from the driving system in the transmitter is sent to the receiver, the receiver synchronizes with the transmitted signal under certain conditions of the system parameters; thus, chaos synchronization is realized. The transmitter must be a chaotic system; however, the receiver itself may or may not be a chaotic system without the transmission of a chaotic signal from the transmitter [2]. As far as the conditional Lyapunov exponents between the transmitter and receiver systems have negative values, we can achieve chaos synchronization. Following the technique, chaos synchronization has been successfully demonstrated in various systems [3], [4]. However, the method is not straightforwardly applicable to laser systems, since we cannot divide the dynamics of laser variables into subsystems. As an alternative technique for such a case, the difference between certain variables in the transmitter and receiver systems can be used as control parameters for the synchronization [5], [6].

It is well known that lasers are chaotic systems with three variables: the field, the polarization of matter, and the population inversion, that are described by the Lorenz–Haken equations [7], [8]. However, most lasers themselves do not show chaotic behaviors, since one or two of the decay rates involved in the laser rate equations are fast compared with the other rates and the system can be written by two or single rate equations. Nevertheless, in such lasers, instabilities occur in their optical outputs by the introduction of additional degrees of freedom to the systems. For example, semiconductor lasers are classified into stable class-B lasers [9] that are described by the rate equations of the field and the population inversion (the carrier density), but they show chaotic behavior for additional perturbations such as optical feedback, external optical injection, and injection current modulation. In class-B lasers, experimental synchronization between two chaotic laser systems has been demonstrated in solid-state lasers [10] and CO_2 lasers [11]. Also, there have been many papers which studied chaos synchronization in semiconductor laser systems [12]–[32].

The semiconductor laser is well suited for a chaotic device, since the internal laser oscillation is easily interfered with a field from external light injection or optical feedback. Here, we are concerned with semiconductor lasers with optical feedback that are described by delay differential equations. In delay-differential systems, there exits a solution for complete chaos synchronization in which the dynamics of the transmitter and receiver systems can be described by the same or equivalent set of the rate equations [12]–[14], [25], [26]. But, it has been proven that delay differential conditions are not essential for complete chaos synchronization and a continuous system (such as a Lorenz system) that is described by differential equations with more than three variables has a solution of complete chaos synchronization [33]. There exists another possibility for the synchronization mechanism in laser systems. It is well known that the laser can show synchronous oscillations by optical injection locking in a master–slave configuration [12]–[24]. The synchronization scheme is completely different from that of complete chaos synchronization. As will be shown later, the two synchronization schemes can be easily distinguished from a time lag between the transmitter and receiver waveforms in

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delay differential systems [25], [26]. Recently, experimental synchronization in semiconductor lasers has also been reported [15], [19]–[21]; however, most experimental results of chaos synchronization were based on optical injection locking phenomena in a master–slave configuration of the transmitter and receiver lasers, which is known as generalized synchronization [34]. A few of the experiments demonstrated complete chaos synchronization [16], [25].

The study of synchronization in chaotic lasers is important in practical applications for optical secure communications. Either digital or analog methods can be applied for that purpose. Among the digital techniques, the method of code scrambling based on chaotic signal generations is used for chaos communications at the code level [35]. Digital methods have various advantages; however, we will not discuss the details, and we focus on the analog methods in the following. Chaos is essentially analog in nature and well suited for analogue modulation. One can embed small messages into chaotic carriers. Chaos is dependent upon hardware and it is not easy to guess or analyze chaotic signals without knowing the chaos keys (chaos parameters). In secure communications based on chaos at the signal level, a message together with a chaotic carrier is sent to a receiver. In the receiver system, only the chaotic carrier from the transmitter is duplicated by chaos synchronization and, thus, the message can easily be decoded.

In laser systems, Colet and Roy first discussed the possibility of secure communications based on chaos synchronization [36]. For encoding and decoding of messages, three typical types of synchronization schemes have been proposed in laser systems: 1) chaos modulation (CMO) [37]–[46]; 2) chaos masking (CMA) [47]–[54]; and 3) chaos shift keying (CSK) [5], [55]–[59]. In the CMO scheme, a carrier is simply modulated by a message, while it is just added to a chaotic carrier in the CMA scheme. On the other hand, in the CSK scheme, two separated states corresponding to bit sequences of a message are sent to a receiver system and the message is decoded based on chaos synchronization in the system with two receivers. The robustness for communications and the allowance for parameter mismatches between the two systems are important issues in chaos communications based on chaos synchronization and such studies have been conducted [12], [55], [59].

In semiconductor laser systems, chaotic secure communications were also investigated theoretically [5], [47], [55] and experimental verification has recently been reported [41], [52]. An optically injected semiconductor laser or semiconductor laser with optical feedback is frequently used as a chaotic laser source, since broad-band chaotic signals can be obtained by optical control. The techniques of CMO, CMA, and CSK can be applied to chaotic secure communications using semiconductor lasers in various system configurations. We can use either complete or generalized synchronization schemes for chaos communications, although the robustness of the communication is different with each approach. The author has already discussed gigahertz synchronization of chaotic oscillations in semiconductor lasers with optical feedback and the possibility of high bit-rate data transmission [19]. Fisher *et al.* have also reported experimental synchronization and a transmission of a sinusoidal signal of 581.5 MHz using similar systems [20]. For chaos synchronization and communications, not only ordinary Fabry–Perot, multi-quantum-well (MQW), or distributed feedback (DFB) lasers, but also various other types of lasers such as self-pulsation lasers and vertical-cavity surface-emitting lasers (VCSELs) are used as laser sources [23], [24].

In this paper, we focus on optical feedback effects in semiconductor lasers. The dynamics of semiconductor lasers with optical feedback have been studied for the past two decades and extensive surveys of bibliographies have been found ([8] and [64]). After a brief introduction to the dynamic properties of semiconductor lasers with optical feedback in Section II, the synchronization of chaotic oscillations in these systems is described. The conditions for complete and generalized synchronization of chaos and the effects of the parameter mismatches between the transmitter and receiver systems are discussed in Section III. Recently, sinusoidal signal transmissions on the order of gigahertz using the CMA technique have been demonstrated [65], [66]. In Section IV, the method of message encoding and decoding for secure communications based on chaos synchronization in semiconductor lasers with optical feedback is presented. We show our result of a sinusoidal signal transmission at a frequency of 1.5 GHz. Section V summarizes the conclusions.

II. Chaotic Dynamics in Semiconductor Lasers With Optical Feedback

A semiconductor laser is quite sensitive to external feedback light. For example, the internal intensity reflectivity of a cleaved facet in an edge emitting semiconductor laser without any antireflection coating is only 0.3, so that the laser is much affected by external perturbations and shows unstable behaviors in the presence of external optical feedback [8], [64]. By optical feedback, the three variables in the laser rate equations—the field amplitude, phase, and carrier density—are coupled with each other, and thus, the laser becomes a chaotic system. Semiconductor lasers with optical feedback are very interesting system not only from the viewpoint of fundamental physics for nonlinear chaotic systems, but also for their potential for applications.

A semiconductor laser with optical feedback shows various interesting dynamic behaviors depending on the system parameters and the instabilities of the laser are categorized into the five regimes, depending on the feedback fraction, according to [67].

- Regime I: Very small feedback (the feedback fraction of the amplitude is less than 0.01%) and small effects. The linewidth of the laser oscillation becomes broad or narrow, depending on the feedback fraction.
- Regime II: Small, but not negligible, effects (less than ~0.1% and the case for $C > 1$, where the C parameter is a measure of instability, discussed later). Generation of the external modes gives rise to mode hopping among internal and external modes.
- Regime III: This is a narrow region around ~0.1% feedback. The mode-hopping noise is suppressed and the laser may oscillate with a narrow linewidth.

Regime IV: Moderate feedback (around 1%). The relaxation oscillation becomes undamped and the laser linewidth is broadened greatly. The laser shows chaotic behavior and sometimes evolves into unstable oscillations in a coherence collapse state. The noise level is enhanced greatly under this condition.

