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Effect of Grain Angle on Complex Young's Modulus E^* of Spruce and Hoo[*1]

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スプルース・ホオの複素弾性率におよぼす繊維傾斜角の影響[*1]

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スプルース・ホオの複素ヤング率$E^*(=E'+iE'')$および力学的損失正接 tan δ におよぼす繊維傾斜角の影響を，減圧状態（20℃，4 mmHg）および気乾状態（20℃，R.H. 65％；スプルースのみ）で振動リード法により検討した。減圧状態では，E', E'' は繊維傾斜角 φ の増加につれて急激に減少し，とくにスプルースでは著しい異方性を示した。これに対して，tan δ は φ の増加につれてわずかに増加するだけで，異方性はきわめて小さかった（Figs. 2，3，4）。

次に E', E'', tan δ と φ との関係を数学的に表示するため，異方性弾性体におけるヤング率と主軸のの傾斜角との関係を複素弾性率の実数部・虚数部にそれぞれ形式的に適用し，(5)，(7)，(8)式より E', E'', tan δ を算出するとFigs. 5，6，8のようになり，計算値は実験値とかなり良い一致を示すことが見いだされた。

Effects of grain angle in L-R plane of Spruce and Hoo on complex Young's modulus $E^*(=E'+iE'')$ and loss tangent tan δ were investigated by the vibrating reed method at vacuumed state (20℃, 4 mmHg) and at humidified state (20℃, R.H. 65%; only Spruce). At the vacuumed state, E' and E'' showed remarkable anisotropy. On the other hand, tan δ increased gradually with increase of grain angle φ and its anisotropy was very small (Figs. 2, 3 and 4). The E', the E'' and the tan δ of Spruce at the humidified state were smaller in E' and larger in E'' and tan δ than those at the vacuumed state, respectively (Figs. 2 and 3).

The mathematical expressions of anisotropies of complex Young's modulus and loss tangent were examined based on the formal applications of transformation formula of strain tensor in elastic body. Then, it was found that the calculated curves of dynamic Young's modulus, loss modulus and loss tangent against grain angle gave reasonably good agreement with the experimental results.

1. INTRODUCTION

An anisotropy of material attracts our attention from the viewpoint of the designs of composite materials or the rational use of materials. Wood also naturally possesses anisotropic properties. This is good for proper utilizations of wood. The anisotropic elasticity of wood and wood based materials has been studied by many researchers and its results have been applied to the fields of laminated woods, plywood panels and shells, wood members of structure, etc. On the other hand, the treatments of dynamic viscoelastic properties of wood have been limited to one dimensional analysis, and the studies of effects of grain angle on the viscoelastic properties also have not been so much reported.[1)-5),7)] Recently we have investigated the anisotropy of mechanical loss tangent concerning the finding of a proper index of the wood for musical instruments.

This paper deals with an experimental study of the

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effects of grain angle on dynamic Young's modulus E', loss modulus E'' and mechanical loss tangent $\tan\delta$ of Spruce and Hoo, and mathematical expressions of E', E'' and $\tan\delta$ against grain angle φ.

2. MATERIALS AND METHODS

2.1 *Materials*

Species used were Spruce (*Picea* sp.) and Hoo (*Magnolia obovata* Thunb.). Sizes, specific gravities and moisture contents of specimens are shown in Table 1. Grain angles of specimens φ (inclination angle between the longitudinal direction of specimens and fiber axis of cell) were 0°, 10°, 20°, 30°, 40°, 45°, 50°, 60°, 70°, 80° and 90° in L-R plane. The numbers of specimens were three at each grain angle.

The length of specimen was controlled to keep the resonance frequency almost constant (about 85 Hz both in Spruce and Hoo) at each grain angle.

2.2 *Methods*

E' and $\tan\delta$ were measured by the vibrating reed method.

The vibrating test was done in a desiccator in which moisture condition was controlled at 20℃, R.H. 65% by saturated NH_4NO_3 solution and at 20℃, atm.p.4 mmHg.

The deflection of specimen was measured by a microscope with a micrometer, and the resonance frequency was measured by an electronic counter.

