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Measurement of Flexural Rigidity of Plywood-Webbed Box-Beams by Vibration Method*1

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振動法による合板ウェブボックスビームの曲げ剛性の測定*1

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振動試験による合板ウェブ・ボックスビームの曲げ剛性（*EI*）の測定法を検討した。

試験体は総数 30 体で，同一製造マニュアルに従って 6 社で製造された。両端を単純支持した梁をゴムハンマで打撃し，支持点に加わる振動の反力の波形から梁の固有振動数を求めた。

ボックスビームを均質梁とみなして振動理論を適用した場合およびボックスビームの構造の不均一性を考慮して伝達マトリクス法を適用した場合について検討した。計算結果に両者の差は認められず，静的曲げ試験による *EI* に比べていずれも平均値で約 15% 過小評価した。

振動試験によって，梁の重量と固有振動数から静的曲げ剛性を推定するモデル式を誘導した。

The flexural rigidity (*EI*) of plywood-webbed box-beams estimated from minimum information in a manufacturing process was investigated.

Thirty simple-supported box-beams were tapped with a hammer, and the resonance frequencies were measured from the free vibrations of the beams.

The elementary-vibration theory of uniform beams and a transfer-matrix method for vibration problems were used to estimate the *EI* of the beams. The results were compared with those by static-bending tests. Both *EI* estimates by the elementary-vibration theory and by the transfer-matrix method were smaller by an average 15% than that by the static-bending tests.

A prediction formula of *EI* by static tests calculated from vibration tests was obtained.

Keywords : box beam, flexural rigidity, vibration method.

1. INTRODUCTION

The shortage of large-size timbers and improvements in gluing techniques have promoted development of composite beams for structural members. In Japan, the utillization of plywood-webbed box-beams is not popular at present, but the basis for their use has been laid for both technical and fundamental aspects. This data has the potential to supply designed beams of large size.

This work aims to determine the feasibility of a vibration technique for estimating the flexural rigidity (*EI*) of plywood-webbed box-beams from the minimum information which can be obtained easily in a manufacturing process; namely, resonance frequency, weight, and dimensions of a beam.

This work was proposed because the design size of wooden beams usually is controlled by their stiffness which is a practical design parameter in the design of wooden structures and because a vibration technique would provide a simple device to serve a practical use

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in factories.

2. MATERIAL AND METHODS

2.1 Plywood-webbed box-beams

Details of beam construction are shown in Figure 1. Hem-Fir or Spruce-Pine-Fir 2″ by 4″ (nominal

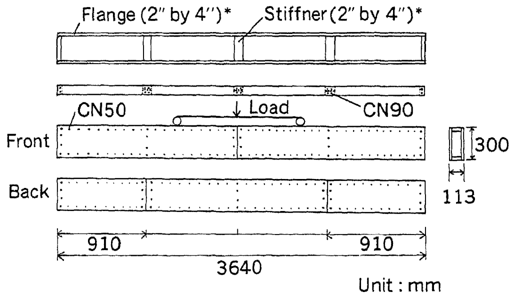

Fig. 1. Construction of plywood-webbed box beams.
* Nominal inch sizes of members.

inch size) No. 2 grade lumber was used for flange and stiffner members. No. 1 grade plywood, 12 mm thick for construction use, was used for web members.

Web members were joined with flange and stiffner members by an aqueous vinyl-polymer solution-isocianate adhesive and CN-50 nails, and the joints between stiffner members and flange members were made with CN-90 nails.

Thirty box-beams were fabricated at six factories ; namely, five beams at each factory, according to the same processing formula.

2.2 Methods

EI was measured by both static and vibration tests. The former were made with a two-point loading at one-third of span length. The latter were made by the free vibration of simple-supported beams with span lengths of 360 cm. Specimens were tapped with a rubber hammer at their mid points, and the vibrations were detected by a load cell which was installed as one of the support blocks. The resonance frequency was measured by a Fast Fourier Transformation Spectrum Analyzer.

