[*Mokuzai Gakkaishi* Vol. 37, No. 1, p. 9-15 (1991) (Original Article)]

Identification of Power Spectrum Peaks of Vibrating Completely-Free Wood Plates and Moduli of Elasticity Measurements*[1]

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全辺自由条件で振動する木材平板のパワースペクトルピークの同定と弾性率測定*[1]

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板の振動を利用して，木材の異方性弾性定数を同時測定する方法について検討した。

まず，板を打撃したときに発生する音のパワースペクトルのピークを振動の各モードに自動帰属させる方法について検討した。スペクトルのパターンは弾性定数によって変化するが，最も周波数の低い振動モードは（0，2）モードまたは（1，1）モードであることがわかった。この結果と高次振動の周波数の規則性を利用し，コンピュータによってスペクトルピークを自動帰属させたところ，木材の弾性定数を決定するために不可欠な4つの振動モードについては，帰属の適中率が約85%であった。

適中率を高めるため，振動の位相を考慮したスペクトル解析を行なったところ，二つのヤング率とせん断弾性率を計算するために必要なピークを100%正しく帰属させることができた。しかし，板振動によるポアソン比の測定は，困難なことがわかった。

A simultaneous determination of orthotropic elastic constants of wood using a plate-vibration technique was studied.

First, the identification of power-spectrum peaks of a vibrating completely-free plate was examined. The spectrum pattern depended on the elastic constants, but the smallest vibration mode was a (0, 2) or (1, 1) mode. The regularities of the resonance frequencies of higher order vibrations were used to identify the four essential modes needed for the calculation of the engineering constants E_1, E_2, G_{12}, and ν_{12}. This procedure gave a degree of correct judgment of about 85%.

By introducing spectrum analysis which takes the separation of signals of bending and twisting vibrations into consideration, the three important peaks of (2, 0), (0, 2), and (1, 1) modes needed to calculate E_1, E_2, and G_{12}, respectively, were identified. A Poisson's ratio, however, could not be determined by the plate-vibration technique.

Keywords: identification, power-spectrum peaks, plate vibration, orthotropic *MOE*, simultaneous measurements.

1. INTRODUCTION

Measurement of elastic constants of wood using a plate sample enables a simultaneous determination of anisotropic constants with one test specimen.

However, the application of a plate-bending and -twisting method by a vibration technique is not simple because identification of the resonance peaks of a power spectrum is a complicated problem, and

*1 Received June 14, 1990. A part of this work was presented at the 37th annual meeting of the Japan Wood Research Society (Kyoto, 1987).

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because a direct confirmation of the nodal pattern of a vibrating plate using the Chladni-figure technique normally is needed for orthotropic materials such as wood and wood-based materials.

In this work, automated identification of power spectrum peaks in a completely-free vibrating wood plate was studied. Furthermore, an improved method for simultaneously determining the anisotropic elastic constants was developed.

2. BASIC CONCEPTS AND PRELIMINARY APPROACH TO IDENTIFICATION OF POWER SPECTRUM PEAKS

The Rayleigh method provides good estimations of the resonance frequencies, $fr(i, j)$s, of orthotropic wood plates.[1,2]

$$fr(i, j)=\frac{1}{2\pi}\sqrt{\frac{S}{\rho h}} \quad (1)$$

$$S=D_{11}\frac{\alpha_1(i, j)}{a^4}+D_{22}\frac{\alpha_2(i, j)}{b^4}$$

$$+2D_{12}\frac{\alpha_3(i, j)}{a^2b^2}+4D_{66}\frac{\alpha_4(i, j)}{a^2b^2}$$

where, ρ is density, h is height, a is length, and b is width. The constants α_1, α_2, α_3, and α_4 are given as shown in Table 1 from the energy equivalence principle.[2] Flexural rigidities D_{11}, D_{22}, D_{12}, and a torsional rigidity D_{66} are related to four engineering constants; two Young's moduli, one shear modulus, and one Poisson's ratio as follows:

$$E_1=\frac{12}{h^3}D_{11}\mu, \quad E_2=\frac{12}{h^3}D_{22}\mu,$$

$$G_{12}=\frac{12}{h^3}D_{66}, \quad \nu_{12}=D_{12}/D_{22}, \quad (2)$$

where, $\mu=1-D_{12}^2/(D_{11}\cdot D_{22})$.