Regime V: Strong feedback regime (higher than 10% feedback). The internal and external cavities behave like a single cavity and the laser oscillates in a single mode. The linewidth of the laser is narrowed greatly.

The investigated dynamics were for a DFB laser with a wavelength of 1.55 μm, so that the feedback fraction corresponding to each dynamics scenario described above is not always true for other lasers. On the other hand, the dynamics for other lasers show similar trends for variation of feedback fraction. We are very interested in regime IV, which shows chaotic dynamics, though it is a small level of the feedback (the intensity fraction of the feedback is only 0.01%). In actual applications of semiconductor lasers, this regime is important because, for example, the feedback fraction of laser amplitude in compact disk systems corresponds to regime IV. Thus, regime IV is important for both the study of nonlinear dynamics and applications.

A semiconductor laser with optical feedback for regime IV is modeled by the Lang–Kobayashi equations that include the optical feedback effects in the laser rate equations [68]–[70]. The instability and dynamics of semiconductor lasers with optical feedback are studied by the nonlinear laser rate equations for the field amplitude, the phase, and the carrier density. Lasers show the same or similar dynamics where the rate equations are written in the same form. Therefore, edge-emitting semiconductor lasers such as Fabry–Perot, MQW, and DFB lasers exhibit similar chaotic dynamics, though the parameter ranges may be different. The measure of the feedback strength is the C parameter, defined by [64]

$$C = \frac{\kappa\tau}{\tau_{\mathbf{in}}}\sqrt{1+\alpha^2} \tag{1}$$

where

- κ feedback fraction defined later;
- τ $= 2L/c$—round-trip time for light in the external cavity, where L is the distance between the laser facet and the external mirror, i.e., the external cavity length and c is the speed of light in vacuum;
- α linewidth-enhancement factor that plays an important role in semiconductor lasers;
- $\tau_{\mathbf{in}}$ round-trip time of light in the internal laser cavity.

If $C > 1$, many modes (external modes and anti-modes) for possible laser oscillations are generated, and the laser becomes unstable. The instabilities of semiconductor lasers much depend on the number of excited modes (or, equivalently, the value of C). The stability and instability of the laser oscillations are theoretically studied by the linear stability analysis around the stationary solutions for the laser variables. A lot of papers have been reported for the study of the dynamic properties in semiconductor laser with optical feedback [71], [72].

The dynamics of semiconductor lasers with optical feedback depend on the system parameters and the important parameters among them can be controlled are the feedback strength κ, the distance L between the front facet of the laser and the external mirror, and the bias injection current J. For variation of the external mirror reflectivity, the laser exhibits a typical chaotic bifurcation very similar to a Hopf bifurcation. But the route to chaos depends on the parameters. It shows sometimes a period-doubling route to chaos like a Hopf bifurcation under certain conditions, while the laser shows a quasiperiodic bifurcation for other cases [64]. It sometimes exhibits behavior completely different from that of ordinary chaotic routes. Another example of the instabilities induced by optical feedback is sudden power dropouts and gradual power recovery in the laser output power so-called low frequency fluctuations (LFFs). However, the physical origin of LFFs does not have a clear theoretical basis [8]. LFFs are typical phenomena observed in a low bias injection current condition [64]. A message transmission based on LFF synchronization is an exception, since the chaotic carrier frequency is much lower than ordinary chaotic oscillations, in the order of gigahertz. The frequency of LFFs depends on the system parameters, but it is usually less than a hundred megahertz. On the other hand, the ordinary chaotic carrier frequency is characterized by the relaxation oscillation frequency of a semiconductor laser. For a while, we discuss the chaotic dynamics in semiconductor lasers with optical feedback and we focus on the dynamics for the feedback corresponding to the regime IV.

Laser oscillation also depends on the bias injection current. In the presence of optical feedback, the laser shows mode hopping in its *L–I* characteristics with an increase of the bias injection current and successive external modes are selected with an increase in the injection current. Within the mode hop for the bias injection current, there exists a chaotic bifurcation depending on the external mirror condition [73]. It is a well-known result that the laser oscillates stably for a higher bias injection current. So rather larger optical feedback strength may be required to destabilize the laser at a higher bias injection current.

The external cavity length also plays an important role in the chaotic dynamics of semiconductor lasers. There are several important scales for the length and change of the external mirror in the dynamics. Chaotic dynamics occur even for a small change of the external mirror position, compatible with the optical wavelength λ [74]. For a small change, the laser output shows periodic undulations (period of $\lambda/2$) and exhibits a chaotic bifurcation within the period. Actually, there is also hysteresis either for an increase or decrease of the external cavity length. On the other hand, when the external reflector is a phase-conjugate mirror, the phase is locked to a fixed value. In this case, the laser is insensitive to a small change in the external mirror length and its dynamics is defined only by the absolute position of the external mirror [72]. This undulation is always observed for every external mirror position as far as the coupling between the external and internal optical field is coherent.

The second case is when the external mirror is positioned within the distance corresponding to the relaxation oscillation frequency (on the order of several centimeters) and the mirror is moved with a scale of millimeters. In this region, the coupling between the internal and external fields is strong (the C parameter is small and the number of modes excited is small) and the laser shows a stable oscillation. Rather larger optical feedback is required to destabilize the laser in this region. For example, power dropouts due to LFFs occur irregularly in time for a large value of C, while periodic LFFs were observed under a large optical feedback at a high injection current [75]. The region of this external mirror position is important from a point of view for practical applications of semiconductor lasers such as optical data storages and optical communications. When the external mirror length is small enough compared with the length of the internal laser cavity, the behavior of the laser oscillation is usually governed by the external mirror.

When the external mirror is positioned over a distance equivalent to the relaxation oscillation frequency of a laser but it is within the coherence length of the laser (on the order of several centimeters to several meters), the laser is greatly affected by the external optical feedback. In this region, the number of modes related to the C parameter is large and the laser shows various dynamical behaviors at moderate feedback rate [74]. This region is also important for the study of fundamental dynamics and their applications, since external feedback length of many practical systems is on the order of several to tens of centimeters and we can easily make chaos devices for various applications.

When the external mirror is positioned at the distance beyond the coherence length of a laser (more than several meters), it still shows chaotic oscillations, but the effects have a partially coherent or incoherent origin [76]. The instabilities and chaos of semiconductor lasers are also induced by incoherent feedback, not only from the laser itself but also from optical injection from another laser source [53].

Here, we discussed the instabilities and chaos only for edge emitting semiconductor lasers, however, there are many kind of structures for semiconductor lasers. For examples self-pulsating semiconductor lasers, vertical-cavity surface-emitting semiconductor lasers, broad area semiconductor lasers, and others. These lasers themselves exhibit chaotic dynamics without the introduction of external perturbations. Furthermore, they also show a variety of chaotic dynamics by optical feedback and injection current modulation. The detailed chaotic dynamics depend on the particular structure, but the same or similar dynamics with edge emitting semiconductor lasers are also observed. Thus, some of them are also used as devices for chaos synchronization and communications. However, in the following we restrict the discussion to edge emitting semiconductor lasers.

III. Chaos Synchronization in Semiconductor Lasers With Optical Feedback

In laser systems, not only master–slave configurations but also mutually injected systems [31] can be used for chaos synchronization systems. However, mutual systems are not suited for chaos communications. So here we treat only master-slave configurations as chaos synchronization systems. In delay differential systems, such as semiconductor lasers with optical feedback, there are two types of synchronization from the standpoint of the physical origins of the phenomena: one is complete chaos synchronization in which the rate equations, both for the transmitter and the receiver, are written by the same or equivalent equations. In complete chaos synchronization, the frequency detuning between the transmitter and receiver lasers must be almost zero and the other parameters must also be nearly identical [77]. Complete chaos synchronization in semiconductor laser systems is realized when the optical injection fraction is small (typically less than a few percent of the chaotic intensity variations). The systems can be considered as very secure from eavesdroppers in communications, since the constraints on the parameter mismatches are very severe.