2.3 *Pre-examination for determining the magnitudes of both the clamp torque of sample holder and the deflection of specimen at vibrating test*

The relation between clamp torque which was regulated with a torque wrench and E' and $\tan\delta$ in L-direction of Hinoki (*Chamaecyparis obtusa* Endl.) was examined. It was proved that E' and $\tan\delta$ were almost constant above the torque 0.2kg·cm. So that, the clamp torque was regulated to 0.25kg·cm in the following experiments.

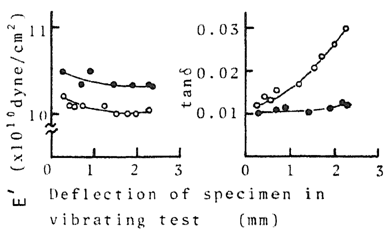

Fig. 1. Effect of deflection of specimen in vibrating test on dynamic Young's modulus E' and loss tangent $\tan\delta$ in Hinoki. ○: 20℃, R.H. 65%, ●: 20℃, 4 mmHg.

The effects of deflection of specimens at vibrating test on E' and $\tan\delta$ of Hinoki are shown in Fig. 1. E' decreases slightly with the increase of the deflection. The $\tan\delta$ in the existence of air is affected very much by the deflection because of the damping effect of the air. The effect of deflection at vacuumed state is very small.

But, the decrease of the deflection of specimen makes the experimental error increase in measuring of deflection, so that the deflection of specimen was regulated to 0.3 mm in this study.

3. RESULTS AND DISCUSSIONS

Effects of the grain angle φ in L-R plane on E', E'' and $\tan\delta$ of Spruce at 20℃, R.H. 65% and at 20℃, vacuumed state are shown in Figs. 2 and 3. The dynamic Young's moduli at both states decrease rapidly in the region $0° < \varphi < 30°$, and over all grain angles the E' at vacuumed state is slightly larger than that at humidified state. The loss moduli also show the same changes as E' against grain angle, but the E'' at vacuumed state is slightly less than that at humidified state. This results

Table 1. Specimens and conditions of measurements

Species	Conditions: Temp. and Relative humidity	Conditions: Pressure	Specific gravity at test	Moisture content at test (%)	Sizes (mm): Length*	Sizes (mm): Width	Sizes (mm): Thickness
Spruce	20℃, R.H. 65%	1 atm.	0.45–0.52	10.9–12.6	100–53	10	1
	20℃, –	4 mmHg	0.44–0.50	1.2–1.7	100–53	10	1
Hoo	20℃, –	4 mmHg	0.39–0.45	0.8–1.5	87–57	10	1

* Length was controlled to keep the resonance frequency constant (about 85Hz).

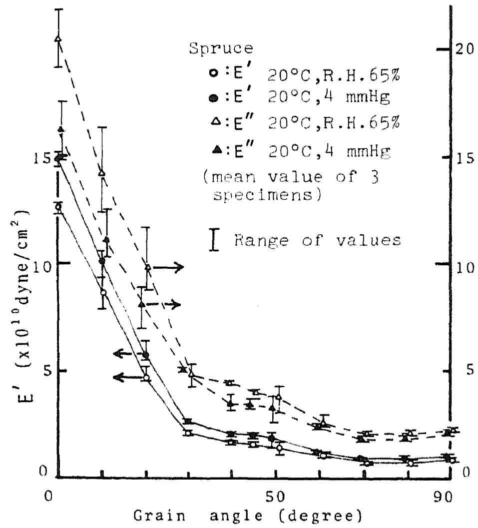

Fig. 2. Relations between dynamic Young's modulus E' or loss modulus E'' and grain angle φ in Spruce.

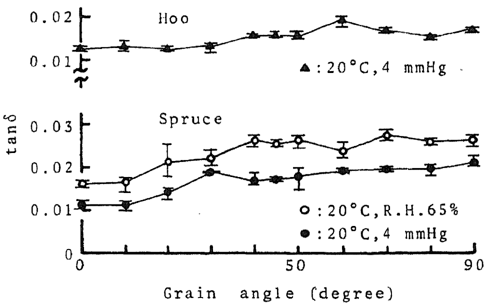

Fig. 3. Relations between loss tangent $\tan\delta$ and grain angle φ in Spruce and Hoo.

from the fact that the $\tan\delta$ at vacuumed state is less than that at humidified state, as shown in Fig. 3.