Therefore, if the frequencies of the four essential vibration modes, (0, 2), (1, 1), (2, 0), and (2, 2), are determined, the engineering constants can be obtained simultaneously from Equations 1 and 2 as described later.

However, the problem is how we automatically identify the spectrum peak of each vibration mode from a power spectrum without making direct observations of nodal patterns.

2.1 *Power-spectrum patterns of a vibrating completely-free rectangular plate*

If the power-spectrum pattern of a vibrating wood plate is unique, the problem would become simple. The following simulation approach was made to examine the regularity of power spectrum patterns of wood plates under the free-edges boundary conditions.

The ninety-five published data sets[3] of the full orthotropic elastic constants of wood, in which the data of forty-seven softwood and forty-eight hardwood were included, were used to calculate the resonance frequency of each vibration mode.

The simulation result proved that the regularity of the spectrum pattern was relatively great, but the pattern was not unique as shown in Table 2. The results on the ten smallest resonance frequencies of calculated spectra are summarized as follows:

(1) The smallest vibration mode was the pure bending mode (0, 2) or the pure twisting mode (1, 1).

(2) In the case of the longitudinal-radial plane specimens, the most probable smallest mode was (1, 1), and the second was (0, 2).

(3) In the case of the longitudinal-tangential plane specimens, the most probable smallest mode was (0, 2), and the second was (1, 1).

Table 1. Constants α_i.

Vibration modes M	N	α_1	α_2	α_3	α_4
1	1	0	0	0	144
0	2	0	500.6	0	0
0	3, 4, …	0	Y^4	0	0
2	0	500.6	0	0	0
3, 4, …	0	X^4	0	0	0
1	2	0	500.6	0	593.76
1	3, 4, …	0	Y^4	0	$12.3Y(Y+6)$
2	1	500.6	0	0	593.76
2	2	500.6	500.6	151.3	2448.3

Note: $X=(M-0.5)\pi$, $Y=(N-0.5)\pi$.

Table 2. Simulation result of the power-spectrum pattern.

(a) LR-plane

Vibration modes	Order of spectrum peaks									
	←Low frequency					High frequency→				
	1	2	3	4	5	6	7	8	9	10
(1, 1)	65	35								
(0, 2)	35	65								
(1, 2)			97	3						
(0, 3)			2	87	9	1				
(2, 0)			1	9	77	13				
(2, 1)					1	59	40			
(1, 3)					13	27	60			
(2, 2)								46	54	
(0, 4)								54	46	
(1, 4)										100

(b) LT-plane

Vibration modes	Order of spectrum peaks									
	←Low frequency					High frequency→				
	1	2	3	4	5	6	7	8	9	10
(1, 1)	7	93								
(0, 2)	93	7								
(1, 2)			67	33						
(0, 3)			33	67						
(2, 0)					45	43	12			
(2, 1)						16	45	34		
(1, 3)					53	32	13	3		
(2, 2)								1	64	35
(0, 4)					2	9	31	57	1	
(1, 4)								5	29	65

Notes: The number in the Table is the probability when the peak of (i, j) mode locates at the n-th order; for example, in the upper table the probability when the peak of the (1, 1) mode locates at the lowest order is 65%.

Legend: L-R: longitudinal-radial. L-T: longitudinal-tangential.

(4) The regularity of the spectrum pattern in the longitudinal-radial plane specimen was greater than that in the longitudinal-tangential plane specimen.

This means that the identification of the spectrum peaks should be made for each specimen.

2.2 Feasibility of automated identification of resonance peaks using a computer analysis.

In this part, the automated identification of the spectrum peaks based on the regularity of the resonance frequencies of a series of higher order vibration modes in pure bending and pure twisting vibrations was examined. The flow chart of the procedure is shown in Figure 1.