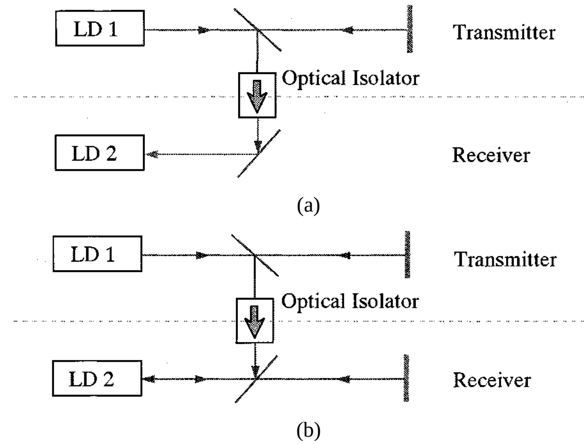

Fig. 1. Schematic diagram of chaos synchronization systems in semiconductor lasers with optical feedback. (a) Optical injection system. (b) Symmetric system.

The other possible scheme is synchronization by optical injection locking. In general, it is not easy to set the oscillation frequencies between the transmitter and receiver lasers the same and a frequency detuning inevitably occurs. However, there exits a frequency pulling effect in the master-slave configuration as long as the detuning is small and the receiver laser shows a synchronous oscillation with the transmitter laser by optical injection locking. Synchronization of chaotic oscillations is also observed in a stable injection locking range in the master-slave configuration; however, it is restricted to a certain region within the ordinary injection locking area. In this case, the restrictions on the parameter mismatch is relaxed relative to that for complete chaos synchronization. Therefore, we can expect chaos communications based on the complete synchronization scheme to have a high level of security, since the constraints on the parameter mismatches for the synchronization is severe. The typical feature of this chaos synchronization is that the synchronization is attained at higher injection rates of several tens of percents for the amplitude fluctuations [77].

We here consider chaos synchronization in semiconductor laser systems with optical feedback. The models under consideration are shown in Fig. 1. We prepare two semiconductor lasers having similar device characteristics as light sources. Two types of synchronization systems with unidirectional light coupling are shown. In Fig. 1(a), the transmitter is a chaotic system, consisting of a semiconductor laser with optical feedback. The system is chaotic under certain parameter conditions. On the other hand, the receiver is a solitary laser and it is a stable system by itself. With chaotic signal injection from the transmitter to the receiver, the receiver synchronizes with the transmitter under appropriate conditions. In Fig. 1(b), the transmitter and the receiver are the same system, a semiconductor laser with optical feedback. Also, a chaotic signal from the transmitter is injected into the receiver laser, then the receiver laser synchronizes with the transmitter laser. As discussed in the following, the system in Fig. 1(a) can be considered as a special case of that in Fig. 1(b).

The systems are described by following set of rate equations for the laser fields $E_{t,r}$, the phases $\varphi_{t,r}$, and the carrier densities $n_{t,r}$ [12]. The subscripts t and r stand for the transmitter and receiver lasers. For the transmitter

$$\frac{dE_t(t)}{dt} = \frac{1}{2}G_{n,t}\{n_t(t) - n_{th,t}\}E_t(t) + \frac{\kappa_t}{\tau_{\text{in},t}}E_t(t-\tau_t)\cos\theta_t(t) \quad (2)$$

$$\frac{d\varphi_t(t)}{dt} = \frac{1}{2}\alpha_t G_{n,t}\{n_t(t) - n_{th,t}\} - \frac{\kappa_t}{\tau_{\text{in},t}}\frac{E_t(t-\tau_t)}{E_t(t)}\sin\theta_t(t) \quad (3)$$

$$\frac{dn_t(t)}{dt} = \frac{J_t}{ed} - \frac{n_t(t)}{\tau_{s,t}} - G_{n,t}\{n_t(t) - n_{0,t}\}|E_t(t)|^2 \quad (4)$$

$$\theta_t(t) = \omega_{0,t}\tau + \phi_t(t) - \phi_t(t-\tau_t) \quad (5)$$

and for the receiver

$$\frac{dE_r(t)}{dt} = \frac{1}{2}G_{n,r}\{n_r(t) - n_{th,r}\}E_r(t) + \frac{\kappa_r}{\tau_{in,r}}E_r(t-\tau_r)\cos\theta_r(t) + \kappa_{\text{inj}}E_t(t-\tau_c)\cos\xi(t) \quad (6)$$

$$\frac{d\varphi_r(t)}{dt} = \frac{1}{2}\alpha_r G_{n,r}\{n_r(t) - n_{th,r}\} - \frac{\kappa_r}{\tau_{in,r}}\frac{E_r(t-\tau_r)}{E_r(t)}\sin\theta_r(t) - \kappa_{\text{inj}}\frac{E_t(t-\tau_c)}{E_r(t)}\sin\xi(t) \quad (7)$$

$$\frac{dn_r(t)}{dt} = \frac{J_r}{ed} - \frac{n_r(t)}{\tau_{s,r}} - G_{n,r}\{n_r(t) - n_{0,r}\}|E_r(t)|^2 \quad (8)$$

$$\theta_r(t) = \omega_{0,r}\tau + \phi_r(t) - \phi_r(t-\tau_r) \quad (9)$$

$$\xi(t) = \omega_{0,t}\tau_c + \phi_r(t) - \phi_t(t-\tau_c) + \Delta\omega t \quad (10)$$

where

- $G_{n,j}$ ($j = t, r$)—linear gain coefficient;
- $n_{th,j}$ threshold carrier density;
- $n_{0,j}$ carrier density at transparency;
- α_j linewidth-enhancement factor;
- κ_j feedback coefficient;
- κ_{inj} injection coefficient from the transmitter to the receiver;
- J_j bias injection current;
- e electron charge;
- d thickness of the active area;
- $\tau_{\text{in},j}$ flight time of light in the internal laser cavity;
- τ_j round-trip time in the external cavity;
- τ_c transmission time of light from the transmitter to the receiver;
- $\omega_{0,j}$ laser oscillation frequency for solitary oscillations;
- $\Delta\omega$ $= \omega_{0,t} - \omega_{0,r}$—detuning between the two lasers.

For small optical feedback, we only consider the effect of a single round-trip of light in the external cavity and the feedback coefficient κ_j is written as [67]

$$\kappa_j = (1 - r_{0,j}^2)\frac{r}{r_{0,j}} \quad (11)$$

where we assumed that the amplitude reflectivities for the front and back facets of the laser cavity are the same ($r_{0,j}$). It is not always true for practical lasers, but the other cases can be calculated in a straightforward manner. r is the reflectivity of the external mirror. The second terms on the right hand side of (2), (3), (6), and (7) are the external feedback effects. The third terms on the right hand side of (6) and (7) are the effects of the optical injection from the transmitter to the receiver. The derived equations are for the model in Fig. 1(b). But, if we put $\kappa_r = 0$ in (6) and (7), the systems reduce to the model in Fig. 1(a).

We investigate possible solutions for chaos synchronization based on the rate equations. To synchronize chaotic waveforms in the two nonlinear systems, the deviations of the corresponding parameters that characterize each system must be small. Therefore, we at first assume the case that all parameters in the transmitter and the receiver have the same values except for the feedback coefficients κ_t and κ_r. Then we can easily obtain the conditions under which the rate equations for the receiver laser are mathematically described by the equivalent delay differential equations as those for the transmitter laser. The conditions are [12]

$$E_r(t) = E_t(t - \Delta t) \quad (12)$$

$$\phi_r(t) = \phi_t(t - \Delta t) - \omega_0\Delta t (\bmod 2\pi) \quad (13)$$

$$n_r(t) = n_t(t - \Delta t) \quad (14)$$

$$\kappa_r = \kappa_t + \kappa_{\text{inj}} \quad (15)$$

$$\Delta t = \tau_c - \tau \quad (16)$$

where $\tau = \tau_t = \tau_r$. Under the above conditions, the rate equations of the transmitter and receiver lasers are mathematically described by the equivalent equations and the receiver laser can synchronize with the transmitter laser. This is so called complete chaos synchronization. In this case, the receiver laser anticipates the chaotic output of the transmitter and it outputs the chaotic signal in advance with time τ as understood from (16), so that the scheme is also called anticipating synchronization. Anticipating chaos synchronization is not only observed in higher dimensional chaotic systems described by delay differential equations but is also observed in low-dimensional continuous systems described by differential equations with more than three variables [33].