The similar results were obtained about Hoo at 20°C, vacuumed state as shown in Figs. 3 and 4, but the decreases of E' and E'' against grain angle at small grain angles were more moderate comparing with that of Spruce.

It is very interesting that there are remarkable differences of anisotropy between E' or E'' and $\tan\delta$. These differences perhaps come from the differences in mechanism of the responses E' or E'' and $\tan\delta$ to mechanical stimulus.

For the expression of anisotropy of dynamic Young's modulus E' of wood, Hearmon[6], Matsumoto[1] and other researchers applied the well-known transformation

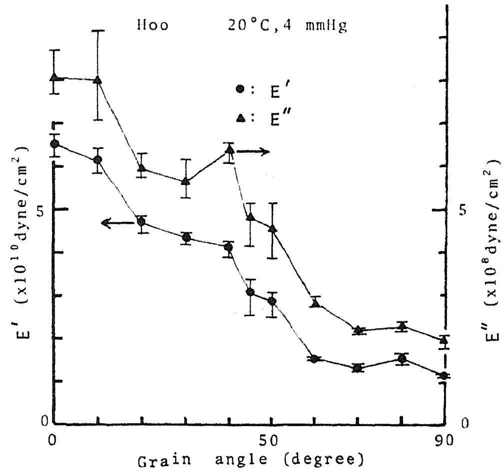

Fig. 4. Relations between E' or E'' and grain angle φ in Hoo.

formula[6),8)] to elastic body

$$S'_{1111} = S_{1111}\cos^4\varphi + S_{2222}\sin^4\varphi + (2S_{1122} + 4S_{1212})\sin^2\varphi\cos^2\varphi, \qquad (1)$$

or

$$\frac{1}{E} = \frac{l^4}{E_X} + \frac{m^4}{E_Y} + l^2 m^2\left(\frac{1}{G_{XY}} - \frac{2\nu_X}{E_X}\right), \qquad (2)$$

to the relation between E' and φ,

$$\frac{1}{E'} = \frac{l^4}{E'_X} + \frac{m^4}{E'_Y} + l^2 m^2\left(\frac{1}{G'_{XY}} - \frac{2\nu'_X}{E'_X}\right), \qquad (3)$$

where, $l = \cos\varphi$, $m = \sin\varphi$, E'_X and E'_Y are dynamic Young's moduli in the direction of principal axes X and Y, respectively, G'_{XY} is dynamic shearing modulus of rigidity and ν'_X is dynamic Poisson's ratio.

The calculated values based on Eq. 2 gave good agreement with their experimental results.

In Eq. 3, if we put $\varphi = 45°$, we get

$$\left(\frac{1}{G'_{XY}} - \frac{2\nu'_X}{E'_X}\right) = \frac{4}{E'_{45}} - \frac{1}{E'_X} - \frac{1}{E'_Y}. \qquad (4)$$

Substituting this relation into Eq. 3, we have

$$\frac{1}{E'} = \frac{l^4}{E'_X} + \frac{m^4}{E'_Y} + l^2 m^2\left(\frac{4}{E'_{45}} - \frac{1}{E'_X} - \frac{1}{E'_Y}\right), \qquad (5)$$

where E'_{45} is the dynamic Young's modulus in the direction 45° from the fiber axis. In Eq. 5, the numbers of independent constants reduce from four in Eq. 3 to three. This means that dynamic Young's modulus at an arbitrary grain angle is predicted with three independent experimental values E'_X, E'_{45} and E'_Y.

The applications of Eq. 5 to our experimental results

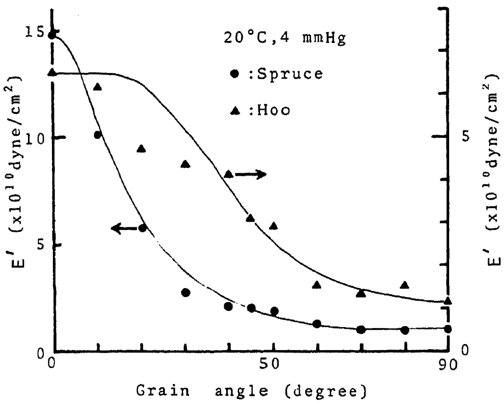

Fig. 5. Comparisons of Eq. 5 (solid lines) with experimental values of E' of Spruce and Hoo at vacuumed state.

