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Large N reduction on coset spaces

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As an extension of our previous work concerning the large N reduction on group manifolds, we study the large N reduction on coset spaces. We show that large N field theories on coset spaces are described by certain corresponding matrix models. We also construct Chern-Simons–like theories on group manifolds and coset spaces, and give their reduced models.

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I. INTRODUCTION

The large N reduction [1] asserts that large N field theories are equivalent to certain corresponding matrix models, which are called the reduced models (for further developments in the large N reduction, see [2–14]). In particular, in the case of gauge theories, these matrix models are obtained by dimensionally reducing the original theories to lower dimensions. The large N reduction is conceptually interesting in the sense that it realizes emergent space-times. It is also practically important because the reduced models can serve as a nonperturbative formulation of large N field theories. However, there is a difficulty. Because of the so-called $U(1)^d$ symmetry breaking [2], some remedy is needed in the case of gauge theories. In particular, no remedy that preserves supersymmetry is known.

While the large N reduction has been studied so far on flat space-times, it is important to generalize it to curved space-times from both the conceptual and practical viewpoints. First, it can provide hints to the problem of describing curved space-times [15] in the matrix models that are conjectured to give a nonperturbative formulation of superstring theory [16–18]. Second, the reduced models on curved space-times are in general free from the $U(1)^d$ symmetry breaking. In particular, the reduced models of supersymmetric gauge theories on curved space-times can serve as their nonperturbative formulation that respects (full) supersymmetry.

Recently, it was shown in [19] that the large N reduction holds on general group manifolds, which are typical curved manifolds.[1] In this paper, we extend it to the case of coset spaces.[2] We give a prescription by which the reduced models of large N field theories on coset spaces are obtained from the reduced models of the corresponding theories on group manifolds. We also generalize Chern-Simons (CS) theories on three-dimensional manifolds to arbitrary group manifolds and coset spaces, and give the corresponding reduced models.

This paper is organized as follows. In Sec. II, as a preparation, we summarize some properties of group manifolds and coset spaces. In Sec. III, we briefly review the results in [19]. In Sec. IV, we study the large N reduction on coset spaces. In Sec. V, we construct CS-like theories on group manifolds and coset spaces, and show that the large N reduction also holds. Section VI is devoted to conclusion and discussion.

II. GROUP MANIFOLDS AND COSET SPACES

In this section, we describe some properties of group manifolds and coset spaces which are needed in our analysis. See also [22]. Let G be a compact connected Lie group and H be a Lie subgroup of G. We put $\dim G = D$ and $\dim H = d$. The dimension of the coset space G/H is given by $D - d$. We use the following indices: $A, B, \ldots$ run from 1 to D, $\alpha, \beta, \ldots$ form 1 to $D - d$, and $a, b, \ldots$ from $D - d + 1$ to D. $M, N, \ldots$ run from 1 to D, $\mu, \nu, \ldots$ from 1 to $D - d$, and $m, n, \ldots$ from $D - d + 1$ to D.

We take a basis of the Lie algebra of G, t_A, such that t_a are a basis of the Lie algebra of H. t_A obey a commutation relation

$$[t_A, t_B] = i f_{ABC} t_C, \tag{2.1}$$

where f_{ABC} are completely antisymmetric. It follows that $f_{ab\alpha} = 0$. Let x^M be coordinates of the group manifold G.

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[1]In [20], a different type of the large N reduction on $SU(2) \simeq S^3$ was also developed. For earlier discussions and further developments, see [21–29].

[2]While noncommutative field theories on coset spaces such as $CP^n(\simeq SU(n+1)/SU(n) \times U(1))$ [30–39] have been constructed in terms of matrix models, our formulation realizes large N field theories on arbitrary coset spaces.

$g(x) \in G$ is locally factorized as

$$g(x) = h(y)L(\sigma), \tag{2.2}$$

where $h(y) \in H$, and y^m and σ^μ are coordinates of H and G/H, respectively. The isometry of G is the $G \times G$ symmetry: one acts on G from the left, while the other from the right. Only the right G symmetry remains as the isometry of G/H.

For $g \in G$, a $D \times D$ matrix $Ad(g)$ is defined by

$$g t_A g^{-1} = t_B Ad(g)_{BA}. \tag{2.3}$$

$Ad(g)$ is an orthogonal matrix, namely,

$$Ad(g)_{AB} Ad(g)_{AC} = \delta_{BC}. \tag{2.4}$$

Note that for $h \in H$

$$Ad(h)_{\alpha a} = Ad(h)_{a\alpha} = 0, \tag{2.5}$$

which implies that

$$Ad(h)_{\alpha\beta} Ad(h)_{\alpha\gamma} = \delta_{\beta\gamma}, \qquad Ad(h)_{ab} Ad(h)_{ac} = \delta_{bc}. \tag{2.6}$$

f_{ABC} is an invariant third-rank tensor:

$$Ad(g)_{AD} Ad(g)_{BE} Ad(g)_{CF} f_{DEF} = f_{ABC}. \tag{2.7}$$

We define the right invariant 1-form E_M^A by

$$\partial_M g(x) g^{-1}(x) = -i E_M^A(x) t_A. \tag{2.8}$$

E_M^A satisfy the Maurer-Cartan equation

$$\partial_M E_N^A - \partial_N E_M^A - f_{ABC} E_M^B E_N^C = 0. \tag{2.9}$$

We also define e_μ^A and $\tilde{e}_m^a$ by

$$\begin{aligned} \partial_\mu L(\sigma) L^{-1}(\sigma) &= -i e_\mu^A(\sigma) t_A, \\ \partial_m h(y) h^{-1}(y) &= -i \tilde{e}_m^a(y) t_a. \end{aligned} \tag{2.10}$$