Fig. 2 shows the results of the numerical simulations satisfying the conditions in (12)–(16). The system we assume is that of Fig. 1(a). Fig. 2(a) illustrates the temporal outputs of the transmitter and receiver. The receiver laser shows an anticipating output with a time lag of $\Delta t = 1$ ns. The optical injection fraction from the transmitter to the receiver is only 1.6% of the amplitude fluctuation (corresponding to a intensity fraction of $\sim 10^{-4}$%). Fig. 2(b) is the correlation plot be-

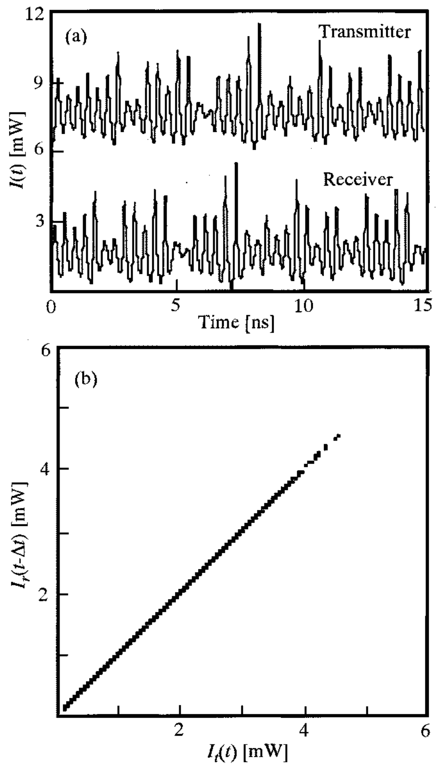

Fig. 2. Numerical result of complete chaos synchronization for the system in Fig. 1(a). (a) Transmitter and receiver outputs. (b) Correlation plot. The external mirror reflectivity is 1.2% and the optical injection to the receiver is also 1.2%. $\tau = 1$ ns, $\tau_c = 0$ ns, and $J = 1.3J_{\rm th}$.

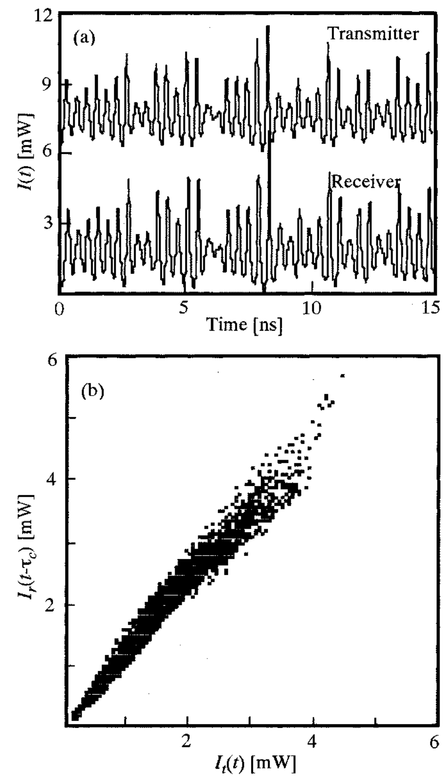

Fig. 3. Numerical result of generalized chaos synchronization for the system in Fig. 1(a). (a) Transmitter and receiver outputs. (b) Correlation plot. The optical injection fraction is 47% and the other parameters have the same values as those in Fig. 2.

tween the two laser outputs. The plot shows the excellent correlation between them and verifies the synchronization. The experimental observation of complete chaos synchronization is difficult since the device parameters of the transmitter and receiver lasers must be matched with each other and the optical injections must be balanced with the external optical feedback. Therefore, few experimental results have been reported. Liu *et al.* [26] reported experimental complete chaos synchronization using similar systems of semiconductor lasers with optical feedback. Also, Tang *et al.* [16] experimentally reported complete chaos synchronization using electrooptic hybrid chaos systems with semiconductor lasers. They observed the time delay and proved the existence of complete chaos synchronization in real lasers. In their experiments, the frequency detuning between the two lasers was essential and was close to zero.

There exits another possibility for the synchronization of chaotic oscillations in semiconductor lasers. An optically injected laser in the receiver system will synchronize with the transmitter laser based on the effects of optical injection locking or amplification due to optical injection. The optical injection locking phenomenon in semiconductor lasers of course depends on the detuning between the frequencies of the master and slave lasers. However, it is usually observed for a rather larger optical injection fraction of several tens of percents of its amplitude fluctuations (corresponding to the intensity injection fraction of several percents or more) with a wide range of the detuning. Under the conditions, the receiver laser shows a synchronous output with the transmitter. The relation between the two laser fields at this synchronization is written by [19]

$$E_r(t) \propto E_t(t-\tau_c). \tag{17}$$

In paticular, the receiver laser responds immediately after it receives the chaotic signal from the transmitter, since the receiver signal always has a time delay with respect to the transmitter signal which is equal to τ_c. The scheme is sometimes called generalized synchronization of chaotic oscillations to distinguish it from complete chaos synchronization. Most experimental results in laser systems including semiconductor lasers reported up to now were based on this type of chaos synchronization.

Fig. 3 shows an example of the numerical results of chaos synchronization based on optical injection locking. Fig. 3(a) shows the output waveforms from the transmitter and receiver lasers. It is noted that the time lag between the two laser outputs is equal to the time τ_c (here, τ_c is set to zero for simplicity) for the transmission of light from the transmitter to the receiver. The detuning of the frequencies between the two lasers is set to be zero and the injection fraction from the transmitter to the receiver is as large as 40% of the amplitude fluctuations. The range of the frequency detuning for the injection locking is small when the injection ratio is small, while it expands over a larger region for a higher injection ratio, so that injection locking is usually observed at a higher injection ratio in experiments. Fig. 3(b) shows the correlation plot corresponding to Fig. 3(a). Though a good linear relation is established, it is not a perfect correlation.

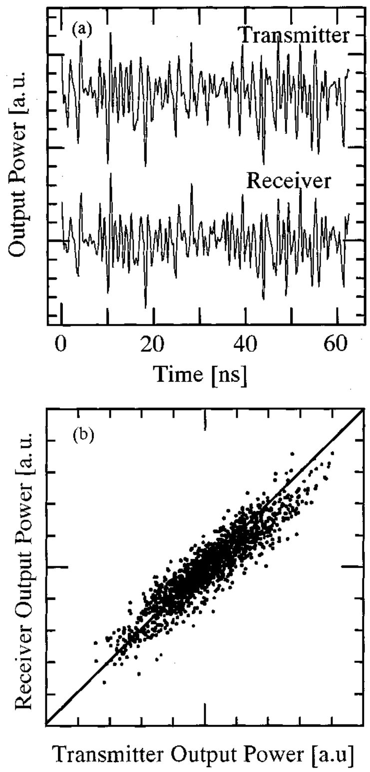

Fig. 4. Experimental result of generalized chaos synchronization for the system in Fig. 1(b). (a) Transmitter and receiver outputs. (b) Correlation plot. The external reflectivities for the master and slave lasers are 0.93% and 0.48% in intensity, respectively. The bias injection currents for the transmitter and receiver lasers are $1.22J_{\rm th}$ and $1.17J_{\rm th}$, respectively. The optical injection to the receiver laser is 4.56%. $\tau = 2$ ns.