The identification procedure was started by using

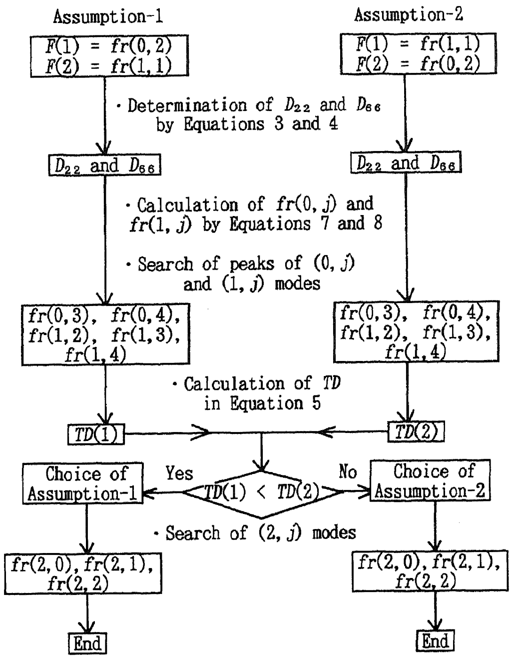

Fig. 1. Automated identification of resonance peaks of a power spectrum.

Note: F(1) and F(2) are the resonance frequencies of the smallest and the second modes, respectively.

the information obtained in the above simulation study.

Assuming that the smallest mode is (0, 2) and the second mode is (1, 1), the flexural rigidity D_{22} and the torsional rigidity D_{66} are given as follows:

$$D_{22}=\frac{k}{\alpha_2(0,\ 2)}fr(0,\ 2)^2 \tag{3}$$

$$D_{66}=\frac{k}{4\alpha_4(1,\ 1)}fr(1,\ 1)^2 \tag{4}$$

where, k is $4\pi^2\rho h a^4$.

Using these provisional values of D_{22} and D_{66}, the five resonance frequencies of (0, 3), (0, 4), (1, 2), (1, 3), and (1, 4) modes were calculated. Then, the most probable resonance peaks of these vibration modes were determined so that the difference of resonance frequency between the corresponding mode and the mode being searched for became a minimum.

Then, changing the order of the assumed (0, 2) and (1, 1) modes, that is, the smallest mode (1, 1) and the second one (0, 2), another series of the above vibration modes was obtained. The series of vibration modes whose TD, as defined in Equation 5, is less was selected as the most proper choice of (0, *j*) and (1, *j*) modes.

$$TD=\sum_{3,4} R(0, j)^2+\sum_{2,3,4} R(1, j)^2 \quad (5)$$

$$R(i, j)=\{fr(i, j)_s-fr(i, j)\}/fr(i, j)\times100 \quad (6)$$

where $fr(i, j)_s$ and $fr(i, j)$ are the resonance frequencies of the peak being searched for and the calculated one, respectively.

Therefore, of the remaining three (2, *j*) modes, the smallest and the largest are assigned to (2, 0) and (2, 2) modes, respectively. Then, D_{11} and D_{12} are given as follows:

$$D_{11}=\frac{k}{\alpha_1(2, 0)}fr(2, 0)^2 \quad (7)$$

$$D_{12}=\frac{1}{2\alpha_3(2, 2)}\{kfr^2(2, 2)-D_{11}\alpha_1(2, 2) - D_{22}\alpha_2(2, 2)-4D_{66}\alpha_4(2, 2)\} \quad (8)$$

The published data sets of the full elastic constants of thirteen species by Tonosaki and Okano[4] then were used to verify the applicability of this identification method.

The results of the $R(i, j)$ of each specimen proved that the mean value and the maximum value of $R(i, j)$ were 2.5% and 9.5%, respectively.

The percentages of the number of the four essential modes which were identified correctly were 92, 92, 92, and 77% for (1, 1), (0, 2), (2, 0), and (2, 2) modes, respectively.

A serious misidentification between (1, 1) and (0, 2) modes occurred in one specimen. The difference of their resonance frequencies was two percent. This condition was probably beyond the ability of the judgment of the proposed method. However, this misidentification has to be avoided because correct identifications of the essential modes are needed to calculate elastic constants. The cause would be due to the heterogeneity of the wood properties in a plate such as a local dispersion of fiber orientation, ring width, or density. This condition would not be inevitable so long as we deal with wood because wood substantially has these heterogeneous properties.

3. IMPROVED METHOD

The following procedure, which took the phase of deflection of a vibrating plate into consideration, gives a substantial resolution of the misidentification of spectrum peaks.