2.3 Calculation of flexural rigidity

2.3.1 Elementary theory of vibration[1]

The *EI* of a simple-supported uniform beam by the elementary-vibration theory is given by

$$EI = 4 \cdot l^3 \cdot fr^2 \cdot w/\pi^2 \qquad (1)$$

where l is length of a beam, fr is the resonance frequency, and w is the weight of the beam.

2.3.2 Transfer-matrix method[2]

A box beam was considered as a beam made of a linear combination of two kinds of elements, spring elements and mass elements.

The field matrix $[T_F]$ and the point matrix $[T_P]$ of each element are expressed as follows ;

$$[T_F] = \begin{pmatrix} 1 & l & l^2/(2EI) & l^3/(6EI) \\ 0 & 1 & l/EI & l^2/(2EI) \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (2)$$

$$\text{and} \quad [T_P] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \underline{M}\omega^2 & 0 & 0 & 1 \end{pmatrix} \qquad (3)$$

, respectively,

where $\underline{M}$ is mass, and ω is circular frequency.

The transfer matrix of an entire beam $[N]$ is given by multiplying the field matrix and the point matrix in turn,

$$\begin{pmatrix} y_n \\ \theta_n \\ M_n \\ Q_n \end{pmatrix} = [T_{Pn}][T_{Fn}][T_{Pn-l}][T_{Fn-l}]\cdots[T_{P1}][T_{F1}] \begin{pmatrix} y_1 \\ \theta_1 \\ M_1 \\ Q_1 \end{pmatrix} \qquad (4)$$

$$= [N] \begin{pmatrix} y_1 \\ \theta_1 \\ M_1 \\ Q_1 \end{pmatrix} \qquad (5)$$

where $[N]$, y, θ, M, and Q are (4×4) matrix, deformation, slope of beam, moment, and shear force, respectively. The subscripts 1 and n are the position numbers of the two ends of the beam.

Then, Equation 5 was solved for the boundary condition of a simple-supported beam ; namely, $y_n = y_1 = 0$ and $M_n = M_1 = 0$.

The accuracy of the solution depends on the number of elements considered. It was examined as a homogeneous rectangular beam. The difference of *EI* between the transfer-matrix method and the elementary-vibration theory decreased with increases in the number of elements. In our work the number of elements were 65. The difference was about 5×10^{-4} %.

3. RESULTS AND DISCUSSIONS

3.1 Application of the elementary-vibration theory

An EI obtained by static-bending tests (EI_S) is not the pure EI of a box-beam because in a beam whose height is large for a box-beam the influence of the shear deformation of the beam can not be ignored. However, this apparent value is used currently as a practical indicator of the mechanical property of composite beams. Our goal is, therefore, how we can estimate the EI_S by vibration techniques.

The relationship between EI by static tests and that by the elementary-vibration theory (EI_V) was investigated.

The EI_V to EI_S ratio averaged 0.849 with a standard deviation of 0.082. The EI_V was smaller by 15 % than the EI_S. Why does such a difference exist between the two methods ?

Considering the role of each element in a beam, the stiffner members contribute not only as structural members but also as a concentrated mass which diminishes the resonance frequency of a beam. In the elementary-vibration theory, the latter effect is averaged by distributing the mass of the stiffner members uniformly over the entire length of the beam. This might be one of the cause of the error in EI estimations. Therefore, the following transfer-matrix method was used to determine the effects of the constitution of a beam on EI estimations.

3.2 Application of the transfer-matrix method

The following assumptions were imposed :
(1) There is no slip between web and flange members.
(2) The stiffner members also work as a concentrated mass. (3) The density of flange and stiffner members is 0.5 g/cm^3. (4) The mass of the nails is taken into consideration, but that of the glue is disregarded.

The difference of EI between the results by the elementary-vibration theory and by the transfer-matrix method was less than 0.1 %. This means that the concentrated mass of stiffner members has no significant effect on the calculation of the EI of a box-beam.