Then, the components of E_M^A are given by

$$\begin{aligned} E_\mu^\alpha(x) &= Ad(h(y))_{\alpha\beta} e_\mu^\beta(\sigma), \\ E_\mu^a(x) &= Ad(h(y))_{ab} e_\mu^b(\sigma), \\ E_m^\alpha(x) &= 0, \qquad E_m^a(x) = \tilde{e}_m^a(y). \end{aligned} \tag{2.11}$$

$\tilde{e}_m^a(y)$ and $e_\mu^\alpha(\sigma)$ are viewed as vierbeins of H and G/H, respectively, and satisfy

$$\begin{aligned} \partial_m \tilde{e}_n^a - \partial_n \tilde{e}_m^a - f_{abc} \tilde{e}_m^b \tilde{e}_n^c &= 0, \\ \partial_\mu e_\nu^\alpha - \partial_\nu e_\mu^\alpha - f_{\alpha AB} e_\mu^A e_\nu^B &= 0. \end{aligned} \tag{2.12}$$

Some algebra gives

$$\begin{aligned} \frac{\partial}{\partial y^m} Ad(h)_{ab} &= \tilde{e}_m^c f_{acd} Ad(h)_{db}, \\ \frac{\partial}{\partial y^m} Ad(h)_{\alpha\beta} &= \tilde{e}_m^a f_{\alpha a\gamma} Ad(h)_{\gamma\beta}. \end{aligned} \tag{2.13}$$

A right and left invariant metric of G is defined by

$$G_{MN} = E_M^A E_N^A. \tag{2.14}$$

It is decomposed as

$$\begin{aligned} ds_G^2 &= G_{MN} dx^M dx^N \\ &= K_{\mu\nu} d\sigma^\mu d\sigma^\nu + (Ad(h)_{ba} \tilde{e}_m^b dy^m + e_\mu^a d\sigma^\mu)^2, \end{aligned} \tag{2.15}$$

where a right invariant metric of G/H, $K_{\mu\nu}$, is given by

$$K_{\mu\nu} = e_\mu^\alpha e_\nu^\alpha. \tag{2.16}$$

When G is viewed as a principal H bundle over G/H, e_μ^a correspond to the connection. The Haar measure of G is defined by

$$dg = d^D x \sqrt{G(x)}, \tag{2.17}$$

which is factorized as

$$dg = d^{D-d}\sigma d^d y \sqrt{K(\sigma)} \det \tilde{e}_m^a(y). \tag{2.18}$$

The right invariant Killing vector $\mathcal{L}_A$ is defined by

$$\mathcal{L}_A = -i E_A^M \frac{\partial}{\partial x^M}, \tag{2.19}$$

where E_A^M are the inverse of E_M^A. It generates the left translation, and is expressed as

$$\begin{aligned} \mathcal{L}_a &= -i \tilde{e}_a^m \frac{\partial}{\partial y^m}, \\ \mathcal{L}_\alpha &= -i Ad(h)_{\alpha\beta} e_\beta^\mu \frac{\partial}{\partial \sigma^\mu} \\ &\quad + i \tilde{e}_b^m e_\nu^c e_\beta^\nu Ad(h)_{bc} Ad(h)_{\alpha\beta} \frac{\partial}{\partial y^m}, \end{aligned} \tag{2.20}$$

where $\tilde{e}_a^m$ and e_β^μ are the inverses of $\tilde{e}_m^a$ and e_μ^β, respectively. We denote the Lie derivative along the Killing vector $\mathcal{L}_A$ by δ_A. For instance, from (2.9), we see that

$$\delta_A E_M^B = E_A^N \partial_N E_M^B + \partial_M E_A^N E_N^B = -f_{ABC} E_M^C, \tag{2.21}$$

$$\delta_A G_{MN} = \delta_A E_M^B E_N^B + E_M^B \delta_A E_N^B = 0. \tag{2.22}$$

The second equation indicates the left invariance of the metric G_{MN}.

The spin connection on G, Ω_M^{AB}, is determined by the equation

$$\partial_M E_N^A - \partial_N E_M^A + \Omega_M^{AB} E_N^B - \Omega_N^{AB} E_M^B = 0. \tag{2.23}$$

Comparing this equation with (2.9), we find that

$$\Omega_M^{AB} = \tfrac{1}{2} f_{ABC} E_M^C. \tag{2.24}$$

It follows from (2.21) that

$$\delta_A \Omega_M^{BC} = -f_{ABD} \Omega_M^{DC} - f_{ACD} \Omega_M^{BD}. \tag{2.25}$$

Equations (2.21) and (2.25) show that the Lie derivative accompanied by the local Lorentz transformation keeps

E^A_M and Ω^{AB}_M invariant. Similarly, the spin connection on G/H, $\omega^{\alpha\beta}_\mu$, is determined by the equation

$$\partial_\mu e^\alpha_\nu - \partial_\nu e^\alpha_\mu + \omega^{\alpha\beta}_\mu e^\beta_\nu - \omega^{\alpha\beta}_\nu e^\beta_\mu = 0. \tag{2.26}$$