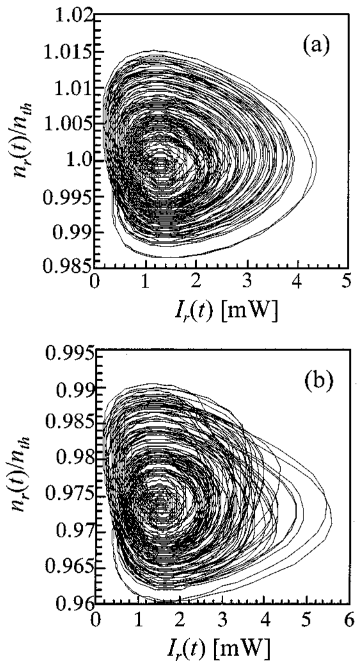

Fig. 5. Chaotic attractors for the intensity and the carrier density in the receiver laser. Receiver waveforms in (a) Fig. 2(a) and (b) Fig. 3(a).

Fig. 4 shows an example of the experimental results for generalized chaos synchronization. The experimental system is the same as that in Fig. 1(b). Without coupling, the transmitter and receiver lasers show different chaotic oscillations with each other and the correlation plot between the two laser outputs spreads over the correlation plane. However, the receiver laser synchronizes with the transmitter when a certain fraction of the transmitter output is injected into the receiver laser. Fig. 4(a) shows an example of the synchronized oscillations between the two lasers. The optical injection to the receiver laser is 4.6% (intensity) of the chaotic fluctuations, which corresponds to the case for generalized chaos synchronization. Fig. 4(b) shows the correlation plot of the waveforms in Fig. 4(a) and it demonstrates that the two lasers synchronize with each other. In the asymmetric system in Fig. 1(a), we also observed synchronization of chaotic oscillations based on the generalized scheme. As a matter of fact, the receiver laser oscillated at a stable mode without optical injection in this configuration. Synchronization of chaotic oscillations is observed either for a system of symmetric or asymmetric configuration in Fig. 1; however, it seems that the allowance for the parameter mismatches in the symmetric system at synchronization is much larger than that in the asymmetric system. Synchronization of chaotic oscillations was also observed in these systems in the low-frequency fluctuation regime [29].

Here, we discuss the differences between complete and generalized chaos synchronization. When a nonlinear oscillator couples with another similar nonlinear oscillator, they synchronize with each other under a certain condition. Under synchronization, the chaotic attractors of the nonlinear systems should be the same. This is true for a complete case. On the other hand, there is a slight difference between the attractors of two nonlinear systems for a generalized case. Fig. 5 shows chaotic attractors in the phase space of the intensity and the carrier density. Fig. 5(a) is a chaotic attractor for the receiver at complete synchronization. Of course, the attractor of the transmitter laser is the same as that in Fig. 5(a), since the rate equations for the two lasers are mathematically written by the same equations. On the other hand, Fig. 5(b) shows a chaotic attractor of the receiver laser at generalized synchronization. The original attractor of the transmitter is the same as that shown in Fig. 5(a). The attractor of the receiver laser in Fig. 5(b) is similar to that in Fig. 5(a); however, there exist differences between them. The optical power of the receiver laser is slightly larger than that of the transmitter and the transmitter signal is amplified. The typical feature of the difference is the reduction of the carrier density of the receiver laser and the gain of the receiver laser is reduced by optical injection. Therefore, the physical origins for perfect and generalized synchronization are completely different from each other.

We investigate possible regions for complete and generalized chaos synchronization in the phase space of the frequency detuning and the optical injection ratio. Fig. 6 shows the result. It is noted that the system we are discussing in the following corresponds to that in Fig. 1(a), but similar comments can be made for the system in Fig. 1(b). The horizontal axis is the detuning and the vertical axis is the optical injection ratio (intensity). The solid curves show the boundaries of the stable and unstable locking areas for ordinary optical injection locking [77]. The dark area shows the region of excellent chaos synchroniza-

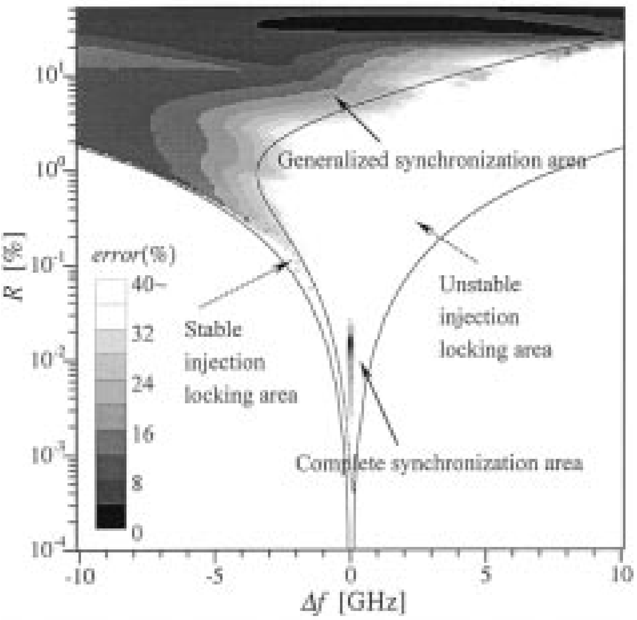

Fig. 6. Calculated areas for complete and generalized chaos synchronization in the phase space of the frequency detuning $\Delta f = \Delta\omega/2\pi$ and the optical injection rate $R = r^2$ (intensity). The condition is the same as that in Fig. 2. The quality of synchronization is represented by gray scales with error rate.

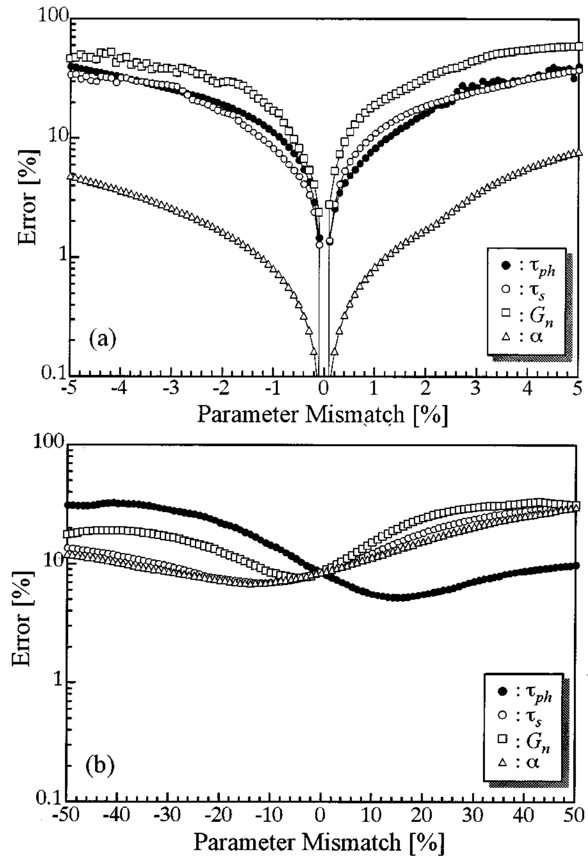

Fig. 7. Calculated synchronization error as a function of parameter mismatches. (a) Complete and (b) generalized chaos synchronization. $\tau_{\rm ph}$ is the photon life time. The other parameter values are the same as those in Fig. 2.

tion, and the definition of synchronization error is given by the deviation of the chaotic intensity fluctuation in the transmitter normalized by that in the receiver. The area of complete synchronization is situated at the unstable injection-locking region with zero detuning and a small optical injection ratio. On the other hand, generalized synchronization of chaotic oscillations is realized over a broad area of phase space, as shown in Fig. 6. The synchronization area is situated within the ordinary injection locking region; however, synchronization of chaotic oscillations does not always occur in that region, but has appropriate conditions for the detuning and the injection ratio.

Finally, the effects of parameter mismatches between the two systems are considered. The model is the same as is the case in Fig. 6. Fig. 7 shows the effects of synchronization errors on parameter mismatches. The horizontal axis is the deviation of the mismatch for each parameter. In the calculations, the mismatches of the internal device parameters are considered. Fig. 7(a) is the case for complete chaos synchronization. In this case, synchronization is attained with excellent quality at very small parameter mismatches and the accuracy rapidly gets worse for an increase in the parameter mismatches. On the other hand, the allowance for the parameter mismatches is rather large for the case of generalized chaos synchronization as shown in Fig. 7(b). In the figure, the frequency detuning was set to be zero and the optical injection ratio was 0.04% in intensity. The accuracy of the synchronization is worse than that for complete chaos synchronization. However, it gradually decreases for increases of the parameter mismatches. It is also noted that the best synchronization is not always attained at zero parameter mismatch.