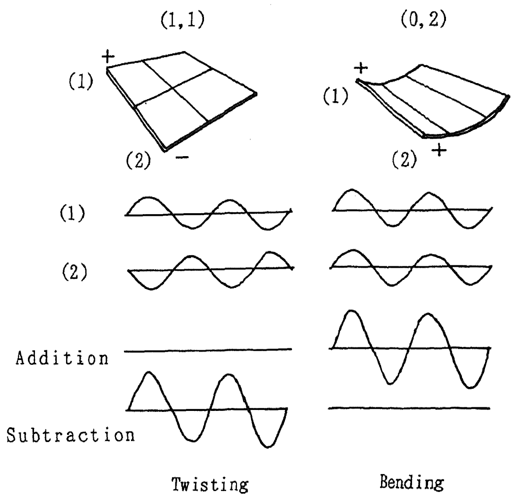

Fig. 2. Addition and subtraction procedures for separating twisting and bending modes.

Figure 2 shows the concept of this procedure. The bending vibration provides the same phase of deflection at both edges, a and b, of a plate. On the other hand, the twisting vibration provides the phase difference of π between both edges. Accordingly, the addition and the subtraction of the deflections at the adjacent edges of an end at any time provide for the doublings of the deflections of the bending vibration and of the twisting vibration, respectively.

By this procedure, the vibration signals can be separated into the following two groups: one is that of twisting vibtations (1, 1), (1, 2), (1, 3), and (1, 4) modes which are separated by the subtraction procedure, and the other is that of bending vibrations (0, 2), (0, 3), (0, 4), (2, 0), (2, 1), and (2, 2) modes separated by the addition procedure.

Accordingly, this procedure enables detection of misidentifications of modes between twisting modes and bending modes in such cases as occurred in the previous section; namely: confusion between (1, 1) and (0, 2) modes, between (1, 2) and (0, 3) modes, between (1, 3) and (0, 4) modes, and so forth.

3.1 Experiments

Five plate samples [(30 cm×30 cm×1 cm; western redceder (*Thuja plicata* Donn), *Agathis* sp., hemlock (*Tsuga* sp.), buna (*Fagus crenata* Bl.), and keyaki (*Zelkowa serrata* Mak.)] were used.

The disposition of a specimen and a pair of microphones is shown in Figure 3. A specimen is supported

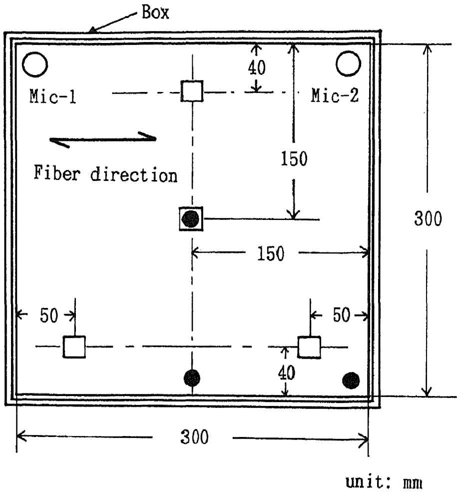

Fig. 3. Support and tapping conditions.
Legend: ○ : microphone, □ : sponge-rubber support, ● : tap position.

on four pieces of sponge rubber. A preliminary experiment proved that the support condition did not affect the resonance frequencies of spectrum peaks to be identified.

The addition and subtraction operations of tap tones were made by additional and differential amplifiers composed of linear operational amplifiers.[5]

A specimen was tapped with a small hammer at three points to enhance the spectrum peaks of the essential vibration modes, as shown in Figure 3, and the tap tones were analyzed by using a Fast Fourier Transformation spectrum analyzer (Ono Sokki Co.; dual-channels analyzer CN-910).

3.2 Results and discussions

Figure 4(a) shows a power spectrum whose signal was detected by a microphone located at a corner of a plate. The ten peaks to be searched for can be distinguished. Figure 4(b) shows the power spectrum isolated by the subtraction procedure. The identification of the twisting modes, (1, *j*) s, were automatically made by assigning the outstanding peaks in the order of the magnitude of the resonance frequencies. Figure 4(c) shows the power spectrum isolated by the addition procedure. Three outstanding peaks of (2, *j*) modes and relatively weak peaks of (0, *j*) modes were resolved. However, seeing both spectra of Figs. 4(b) and 4(c) in detail, a little contamination was found between both spectra, but the identification of each