From (2.12), we find that

$$\omega^{\alpha\beta}_\mu = \tfrac{1}{2} f_{\alpha\beta\gamma} e^\gamma_\mu + f_{\alpha\beta a} e^a_\mu. \tag{2.27}$$

III. LARGE N REDUCTION ON GROUP MANIFOLDS

In this section, we briefly review the results in [19]. The statement of the large N reduction on G we showed in [19] is as follows. Let a large N matrix field theory be defined on G. Its action is given by integration of a Lagrangian density over G with the Haar measure (2.17). We assume that the theory possesses the right G symmetry. In other words, the Lagrangian has no explicit dependence on the coordinates of x^M of G if all the derivatives are expressed in terms of $\mathcal{L}_A$ (2.19). Then, the planar limit of the theory is described by the reduced matrix model that is obtained by dropping the coordinate dependence of the fields and replacing $\mathcal{L}_A$ by the commutator with the matrix $\hat{L}_A$ given explicitly below. We emphasize here that the left G symmetry is not necessary for the large N reduction to hold. As we will see in the next section, this fact is crucial in generalizing the large N reduction to the case of coset space G/H.

In what follows, we illustrate the large N reduction on group manifolds by considering $U(N)$ Yang-Mills (YM) theory on G with a real scalar and a Dirac fermion in the adjoint representation. The action is given by[3]

$$S = S_{\rm YM} + S_s + S_f, \tag{3.1}$$

$$S_{\rm YM} = \frac{1}{4\kappa^2}\int d^D x \sqrt{G} G^{MP} G^{NQ} \,\mathrm{Tr}(F_{MN} F_{PQ}), \tag{3.2}$$

$$S_s = \frac{1}{\kappa^2}\int d^D x\sqrt{G}\Big(\frac{1}{2} G^{MN}(\partial_M\phi + i[A_M,\phi]) \times (\partial_N\phi + i[A_N,\phi]) + \frac{1}{2} m_s^2\phi^2 + \frac{1}{4}\phi^4\Big), \tag{3.3}$$

$$S_f = -\frac{1}{\kappa^2}\int d^D x\sqrt{G}\Big(\bar\psi\gamma^A E^M_A\Big(\partial_M\psi + i[A_M,\psi] + \frac{1}{4}\Omega^{BC}_M\gamma_{BC}\psi\Big) + m_f\bar\psi\psi\Big), \tag{3.4}$$

where A_M, ϕ, and ψ are $N\times N$ matrix fields, and $F_{MN} = \partial_M A_N - \partial_N A_M + i[A_M, A_N]$. By expanding A_M as

$$A_M = E^A_M X_A \tag{3.5}$$

and using the equations described in the previous section, we rewrite (3.2) as

$$S_{\rm YM} = -\frac{1}{4\kappa^2}\int dg\, \mathrm{Tr}(\mathcal{L}_A X_B - \mathcal{L}_B X_A - i f_{ABC} X_C + [X_A, X_B])^2. \tag{3.6}$$

In a similar manner, (3.3) and (3.4) are rewritten as

$$S_s = \frac{1}{\kappa^2}\int dg\,\mathrm{Tr}\Big(-\frac{1}{2}(\mathcal{L}_A\phi + [X_A,\phi])^2 + \frac{1}{2}m_s^2\phi^2 + \frac{1}{4}\phi^4\Big), \tag{3.7}$$

$$S_f = -\frac{1}{\kappa^2}\int dg\,\mathrm{Tr}\Big(i\bar\psi\gamma^A(\mathcal{L}_A\psi + [X_A,\psi]) + \frac{1}{8} f_{ABC}\bar\psi\gamma^{ABC}\psi + m_f\bar\psi\psi\Big). \tag{3.8}$$

The theory possesses the $G\times G$ symmetry, while the left G symmetry is not necessary for the large N reduction. We take the planar ('t Hooft) limit in which

$$N\to\infty, \qquad \kappa\to 0 \quad \text{with} \quad \kappa^2 N = \lambda \text{ fixed}, \tag{3.9}$$

where λ is the 't Hooft coupling.

To obtain the reduced model, we first define an n-dimensional vector space V_n by truncating the space of the regular representation of G as follows. We label the irreducible representations of G by r, and denote the representation space of the representation r by $V^{[r]}$ and its dimension by d_r. We define a set of the irreducible representations, I_Λ, for a positive number Λ:

$$I_\Lambda = \{r; C_2(r) < \Lambda^2\}, \tag{3.10}$$

where $C_2(r)$ is the second-order Casimir of the representation r. Then, V_n is defined by

$$V_n = \bigoplus_{r\in I_\Lambda} \underbrace{V^{[r]}\oplus\ldots\oplus V^{[r]}}_{d_r}. \tag{3.11}$$

Note that the dimension of V_n is given by

$$n = \sum_{r\in I_\Lambda} d_r^2. \tag{3.12}$$

Indeed, the space of the regular representation is obtained by taking the $\Lambda\to\infty$ limit in (3.11). The $\Lambda\to\infty$ limit corresponds to the $n\to\infty$ limit, and Λ plays the role of a ultraviolet cutoff. We next introduce a k-dimensional vector space W_k and consider the tensor product space

$$\mathcal{V}_N = V_n \otimes W_k, \tag{3.13}$$

where $N = nk$ is the dimension of $\mathcal{V}_N$.

[3] We can consider other terms such as higher derivative terms and the Yukawa interaction term.