Chaos synchronization is the essential technique for chaos communications. Either a complete or generalized synchronization scheme can be used for chaos communications. However, the degree of the security for communications may be different, since the effects of the mismatches of the system parameters are much different for the respective schemes. Complete chaos synchronization is suited for such a purpose, but the condition for complete chaos synchronization is quite difficult to realize in actual systems. The robustness of the two schemes is still the important issue in chaos synchronization and communications.

IV. Chaotic Data Transmission

Several analogue schemes for data transmission with chaotic carriers in semiconductor lasers have been proposed for secure communications. Abarbanel *et al.* reported a CMO scheme in the system of a semiconductor laser with optoelectronic feedback [45]. However, for chaotic communications, the fundamental technique in semiconductor lasers with optical feedback we are concerned with here is the method of CMA. The schematic model for that system is shown in Fig. 8. The system is almost the same as that in Fig. 1(b) and the system without transmission of a message is described by the same rate equations [(2)–(11)]. The signal of a message, together with the chaotic carrier from the transmitter, is fed into the receiver laser and the chaotic output from the receiver laser is compared with the transmitted signal. In a CMA system, the message to be transmitted is directly given by a modulation of the injection current or by external modulation using, for examples an electro-optic modulator. The amplitude of the message must be

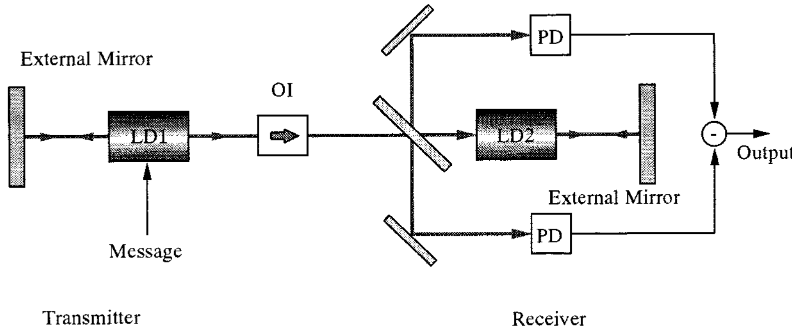

Fig. 8. Schematic diagram of CMA system.

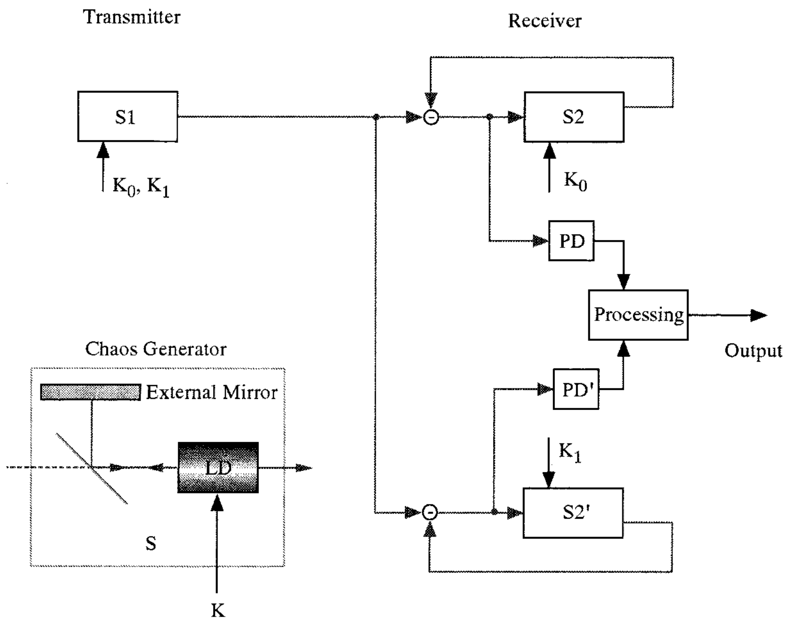

Fig. 9. Schematic diagram of CSK system.

much smaller than the average of the chaotic carrier variations for secure communications. It is usually less than a few percent of the amplitude fluctuations. In the receiver system, only the chaotic carrier is duplicated under certain conditions. Then the message is obtained by simply subtracting the receiver output from that of the transmitter. In the presence of a message in a chaotic signal, this selective synchronization or amplification of the chaotic signal in the receiver system is not self-evident, but a reasonable explanation has not yet been given. The security of data transmission based on chaos synchronization depends to a large degree on the chaos dimension of the system. Since a delay differential system gives rise to high-dimensional chaos, a semiconductor laser with optical feedback is an excellent chaos device for such a purpose.

In the above discussion, we assume that both the optical feedback and optical injection are coherently coupled with the internal laser field. The detuning of the frequencies between the two lasers plays an essential role for synchronization. However, it is not easy to tune the frequencies for real leasers and a drift of the frequencies due to the external perturbations such as a temperature drift may destroy the synchronization. Therefore, systems based on incoherent optical feedback and injection have been proposed in semiconductor laser systems [53]. These systems are almost the same as that in Fig. 1(b) except for some polarization optics (such as a quarter-wave plate) in the optical paths to make the feedback and the optical injection incoherent. In this system, chaos synchronization and message transmission were successfully demonstrated without paying attention to the frequency detuning.

A semiconductor laser with optical feedback is also used as a chaotic generator for a system of CSK. Fig. 9 shows a schematic diagram of the systems. In CSK, two chaotic states according to binary message sequences are generated in time as the two values for a certain system parameter and they are transmitted to the receiver system. For example, the bias injection current is selected as a parameter and two chaotic states corresponding to two different bias injection currents are used. In the receiver system, two chaotic systems are prepared and each system responds and synchronizes with the corresponding chaotic state of the transmitter. Then the message is decoded by the comparison of the signals from the outputs in the two systems. Usually, chaos synchronization has a transient time to give exactly the synchronized waveform with that of the transmitter. Therefore, several cycles of the time corresponding to the typical chaotic carrier frequency are required for exact

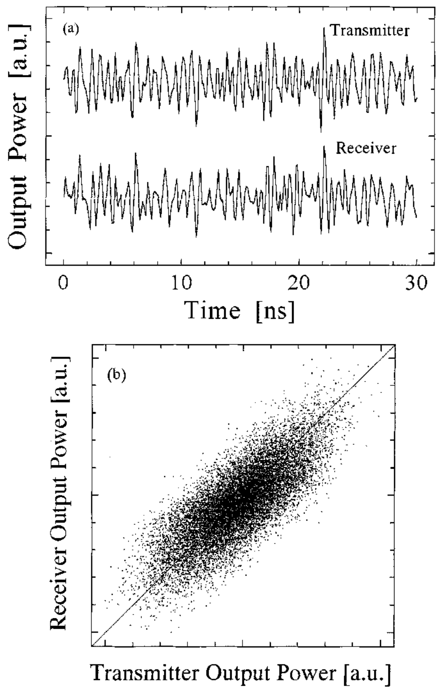

Fig. 10. Experimental results of message transmission. (a) Waveforms for the transmitter and receiver laser outputs in the presence of a message of a 1.5-GHz modulation. The synchronization system corresponds to that in Fig. 1(a). The bias injection currents for the transmitter and receiver lasers are $J = 1.50 J_{\rm th}$ and $1.56 J_{\rm th}$, respectively. The feedback fraction in the transmitter system is 3.75% (intensity) and the optical injection is 6.54% (intensity), τ is 2.3 ns. (b) Correlation plot.