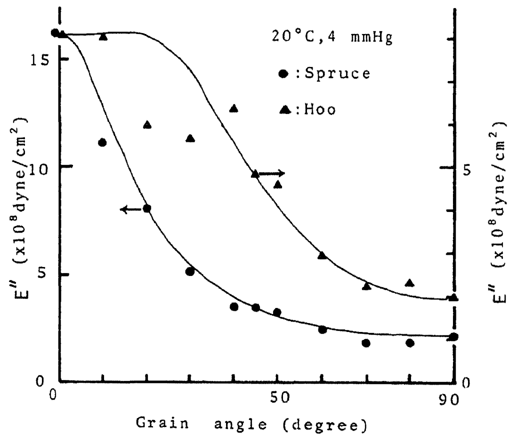

Fig. 6. Comparisons of Eq. 7 (solid lines) with experimental values of E'' of Spruce and Hoo at vacuumed state.

of E' are shown in Fig. 5 as solid lines, and the applicabilities are reasonably good.

On the other hand, an application of transformation formula to the relation between loss modulus E'' and grain angle has not been tried for wood.

As may be seen in Figs. 2 and 4, the data points of E'' show similar changes as those of E' against grain angle, so that a formal application of Eq. 2 to the E'' was examined, that is,

$$\frac{1}{E''}=\frac{l^4}{E''_X}+\frac{m^4}{E''_Y}+l^2m^2\left(\frac{1}{G''_{XY}}-\frac{2\nu''_X}{E''_X}\right), \tag{6}$$

or substituting $\varphi=45°$ in Eq. 6,

$$\frac{1}{E''}=\frac{l^4}{E''_X}+\frac{m^4}{E''_Y}+l^2m^2\left(\frac{4}{E''_{45}}-\frac{1}{E''_X}-\frac{1}{E''_Y}\right). \tag{7}$$

where E''_{45} is the loss modulus in the direction 45° from the fiber axis. The solid lines in Fig. 6 show the curves of E'' calculated from Eq. 7. There is reasonable good agreement between data points and the curves for Spruce and Hoo.

As further checks on the validity of these formal applications, the experimental results of E'' of Sugi (*Cryptomeria japonica* D. Don) obtained by Matsumoto[1]) were examined (where E'' was calculated from his data of E' and logarithmic decrement $\lambda=\pi\cdot\tan\delta$ by the method of free-free beam vibration). Fig. 7 shows good fit of the data to Eq. 7, whereas the fit in Sugi III is poor at small grain angles. This may be due to the poorness of the fit of Eq. 2 to the experimental results of E' at small grain angles.

These facts above represent the validity of formal

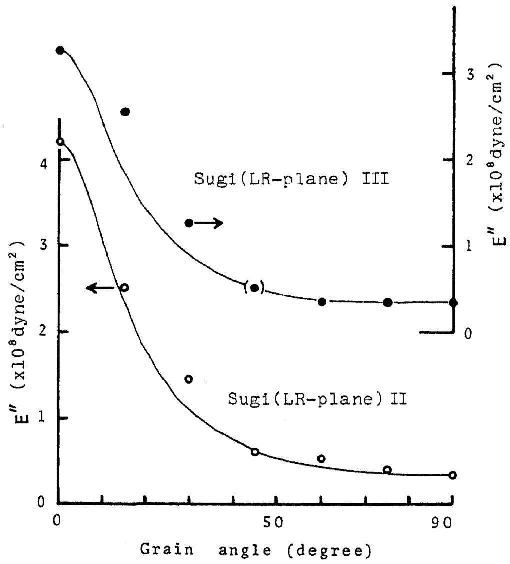

Fig. 7. Comparisons of Eq. 7 (solid lines) with experimental values of E'' of Sugi. ○, ●: experimental values obtained by Matsumoto[1]).

application of Eq. 2 to E'' as well as E' within the limit of experiments.