The rule to obtain the reduced model is

$$X_A(g) \to \hat{X}_A, \qquad \phi(g) \to \hat{\phi}, \qquad \psi(g) \to \hat{\psi}, \\ \mathcal{L}_a \to [\hat{L}_a,], \qquad \int dg \to v, \tag{3.14}$$

where $\hat{X}_A$, $\hat{\phi}$, $\hat{\psi}$, and $\hat{L}_A$ are $N \times N$ Hermitian matrices that are linear operators acting on $\mathcal{V}_N$. $\hat{L}_A$ take the form

$$\hat{L}_A = \Big(\bigoplus_{r\in I_\Lambda} \underbrace{L_A^{[r]} \oplus \ldots \oplus L_A^{[r]}}_{d_r}\Big) \otimes 1_k, \tag{3.15}$$

where $L_A^{[r]}$ are the representation matrices of t_A in the representation r. v is given by

$$v = V/n, \tag{3.16}$$

where V is the volume of G:

$$V = \int dg. \tag{3.17}$$

Applying (3.14) to (3.6), (3.7), and (3.8), we obtain the reduced model of (3.1),

$$S_r = S_{YM,r} + S_{s,r} + S_{f,r}, \tag{3.18}$$

$$S_{\mathrm{YR},r} = -\frac{v}{4\kappa^2}\mathrm{Tr}([\hat{L}_A, \hat{X}_B] - [\hat{L}_B, \hat{X}_A] - if_{ABC}\hat{X}_C \\ + [\hat{X}_A, \hat{X}_B])^2, \tag{3.19}$$

$$S_{s,r} = \frac{v}{\kappa^2}\mathrm{Tr}\Big(-\frac{1}{2}([\hat{L}_A, \hat{\phi}] + [\hat{X}_A, \hat{\phi}])^2 + \frac{1}{2}m_s^2\hat{\phi}^2 + \frac{1}{4}\hat{\phi}^4\Big), \tag{3.20}$$

$$S_{f,r} = -\frac{v}{\kappa^2}\mathrm{Tr}\Big(i\hat{\bar{\psi}}\gamma^A([\hat{L}_A, \hat{\psi}] + [\hat{X}_A, \hat{\psi}]) \\ + \frac{1}{8}f_{ABC}\hat{\bar{\psi}}\gamma^{ABC}\hat{\psi} + m_f\hat{\bar{\psi}}\hat{\psi}\Big). \tag{3.21}$$

Making a redefinition

$$\hat{L}_A + \hat{X}_A \to \hat{X}_A \tag{3.22}$$

leads to

$$S'_r = S'_{YM,r} + S'_{s,r} + S'_{f,r}, \tag{3.23}$$

$$S'_{\mathrm{YM},r} = -\frac{v}{4\kappa^2}\mathrm{Tr}([\hat{X}_A, \hat{X}_B] - if_{ABC}\hat{X}_C)^2, \tag{3.24}$$

$$S'_{s,r} = \frac{v}{\kappa^2}\mathrm{Tr}\Big(-\frac{1}{2}[\hat{X}_A, \hat{\phi}]^2 + \frac{1}{2}m_s^2\hat{\phi}^2 + \frac{1}{4}\hat{\phi}^4\Big), \tag{3.25}$$

$$S'_{f,r} = -\frac{v}{\kappa^2}\mathrm{Tr}\Big(i\hat{\bar{\psi}}\gamma^A[\hat{X}_A, \hat{\psi}] + \frac{1}{8}f_{ABC}\hat{\bar{\psi}}\gamma^{ABC}\hat{\psi} \\ + m_f\hat{\bar{\psi}}\hat{\psi}\Big). \tag{3.26}$$

Note that S'_r is identical to the dimensional reduction of (3.1) to zero dimension. $\hat{X}_A = \hat{L}_A$ is a classical solution of S'_r, around which we expand S'_r to obtain S_r.

The statement of the large N reduction is as follows. Here we assume that G is semisimple. If we expand (3.23) around $\hat{X}_A = \hat{L}_A$, the planar limit of (3.1) is retrieved in the limit in which

$$n \to \infty, \qquad k \to \infty, \\ \kappa \to 0, \quad \text{with} \quad \lambda = \kappa^2 N = \kappa^2 nk \text{ fixed}. \tag{3.27}$$

For instance, the correspondence for the free energy is given by

$$\frac{F}{N^2 V} = \frac{F_r}{N^2 v}, \tag{3.28}$$

where F and F_r are the free energies of the original theory and the reduced model, respectively. For the correspondence for the correlation functions, see [19]. The reduced model (3.23) respects the $G \times G$ symmetry and the gauge symmetry of the original theory. The latter corresponds to the symmetry given by

$$\hat{X}' = U\hat{X}U^{-1} \tag{3.29}$$

for an arbitrary $N \times N$ unitary matrix U, where $\hat{X}$ stands for $\hat{X}_A$ or $\hat{\phi}$ or $\hat{\psi}$ or $\hat{\bar{\psi}}$.

If G is not semisimple, the above statement does not hold as it stands.[4] The zero-dimensional massless modes around the background $\hat{X}_A = \hat{L}_A$ in (3.23) makes the background unstable. To resolve this problem, we need a remedy such as the quenching [2,4] or the twisting [6].

IV. LARGE N REDUCTION ON G/H

A. Theories on G/H obtained by the dimensional reduction of $G \times G$ symmetric theories on G

In this subsection, we study the large N reduction for theories on G/H that are obtained by the dimensional reduction of $G \times G$ symmetric theories on G. For the dimensional reduction of such theories, see also [22].