The effect of the densities of the flange and stiffner members was investigate with respect to the range from $\rho=0.40$ to 0.60 g/cm^3 considering the case of Hem-Fir and S-P-F lumber. Density had no influence.

The effect of the density of web members also was investigated with respect to the range from $\rho=0.5$ to 0.7 g/cm^3. No influence existed either.

3.3 The other causes affecting EI estimation

A rotatory inertia of beams affects a resonance frequency.[3] This effect, when the stiffner members were disregarded in the calculation, was estimated to be about 0.5 %. This factor also was minor in EI estimation.

The shear deformation of thick beams affects their apparent deflection. The total deflection of a beam (y) is the sum of the deflection by a moment (y_M) and that by a shear force (y_Q).

$$y=y_M+y_Q=y_M(1+c) \qquad (6)$$

The y_Q to y_M ratio (c) is given by[4]

$$c=\left(\frac{E}{G}\right)_l \cdot K_L \cdot K_F \cdot \left(\frac{h}{l}\right)^2 \qquad (7)$$

where K_L is a load factor, K_F is a dimensionless shear factor, $(E/G)_l$ is the equivalent E to G ratio of a beam, and h/l is the height-to-length ratio.

Of these factors, K_L[4] affects the difference between EI_V and EI_S because there is a difference in the manner of loading ; namely, a two-point loading for static tests and a sinusoidally distributed loading for vibration tests. Therefore, $K_L=9.39$ for static tests, and $K_L = 9.87$ for vibration tests. This means that the vibration test induces an increase of K_L by about 5 % more than the static test. Finally, the vibration test estimates the EI 1.2 % less than does the static test.

There still exists a difference of EI by 13 to 14 % between the static tests and vibration tests. The cause of this difference is not clear at this time, and further investigation is not planned because our final target is only how to estimate the EI from the minimum information on beams which is obtained easily in the manufacturing process.

3.4 Prediction formula of EI_S from vibration tests

The relationship between EI_S and EI_V is shown in Figure 2. The statistical results have shown that EI_V/EI_S is 0.849 on average and that its standard deviation is 0.082.

Accordingly, the following equation will predic EI_S from EI_V :

$$EI_S=1.18\ EI_V \qquad (8$$

However, this equation overestimates the EI v of the plots below this prediction line. This ove

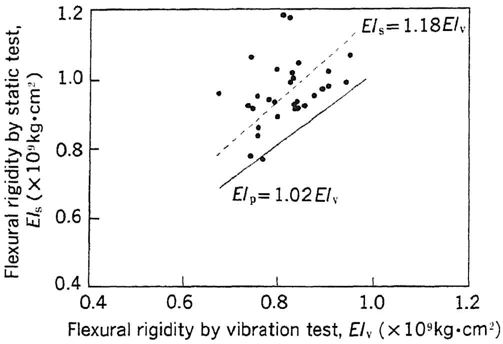

Fig. 2. Prediction formula of *EI* from vibration tests.
Note: EI_p is 95 % lower confidence limit of EI_S vs EI_V line.

timation is not acceptable from the viewpoint of structural design because it estimates the deflection of a beam as being less than the actual deflection.

Then, the lower confidence limit of 95 % was used as appropriate to reduce the acceptable risk level,

$$EI_p = 1.02\ EI_V \qquad (9)$$

This prediction line also is shown in Figure 2.

4. CONCLUSION

A vibration method for measuring the *EI* of plywood-webbed box-beams was investigated.

The elementary-vibration theory of uniform beams gave smaller values of *EI* by 15 % on average than those by static bending tests.

The result of the transfer-matrix method proved that the concentrated mass of stiffner members had no influence on the calculation of *EI*.

Other factors affecting *EI* calculations were examined; namely, densities of stiffner and web members, a rotatory inertia, and a shear deformation, but they were minor factors.

A prediction formula (EI_p) of EI_S from EI_V was arrived at in Equation 9.

In conclusion, the proposed vibration method is a feasible practical way for predicting the *EI* of plywood-webbed box-beams.

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