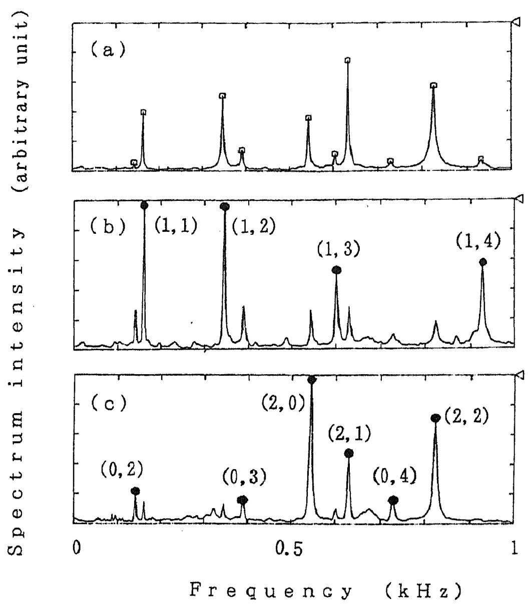

Fig. 4. Typical power spectra (western redcedar); (a) detected by a microphone, (b) separated by subtraction procedure, and (c) separated by addition procedure.

mode could be made easily by comparing both spectra. In this case, the identification of the (0, 2) and (1, 1) modes, which are located closely, can be made easily because the intensity of the (1, 1) peak is greater than that of the (0, 2) peak in the subtraction spectrum, is very weak in the addition spectrum, and the intensity of the (0, 2) peak is more than that of the (1, 1) mode in the addition spectrum, and is weaker than that of the (1, 1) mode in the subtraction spectrum.

Figure 5 shows an example where a clear isolation was made.

Figure 6 shows the case when misidentifications occurred in the *Agathis* sp. plate. The identifications of the (2, 2) and (0, 4) modes were difficult in that the judgments were made by only a comparison of both addition and subtraction spectra because both modes belonged to the bending vibration modes, their spectrum peaks were located closely, and because their intensities were nearly equal. Here, the identifications of (2, 2) and (0, 4) modes were confirmed by Chladni-figure observations.

In conclusion, there was one case of misidentification between the (2, 2) mode and the (0, 4) mode, although the combination of the phase-isolation

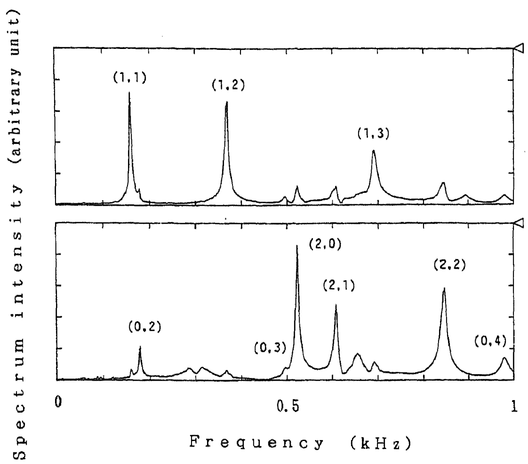

Fig. 5. Typical clear isolation of spectra (keyaki).
Note: Upper is subtraction; lower is addition.

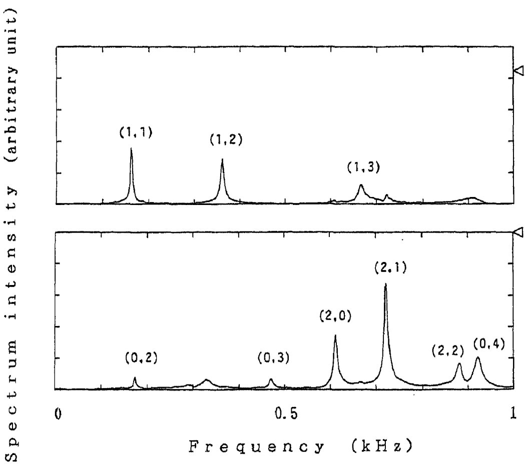

Fig. 6. Isolated spectra (Agathis sp.).
Note: Misidentification between (2, 2) and (0, 4) modes occurred in the computer identification. Upper is subtraction; lower is addition.

method and the computer identification based on the vibration theory was applied. However, of the four essential modes, the three modes, that is (2, 0), (0, 2), and (1, 1), which were required to calculate E_1, E_2, and G_{12}, respectively, were identified correctly.