Here, as an illustration, we examine the theory (3.1). As explained in detail below, the dimensional reduction to G/H is achieved by imposing the constraints

$$\mathcal{L}_a X_A = if_{aAB}X_B, \tag{4.1}$$

$$\mathcal{L}_a \phi = 0, \tag{4.2}$$

$$\mathcal{L}_a\psi = \frac{i}{4}f_{aAB}\gamma^{AB}\psi, \qquad \mathcal{L}_a\bar{\psi} = -\frac{i}{4}f_{aAB}\bar{\psi}\gamma^{AB}, \tag{4.3}$$

[4]There is no problem for matter fields even if G is not semisimple. In fact, (3.20) and (3.21) without $\hat{X}_A$ retrieve the planar limit of (3.3) and (3.4) without the gauge field, respectively.

on the theory. These constraints are, for instance, realized by adding

$$\int dg\,\mathrm{Tr}\Big(M_g^2(\mathcal{L}_a X_B - if_{aBC}X_C)^2 + M_s^2(\mathcal{L}_a\phi)^2 + M_f\Big(\mathcal{L}_a\bar{\psi} + \frac{i}{4}f_{aAB}\bar{\psi}\gamma^{AB}\Big)\Big(\mathcal{L}_a\psi - \frac{i}{4}f_{aCD}\gamma^{CD}\psi\Big)\Big) \tag{4.4}$$

to the action and taking the M_g, M_s, $M_f \to \infty$ limit. Because of these constraints, the $G\times G$ symmetry of (3.1) is broken to the right G symmetry. As emphasized in the beginning of Sec. III, the right G symmetry is sufficient for the large N reduction to hold. The large N reduction, therefore, holds for the theory obtained by dimensionally reducing (3.1) to G/H as follows. Applying the rule (3.14) to the theory (3.1) with (4.1), (4.2), and (4.3), leads to imposing constraints

$$[\hat{L}_a, \hat{X}_B] = if_{aBC}\hat{X}_C, \tag{4.5}$$

$$[\hat{L}_a, \hat{\phi}] = 0, \tag{4.6}$$

$$[L_a, \psi] = \frac{i}{4}f_{aCD}\gamma^{CD}\psi, \qquad [\hat{L}_a, \hat{\bar{\psi}}] = -\frac{i}{4}f_{aAB}\hat{\bar{\psi}}\gamma^{AB}, \tag{4.7}$$

on (3.18) or (3.23). Note that the redefinition $\hat{L}_\alpha + X_\alpha \to \hat{X}_\alpha$ keeps the constraint (4.5) invariant. For instance, these constrains are realized by adding

$$\mathrm{Tr}\Big(M_g^2([\hat{L}_a.\hat{X}_B] - if_{aBC}\hat{X}_C)^2 + M_s^2[\hat{L}_a, \hat{\phi}]^2 + M_f\Big([\hat{L}_a, \hat{\bar{\psi}}] + \frac{i}{4}f_{aAB}\hat{\bar{\psi}}\gamma^{AB}\Big)\Big([L_a, \psi] - \frac{i}{4}f_{aCD}\gamma^{CD}\psi\Big)\Big) \tag{4.8}$$

to (3.18) or (3.23) and taking the M_g, M_s, $M_f \to \infty$ limit. $\hat{X}_A = \hat{L}_A$ satisfies (4.5), (4.6), and (4.7), and is a classical solution of (3.23) with (4.8). We expand (3.23) with (4.8) around the classical solution to obtain (3.18) with (4.8). To summarize, the reduced model of the theory on G/H is the matrix model (3.23) with the constraints (4.5) and (4.6). The reduced model retrieves the planar limit of the theory on G/H in the limit (3.27). It respects the right G symmetry of the theory on G/H. It also has the gauge symmetry (3.29) with the constraint

$$[L_a, U] = 0 \tag{4.9}$$

satisfied. This corresponds to the gauge symmetry of the theory on G/H.

In what follows, we see that imposing (4.1), (4.2), and (4.3), on (3.1) indeed yields the dimensional reduction to G/H. The left G symmetry corresponds to the invariance of (3.1) under the transformation

$$A_M \to A_M + \epsilon\delta_A A_M, \tag{4.10}$$

$$\phi \to \phi + \epsilon\delta_A\phi, \tag{4.11}$$

$$\psi \to \psi + \epsilon\Big(\delta_A\psi + \frac{1}{4}f_{ABC}\gamma^{BC}\psi\Big), \qquad \bar{\psi} \to \bar{\psi} + \epsilon\Big(\delta_A\bar{\psi} - \frac{1}{4}f_{ABC}\bar{\psi}\gamma^{BC}\Big). \tag{4.12}$$

This invariance follows from (2.21), (2.22), and (2.25). Note that the transformation of the fermion includes the local Lorentz transformation as well as the Lie derivative, because E_M^A and Ω_M^{AB} are invariant under such a transformation. By using $\mathcal{L}_A$, (4.10), (4.11), and (4.12) are expressed as

$$X_B \to X_B + \epsilon(i\mathcal{L}_A X_B + f_{ABC}X_C), \tag{4.13}$$

$$\phi \to \phi + \epsilon i\mathcal{L}_A\phi, \tag{4.14}$$

$$\psi \to \psi + \epsilon\Big(i\mathcal{L}_A\psi + \frac{1}{4}f_{ABC}\gamma^{BC}\psi\Big), \qquad \bar{\psi} \to \bar{\psi} + \epsilon\Big(i\mathcal{L}_A\bar{\psi} - \frac{1}{4}f_{ABC}\bar{\psi}\gamma^{BC}\Big). \tag{4.15}$$

Hence, by imposing the constraints (4.1), (4.2), and (4.3) on (3.1), we can make a dimensional reduction from G to G/H, which is the so-called consistent truncation. Namely, every solution to the equation of motion in the dimensionally reduced theory is also a solution to the equation of motion in the original theory.