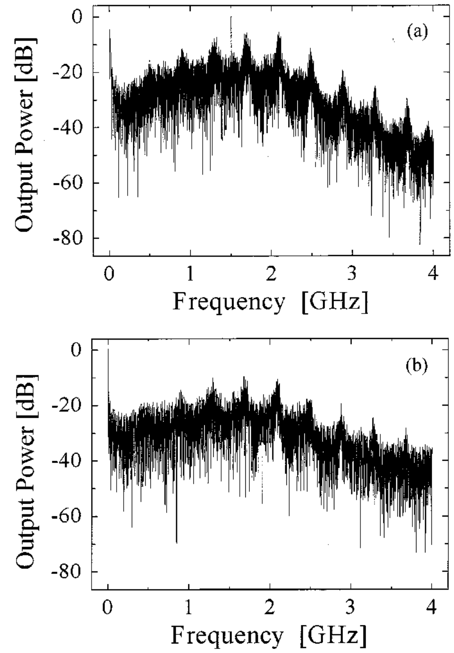

Fig. 11. RF spectra corresponding to the waveforms in Fig. 10. (a) Transmitter output. (b) Receiver output.

synchronization and the transmission rate of messages becomes much lower than the characteristic time of the chaotic carrier. Two chaotic systems for a receiver are usually necessary in CSK. However, the receivers are replaced by a single chaotic system. In that case, the receiver system synchronizes only for one state of the transmitted signals. If a transmitted message is a binary nature, the message can be decoded by a single chaotic system, depending on synchronized or nonsynchronized state of the receiver output. CSK and CMA are essentially different techniques; however, they are not clearly divided. Actually, Mirasso *et al.* [58] proposed the theoretical model of CSK using the same system as that of Fig. 8. In their method, a sequence of binary codes by the injection current modulation was transmitted and the receiver laser synchronized either states of the binary signals. Therefore, the classifications for CMA and CSK are done for the method of decoding for the purpose of convenience.

Next, we show some results for CMA in our experiments. The chaos generator is a semiconductor laser with optical feedback. Fig. 10 shows an example of the experimental results of message transmission. The systems are asymmetrical ones corresponding to Fig. 1(a). A sinusoidal modulation was applied to the injection current of the transmitter laser as a message. The message was a 1.5-GHz sinusoidal wave and the modulation amplitude was 1.5% of the bias injection current (about 4% of the chaotic intensity fluctuations). The optical injection to the receiver laser was 6.45% (intensity fraction) and the frequency detuning between the two lasers at solitary oscillations was 3.1 GHz, so that the scheme was a generalized synchronization that originated from the optical injection locking. Fig. 10(a) is the waveforms in the transmitter and receiver lasers, and Fig. 10(b) is the corresponding correlation plot. The relaxation oscillation frequencies for the two lasers at the solitary oscillations when the bias injection currents were equal and they were about 4 GHz. This frequency plays the role for the maximum chaotic carrier frequency. In Fig. 10(a) and (b), two waveforms are alike one another, despite the presence of the message. From the comparison between Figs. 4(b) and 10(b), the deviations from the exact correlation in Fig. 10(b) are small in spite of the inclusion of the message.

Fig. 11 shows the radio-frequency (RF) spectra corresponding to the previous figure. Fig. 11(a) is the spectrum for the transmitter output. Besides the broad spectral peaks of the external cavity mode and its higher harmonics, a sharp spectral peak for the message of 1.5 GHz is clearly visible in the spectrum. Judging from the spectral distributions, the laser was operating in a weak chaotic state close to a quasiperiodic oscillation. In our experiments, good chaos synchronization was realized when the systems were under weak chaotic oscillations. On the other hand, the spectrum of the receiver output did not show any distinct spectral peak corresponding to the message in Fig. 11(a). However, the overall structure of the spectrum of the receiver output well resembles that of the transmitter except for the message component. Thus, the only chaotic carrier was copied in the receiver laser and the message component involved in the transmitter signal was much suppressed. This is true as far as the message has a small amplitude of less than several percent of the chaotic fluctuations. However, the physical origin of this selective amplification or synchronization has not yet been clarified. A similar trend was also reported in the systems for complete chaos synchronization.

Fig. 12 shows the filtered waveforms for the decoded message, the transmitted signal, and the receiver output from

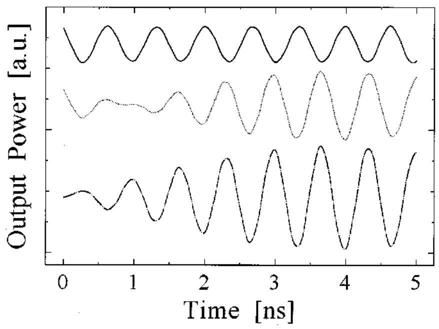

Fig. 12. Narrow bandpass filtered signals for the decoded message (upper trace) and the transmitter (middle trace) and receiver (bottom trace) outputs. The bandwidth of the filter is ±100 MHz centered at the message frequency of 1.5 GHz.

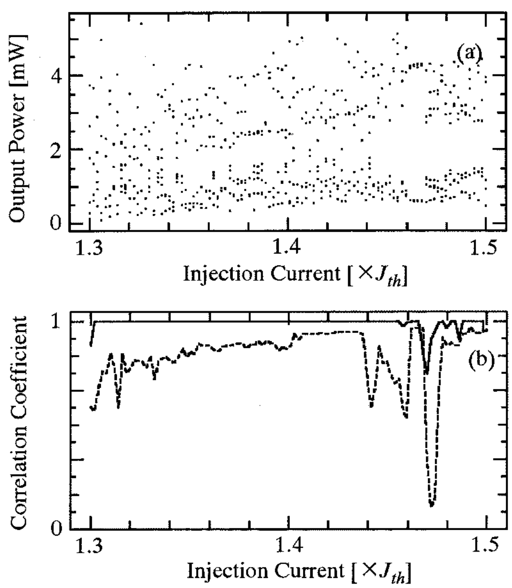

Fig. 13. Chaotic bifurcation for the (a) bias injection current and (b) synchronization errors in the presence of a message. The system is for that in Fig. 1(b). The laser is directly modulated by a sinusoidal wave of 500 MHz through the injection current. The modulation depth is 2% and τ is 2 ns. (b) The solid and broken lines are the correlations without and with the message, respectively.

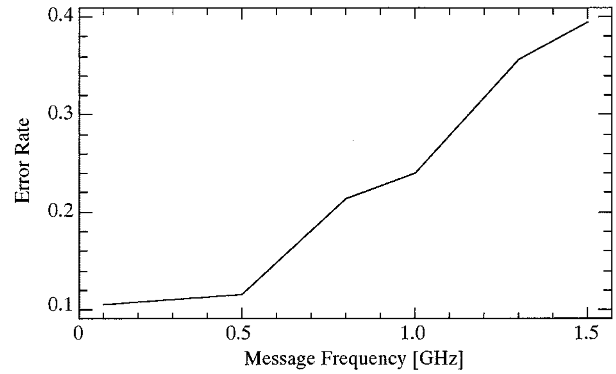

Fig. 14. Accuracy of recovered signal for the message frequency. The relaxation oscillation frequency (carrier frequency) of the laser is 2.6 GHz. The conditions are the same as those in Fig. 13.

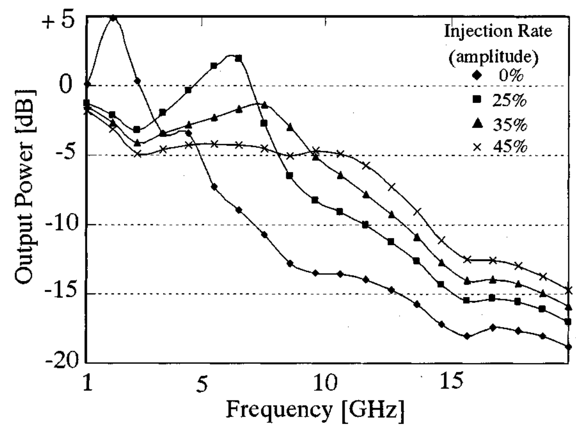

Fig. 15. Numerical result of bandwidth enhancement by a strong optical injection. The injection currents both for the master and slave lasers are the same as $1.3J_{\rm th}$.

the top to the bottom, respectively. The waveforms are the results for a narrow bandpass filter of ±100 MHz centered at the message frequency of 1.5 GHz. The decoded message was a simple subtraction of the receiver output from that of the transmitter. The decoded message shows excellent sinusoidal oscillation, but the filtered waveforms for the transmitter and receiver outputs are not good harmonic signals and even they are not in-phase with the message signal. The degree of the secure communications in the systems must be evaluated by data transmission and decoding for simulated bit sequences, but the obtained results show some of the evidence for the security in the present systems. Since the amplitude modulation was 4% of the averaged chaotic intensity fluctuations in the experiment, and this level was slightly larger than the assumption made for a small perturbation of a message encoding in CMA, the question of the security of the data transmission still arose. A number of problems have been left as future issues for the investigation of secure communications; for example, the bandwidth of signal transmission, the stability of synchronization, and the degree of security.