Accordingly, if a comparison of the addition and subtraction spectra is introduced into the computer program for the identification of power spectrum peaks, automated simultaneous measurements of the elastic constants of wood will become possible.

Four engineering constants were calculated from Equations 2 to 4 and 7 to 8, but some results of E_1, E_2, and ν_{12} were negative. This contradicts the physical meanings of elastic constants. This contradiction was caused by a negative value of $\mu(=1-D_{12}^2/D_{11}D_{22})$; namely: the overestimated value of D_{12} due to the peak shift of the (2, 2) mode which would be caused by the irregularity of a physical property in a plate.

Here, we note that in Equation 2 the product of ν_{12} and ν_{21} is negligibly small compared with 1. Accordingly, if we give proper values to ν_{12} and ν_{21}, the effect of the presumed values of the Poisson's ratio on the calculation of E_1 and E_2 would be within acceptable error. Then, ν_{12} and ν_{21} were given as 0.4 and 0.04, respectively; namely: $\mu=0.984(=1-0.4\times0.04)$. A simulation result proved that the effect of this assumption on the calculation of E_1 and E_2 was less than 1.5% (0.43% on average) for the range of ν_{12}

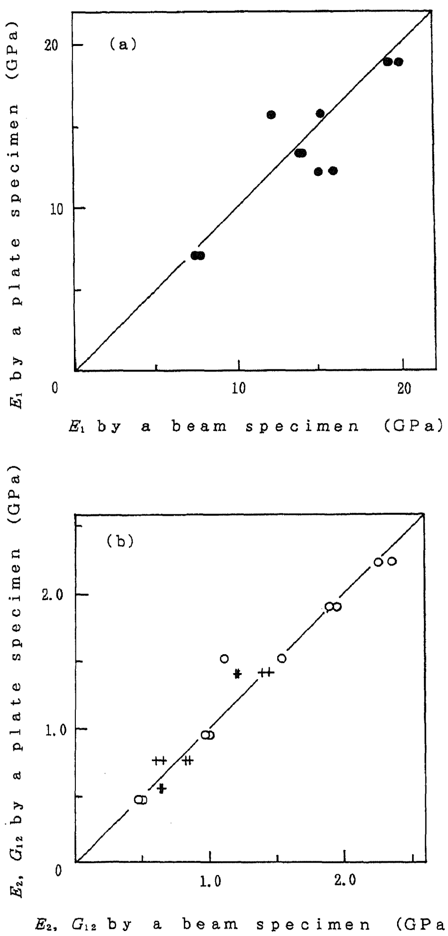

Fig. 7. Relationships between E_1, E_2, and G_{12} by plate specimens and by beam specimens.
Legend: ●: E_1, ○: E_2, and +: G_{12}.

through 0.3 to 0.5.[6] Accordingly, this procedure can be used in the usual measurements of E_1 and E_2.

Figure 7 shows the relationships between E_1, E_2, and G_{12} by plate specimens and by short-beam specimens prepared from each plate sample. The mean values of the ratios of E_1, E_2, and G_{12} between those by plate specimens and those by beam specimens were 0.97 (standard deviation (s.d.) : 0.15), 1.03 (s.d.: 0.13), and 1.05 (s.d.: 0.17), respectively.

A little difference in the elastic constants between a plate specimen and a beam specimen would be caused by irregularities of physical properties in a plate, because a plate specimen was prepared by gluing two or three pieces of narrow lumber.

4. CONCLUSION

Identification of power spectrum peaks of a vibrating completely-free wood plate was examined by using a Fast Fourier Transformation analyzer and a micro-computer.

The proposed spectrum peak identification method which takes the phases of deflections of bending and twisting vibrations at neighboring corners of a plate into consideration was efficient for the identification of near spectrum peaks.

The three spectrum peaks which are needed for calculating the orthotropic constants E_1, E_2 and G_{12}, that is, the spectrum peaks of (2, 0), (0, 2) and (1, 1) modes, respectively, were identified correctly by introducing a spectrum analysis which considered the phases of deflections, but without making Chladni-figure observations. Then, the three engineering constants were determined simultaneously.

However, a Poisson's ratio could not be determined by the plate-vibration method.

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