Let us obtain the explicit form of the resultant theory on G/H. Using the equations described in Sec. II, we solve (4.1) as

$$X_\alpha = Ad(h(y))_{\alpha\beta}e^\mu_\beta(\sigma)a_\mu(\sigma), \tag{4.16}$$

$$X_a = -Ad(h(y))_{ab}\phi_b(\sigma). \tag{4.17}$$

Similarly, (4.2) is solved as

$$\phi = \phi(\sigma). \tag{4.18}$$

To solve (4.3), we introduce $\rho(t_A)$ defined by

$$\rho(t_A) = -\frac{i}{4}f_{ABC}\gamma^{BC}. \tag{4.19}$$

It satisfies

$$[\rho(t_A), \rho(t_B)] = if_{ABC}\rho(t_C). \tag{4.20}$$

Then, we can solve (4.3) as

$$\psi = e^{i\theta^a(y)\rho(t_a)}\chi(\sigma), \qquad \bar{\psi} = \bar{\chi}(\sigma)e^{-i\theta^a(y)\rho(t_a)}, \tag{4.21}$$

where $\theta^a(y)$ is defined by

$$h = e^{i\theta^a(y)t_a}. \tag{4.22}$$

Substituting (4.16) and (4.17) into (3.2) leads to

$$
\begin{aligned}
S^{G/H}_{YM} = \frac{w}{\kappa^2}\int d^{D-d}\sigma\sqrt{K}\,\mathrm{Tr}\Big(&\frac{1}{4}(f_{abc}\phi_c + i[\phi_a,\phi_b])^2 \\
&+\frac{1}{2}K^{\mu\nu}(\partial_\mu\phi_a + i[a_\mu,\phi_a] - e^b_\mu f_{abc}\phi_c) \\
&\times(\partial_\nu\phi_a + i[a_\nu,\phi_a] - e^d_\nu f_{ade}\phi_e) \\
&+\frac{1}{4}K^{\mu\lambda}K^{\nu\rho}(f_{\mu\nu} - b^a_{\mu\nu}\phi_a)(f_{\lambda\rho} - b^b_{\lambda\rho}\phi_b)\Big),
\end{aligned} \tag{4.23}
$$

where w is the volume of H, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu + i[a_\mu, a_\nu]$, and $b^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu - f_{abc}e^b_\mu e^c_\nu = f_{a\alpha\beta}e^\alpha_\mu e^\beta_\nu$. The final expression is indeed independent of y. We have obtained YM theory coupled to d Higgs fields on G/H. This result agrees with the one in [22]. Similarly, substituting (4.16), (4.17), and (4.18), into (3.3), we obtain

$$
\begin{aligned}
S^{G/H}_{s} = \frac{w}{\kappa^2}\int d^{D-d}\sigma\sqrt{K}\,\mathrm{Tr}\Big(&\frac{1}{2}K^{\mu\nu}(\partial_\mu\phi + i[a_\mu,\phi]) \\
&\times(\partial_\nu\phi + i[a_\nu,\phi]) - \frac{1}{2}[\phi_a,\phi]^2 \\
&+\frac{1}{2}m_s^2\phi^2 + \frac{1}{4}\phi^4\Big).
\end{aligned} \tag{4.24}
$$

Finally, by using (4.16), (4.17), and (4.21), and the equation

$$
e^{-i\theta^a(y)\rho(t_a)}\gamma_A e^{i\theta^b(y)\rho(t_b)} = Ad(h)_{AB}\gamma_B, \tag{4.25}
$$

(3.4) becomes

$$
\begin{aligned}
S^{G/H}_{f} = -\frac{w}{\kappa^2}\int d^{D-d}\sigma\sqrt{k}\,\mathrm{Tr}\Big(&e^\mu_\alpha\bar{\chi}\gamma^\alpha\Big(\partial_\mu\chi + \frac{1}{4}\omega^{\beta\gamma}_\mu\gamma_{\beta\gamma}\chi \\
&+ i[a_\mu,\chi]\Big) - i\bar{\chi}\gamma^a[\phi_a,\chi] + \frac{1}{4}f_{abc}e^\mu_\alpha e^c_\mu\bar{\chi}\gamma^{ab\alpha}\chi \\
&-\frac{1}{8}f_{abc}\bar{\chi}\gamma^{abc}\chi + \frac{1}{8}f_{a\alpha\beta}\bar{\chi}\gamma^{a\alpha\beta}\chi + m_f\bar{\chi}\chi\Big).
\end{aligned} \tag{4.26}
$$

We have obtained $2^{d/2}$-flavor fermions for even d, $2^{(d+1)/2}$-flavor fermions for odd d and even D, and $2^{(d-1/2)}$-flavor fermions for odd d and odd D.