In the presence of a message in the chaos-masking technique, the accuracy of the synchronization is affected by the message signal. As shown by Mirasso *et al.* for the extended case of the system in Fig. 1(a) to CSK [58], the deviations from the exact linear relation in the correlation plot increase with increase of the modulation amplitude of the message. The degree of synchronization also depends on other parameters [58]. Fig. 13 shows the degree of chaos synchronization for the bias injection current obtained by the numerical simulations. Fig. 13(a) is a calculated bifurcation diagram of the output of the transmitter laser as a function of the bias injection current. The system considered is the same as that in Fig. 1(a). The transmitter laser shows chaotic behavior throughout the range of the bias injection current except for a small window. Under this bifurcation, the degree of the synchronization based on the CMA technique is shown in Fig. 13(b). Without a message (solid line), the correlation coefficient is almost unity within the injection current range and the receiver laser shows excellent synchronous output with the transmitter laser. It is noted that the synchronization scheme is a complete one. With a message consisting of a 500-MHz sinusoidal modulation to the injection current, the calculated correlation coefficient is shown as a dotted line. The depth for the bias injection current modulation is about 2%

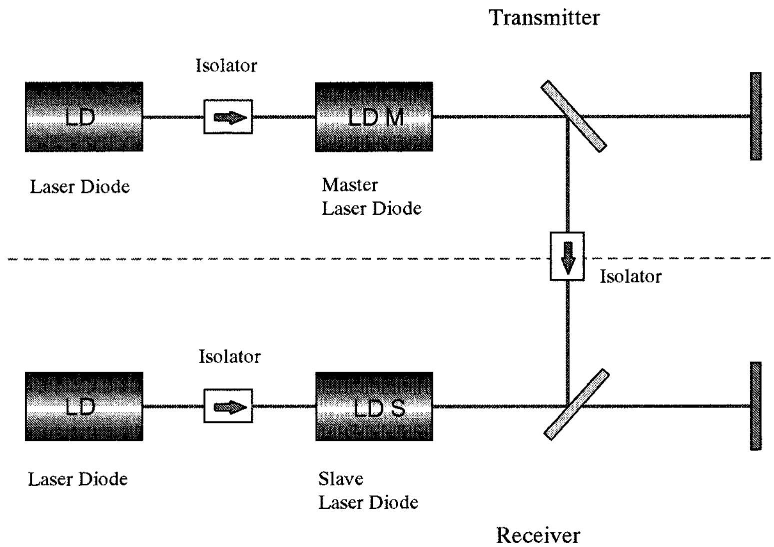

Fig. 16. Schematic diagram of broad-band chaotic communications based on CMA.

(the corresponding intensity modulation is about 4% of the averaged chaotic intensity fluctuations). The correlation coefficient is defined by the normalized covariance for deviations from the line in the correlation plot such as that shown in Fig. 10. The correlation coefficient in the presence of a message is always less than that without the message, but it depends on the bias injection current, and there is a suitable range for the best synchronization, even if the degree of synchronization is degraded in the presence of the message. Usually, the injection current undulation degrades the degree of synchronization to some extent as shown in Fig. 13. So, the external modulation of chaotic intensity by such as an electrooptic modulator is sometimes used instead of injection current modulation. Better synchronization is expected by use of an external modulator.

The quality of the reconstructed signal depends strongly on the maximum carrier frequency in the chaos-masking technique. Fig. 14 shows the dependence of the accuracy for decoded messages on the modulation frequency. The system and the parameters are the same as those in Fig. 13. In the numerical calculation, the bias injection current was set to be $1.42J_{\mathrm{th}}$, where the best synchronization is attained at the message frequency of 500 MHz. The relaxation oscillation frequency of the modeled laser was 2.6 GHz at the operating bias injection current. For frequency modulations below 500 MHz, the error of the decoded message is small. However, the error rapidly increases above that frequency. In the calculations, a low-pass filter with double the message frequency was applied. Here, the error rate is simply defined by the normalized deviation between the original and reconstructed waveforms. The error rate, of course, depends on the filter bandwidth and the error grows as the modulation frequency reaches the relaxation oscillation frequency (carrier frequency). Thus, a higher chaotic carrier frequency is essential for a broad-band data transmission based on the chaotic masking method.

For a small modulation of the bias injection current, the semiconductor laser typically shows a resonant oscillation at a frequency of several gigahertz. Above the relaxation oscillation frequency, the modulation efficiency rapidly decreases with an increase of the modulation frequency. Therefore, the relaxation oscillation frequency is one of the measures for modulation performance of semiconductor lasers at solitary oscillations. The relaxation oscillation frequency depends on the bias injection current and the temperature. However, the relaxation oscillation frequency can be increased by strong optical injection from an external laser source. For the purpose of the enhancement of the modulation bandwidth in semiconductor lasers, the method of strong optical injection has been proposed. The enhancement of the modulation bandwidth is realized in ordinary optical injection locking region in semiconductor lasers [60]–[63]. Fig. 15 shows theoretically calculated modulation properties under strong optical injection. In this simulation, the modulation bandwidth of about 3 GHz at the solitary oscillation is expanded to 11 GHz by a strong optical injection ratio of 45% (field amplitude). In Fig. 15, the parameter ranges in the phase space of the frequency detuning and the optical injection are within the ordinary optical injection-locking region.

As discussed above, the relaxation oscillation frequency is essential for higher data transmission in analog chaos communications. Based on the expansion of modulation bandwidth by strong optical injection, the systems for higher data transmission in the CMA technique are proposed. Fig. 16 shows the schematic diagram of such systems in semiconductor lasers with optical feedback. In the system, the transmitter and receiver lasers are optically injected by strong intensities. Then, the injected lasers are used for light sources of the CMA system consisting of semiconductor lasers with optical feedback. From the preliminary experiments and theoretical calculations, oscillations of maximum chaotic carrier frequency over the bandwidth of 15 GHz were obtained for an initial chaos carrier frequency of 3 GHz by strong optical injections. The detailed results will be reported elsewhere.

V. Conclusion

Synchronization of chaotic oscillations in semiconductor lasers with optical feedback has been discussed and its application to secure communications has also been presented. There are two types of synchronization schemes: 1) complete chaos synchronization, which has the same or equivalent rate equations for the transmitter and receiver lasers and 2) generalized synchronization of chaotic oscillations by optical injection locking. The robustness of the synchronization for each case has been investigated, but further detailed surveys of the synchronization characteristics are still required to fully understand synchronization phenomena in semiconductor lasers with optical feedback, namely, the stability of the communications, the maximum bandwidth of data transmission, and so on.

Communications based on chaos synchronization has been demonstrated in a system of semiconductor lasers with optical feedback. Only the chaotic carrier has been synchronized in the receiver system and a message has been successfully decoded by subtracting the chaotic oscillation of the receiver from the transmitted signal in the CMA method. We have demonstrated that the recovered message was quite different from the narrow bandpass filtered waveforms of the transmitter and synchronized receiver outputs. Thus, we can conduct a secure transmission of a message with a very high data-bit rate based on chaos synchronization. In either case, complete or generalized synchronization can be applied to systems for chaotic communications; however, security problems—including the degree of the security and which synchronization scheme is totally secure—still remain future issues.

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