Then we would calculate the $\tan\delta$ from the relation

$$\tan\delta=E''/E', \tag{8}$$

where E' and E'' are also substituted by Eqs. 5 and 7, respectively. Loss tangent $\tan\delta$ is considered as one of the indices to select suitable woods for musical instruments, and is often used rather than E''.

Fig. 8 shows comparisons of $\tan\delta$ calculated from Eqs. 5, 7 and 8 with experimental results of Spruce and Hoo. Fig. 9 shows the same applications to the experimental results of $\tan\delta$ by Matsumoto[1], where $\tan\delta$ was calculated from his data of logarithmic decrement λ. There is reasonably good agreement between the experimental results and the calculated curves.

The fact that dynamic Young's modulus E' and loss modulus E'' against grain angle can be expressed by the same type of the transformation formula Eq. 1 or 2 is very interesting and perhaps has an important physical meaning, though it is unknown yet. Theoretical substantiations of the formal applications of the formula should be established in future, but the results of our investigations would give us some clues to extend the dynamic viscoelastic laws from one to two dimensions.

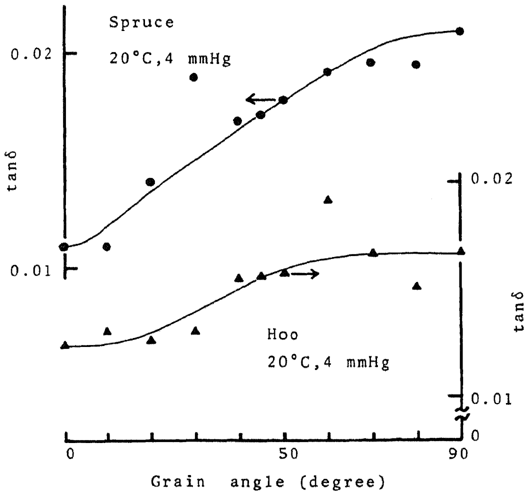

Fig. 8. Comparisons of Eq. 8 (solid lines) with experimental values of $\tan\delta$ of Spruce and Hoo at vacuumed state.

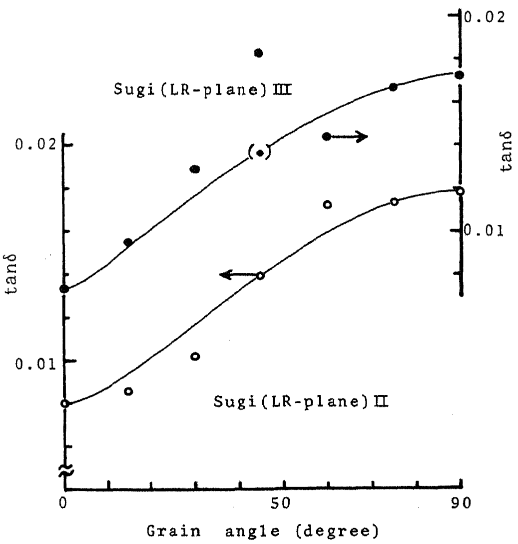

Fig. 9. Comparisons of Eq. 8 (solid lines) with experimental values of $\tan\delta$ of Sugi. ○, ●: experimental values obtained by Matsumoto[1].

4. CONCLUSION

It is known that experimental values of dynamic Young's moduli against grain angle agree fairly well with the curve based on the transformation formula of elastic body, and that the anisotropy of $\tan\delta$ is fairly less than that of dynamic Young's modulus. The same results were also found in our experiments.

The values of loss moduli against grain angle showed the similar changes as those of dynamic Young's moduli. Then the formal application of the transformation formula to loss moduli was examined and the results showed reasonably good agreement between the observed and the calculated values.

A similar expression of loss tangent against grain angle was also tried through applying the transformation formula formally to dynamic Young's modulus and loss modulus, and reasonable agreement between the observed and the calculated values was obtained.

It follows that complex Young's modulus and loss tangent at any grain angle can be expressed by the formal applications of the transformation formula to each component of the complex Young's modulus.

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