B. Minimal theories on G/H

In the previous subsection, we obtained the reduced model of (4.23), which is YM theory with d Higgs scalars on G/H originating from the consistent truncation of pure YM theory on G. We also obtained the reduced model of multiflavor fermions on G/H originating from one-flavor fermion on G. In this subsection, we will study the large N reduction for "minimal" theories on G/H: pure YM theory on G/H and one-flavor fermion on G/H.

We first study pure YM theory on G/H. As explained below, it is equivalent to a theory on G

$$
\begin{aligned}
S^{\min}_{\mathrm{YM}} = -\frac{1}{4\kappa^2}\int dg\,\mathrm{Tr}(&\mathcal{L}_\alpha X_\beta - \mathcal{L}_\beta X_\alpha - if_{\alpha\beta\gamma}X_\gamma \\
&+[X_\alpha, X_\beta])^2,
\end{aligned} \tag{4.27}
$$

with the constraint

$$
\mathcal{L}_a X_\alpha = if_{a\alpha\beta}X_\beta. \tag{4.28}
$$

The theory (4.27) with (4.28) possesses the right G symmetry. Hence, the large N reduction holds for pure YM theory on G/H. Applying the rule (3.14) to (4.27) with (4.28), we obtain the reduced model of pure YM on G/H which is a matrix model

$$
S^{\min}_{\mathrm{YM},r} = -\frac{v}{4\kappa^2}\mathrm{Tr}([\hat{L}_\alpha,\hat{X}_\beta] - [\hat{L}_\beta,\hat{X}_\alpha] - if_{\alpha\beta\gamma}\hat{X}_\gamma)^2, \tag{4.29}
$$

with the constraint

$$
[\hat{L}_a,\hat{X}_\alpha] = if_{a\alpha\beta}\hat{X}_\beta. \tag{4.30}
$$

The reduced model retrieves the planar limit of pure YM theory on G/H in the limit (3.27). As before, the redefinition $\hat{L}_\alpha + \hat{X}_\alpha \to \hat{X}_\alpha$ in (4.29) yields

$$
S^{\min}_{\mathrm{YM},r}{}' = -\frac{v}{4\kappa^2}\mathrm{Tr}([\hat{X}_\alpha,\hat{X}_\beta] - if_{\alpha\beta\gamma}\hat{X}_\gamma - if_{\alpha\beta a}\hat{L}_a)^2. \tag{4.31}
$$

Note again that the redefinition keeps the constraint (4.30) invariant. Hence, the reduced model of pure YM theory on G/H is also given by the matrix model (4.31) with the constraint (4.30). $\hat{X}_\alpha = \hat{L}_\alpha$ satisfies the constraint (4.30) and is a classical solution of the reduced model, (4.31) with (4.30). We expand the reduced model around the classical solution and take the limit (3.27) to obtain the planar limit of pure YM theory on G/H. Note that (4.31) with (4.30) is obtained by putting $\hat{X}_a = \hat{L}_a$ in (3.24) with (4.5). The reduced model, (4.31) with (4.30), respects the right G symmetry and the gauge symmetry of pure YM theory on G/H.

Now let us see that (4.27) with (4.28), indeed, yields pure YM theory on G/H. Equation (4.27) is obtained by putting

$$
X_a = 0 \tag{4.32}
$$

and (4.28) in (3.6). Recall that (3.6) is invariant under the transformation (4.13) with $A = a$. Note also that (4.32) and (4.28) are invariant under this transformation. Hence, (4.27) has the symmetry given by

$$
X_\alpha \to X_\alpha + \epsilon(i\mathcal{L}_a X_\alpha + f_{a\alpha\beta}X_\beta). \tag{4.33}
$$

This implies that we can impose the constraint (4.28) on (4.27) to truncate (4.27) consistently to a theory on G/H. The solution of the constraint (4.28) is given in (4.16). By substituting the solution into (4.27), we indeed obtain pure YM theory on G/H

$$S_{\rm YM}^{\rm min}=\frac{w}{4\kappa^2}\int d^{D-d}\sigma\sqrt{K}K^{\mu\lambda}K^{\nu\rho}\,{\rm Tr}(f_{\mu\nu}f_{\lambda\rho}). \qquad (4.34)$$

Next, we study one-flavor fermion on G/H. Instead of (3.8), we consider the following theory on G:

$$S_f'=-\frac{1}{\kappa^2}\int dg\,{\rm Tr}\bigg(i\bar\psi\gamma^\alpha(\mathcal{L}_\alpha\psi+[X_\alpha,\psi]) + \frac{1}{8}f_{\alpha\beta\gamma}\bar\psi\gamma^{\alpha\beta\gamma}\psi+m_f\bar\psi\psi\bigg), \qquad (4.35)$$

with the constraints (4.28) and

$$\mathcal{L}_a\psi=\frac{i}{4}f_{a\alpha\beta}\gamma^{\alpha\beta}\psi, \qquad \mathcal{L}_a\bar\psi=-\frac{i}{4}f_{a\alpha\beta}\bar\psi\gamma^{\alpha\beta}, \qquad (4.36)$$

where ψ and $\bar\psi$ are a $2^{(D-d)/2}$-component fermion for even $D-d$ and a $2^{(D-d-1)/2}$-component fermion for odd $D-d$. Indeed, while (4.35) is a theory on G, γ^α are the gamma matrices in $D-d$ dimensions. As we will see below, the theory (4.35) with these constrains represents one-flavor fermion on G/H. It possesses the right G symmetry, so that the large N reduction holds for it as in the case of pure YM theory on G/H. Applying the rule (3.14) to (4.35) with (4.28) and (4.36) and making the redefinition $\hat L_\alpha+\hat X_\alpha\to\hat X_\alpha$, we obtain the reduced model of one-flavor fermion on G/H which is a matrix model

$$S_{f,r}^{\rm min}=-\frac{v}{\kappa^2}{\rm Tr}\bigg(i\bar\psi\gamma^\alpha[\hat X_\alpha,\psi]+\frac{1}{8}f_{\alpha\beta\gamma}\bar\psi\gamma^{\alpha\beta\gamma}\psi + m_f\bar\psi\psi\bigg), \qquad (4.37)$$

with the constraints (4.30) and

$$[\hat L_a,\psi]=\frac{i}{4}f_{a\alpha\beta}\gamma^{\alpha\beta}\psi, \qquad [\hat L_a,\bar\psi]=-\frac{i}{4}f_{a\alpha\beta}\bar\psi\gamma^{\alpha\beta}. \qquad (4.38)$$

We expand the reduced model around a classical solution $\hat X_\alpha=\hat L_\alpha$ and take the limit (3.27) to retrieve the planar limit of one-flavor fermion on G/H. The reduced model respects the right G symmetry and the gauge symmetry of one-flavor fermion on G/H.

Finally, let us see that the theory (4.35) with the constraints (4.28) and (4.36) is indeed one-flavor theory on G/H. It is easy to verify that (4.35) is invariant under the transformation

$$X_\alpha\to X_\alpha+\epsilon(\mathcal{L}_aX_\alpha-if_{a\alpha\beta}X_\beta), \qquad (4.39)$$

$$\begin{aligned}\psi&\to\psi+\epsilon\bigg(\mathcal{L}_a\psi-\frac{i}{4}f_{a\alpha\beta}\gamma^{\alpha\beta}\psi\bigg),\\ \bar\psi&\to\bar\psi+\epsilon\bigg(\mathcal{L}_a\bar\psi+\frac{i}{4}f_{a\alpha\beta}\bar\psi\gamma^{\alpha\beta}\bigg).\end{aligned} \qquad (4.40)$$

We can, therefore, impose (4.28) and (4.36) on (4.35) to truncate (4.35) consistently to a theory on G/H. We will check below that the resulting theory is the one with one-flavor Dirac fermion on G/H. We define $\tilde\rho(t_a)$ by

$$\tilde\rho(t_a)=-\frac{i}{4}f_{a\alpha\beta}\gamma^{\alpha\beta}. \qquad (4.41)$$

$\tilde\rho(t_a)$ satisfies

$$[\tilde\rho(t_a),\tilde\rho(t_b)]=if_{abc}\tilde\rho(t_c), \qquad (4.42)$$

$$e^{-i\theta^a(y)\tilde\rho(t_a)}\gamma_\alpha e^{i\theta^b(y)\tilde\rho(t_b)}=Ad(h)_{\alpha\beta}\gamma_\beta. \qquad (4.43)$$

We can solve (4.36) as

$$\psi=e^{i\theta^a(y)\tilde\rho(t_a)}\chi(\sigma), \qquad \bar\psi=\bar\chi(\sigma)e^{-i\theta^a(y)\tilde\rho(t_a)}. \qquad (4.44)$$

Substituting (4.16) and (4.44) into (4.35) indeed yields

$$S_f^{\rm min}=-\frac{w}{\kappa^2}\int d^{D-d}\sigma\sqrt{K}\,{\rm Tr}\bigg(e_\alpha^\mu\bar\chi\gamma^\alpha \times\bigg(\partial_\mu\chi+\frac{1}{4}\omega_\mu^{\beta\gamma}\gamma_{\beta\gamma}\chi+i[a_\mu,\chi]\bigg)+m_f\bar\chi\chi\bigg). \qquad (4.45)$$

V. CS-LIKE THEORIES ON G AND G/H

In this section, we construct CS-like theories on G and G/H and give their reduced models. The CS 3-form on G is defined by

$$\omega_3={\rm Tr}\bigg(A\wedge dA+\frac{2i}{3}A\wedge A\wedge A\bigg). \qquad (5.1)$$

For an arbitrary $N\times N$ unitary matrix, the gauge transformation is given by

$$A'=idUU^{-1}+UAU^{-1}. \qquad (5.2)$$

As is well known, the CS 3-form is transformed under the gauge transformation as

$$\omega_3'=\omega_3-id\,{\rm Tr}(U^{-1}dU\wedge A) -\tfrac{1}{3}{\rm Tr}(dUU^{-1}\wedge dUU^{-1}\wedge dUU^{-1}). \qquad (5.3)$$

The 3-form in the third term of the right-hand side is closed:

$$d\,{\rm Tr}(dUU^{-1}\wedge dUU^{-1}\wedge dUU^{-1})=0, \qquad (5.4)$$

which means that the 3-form belongs to $H^3(G)$.

We define a 3-form f on G in terms of the structure constant f_{ABC}:

$$f=\frac{1}{3!}f_{ABC}E^A\wedge E^B\wedge E^C. \qquad (5.5)$$

It is easy to show that

$$df=0, \qquad (5.6)$$

$$d*f=0, \qquad (5.7)$$

which means that f and $*f$ are harmonic forms so that f

and $\tilde{f}$ are nonzero elements of $H^3(G)$ and $H^{D-3}(G)$, respectively. We define the CS-like theory on G

$$S = \frac{1}{\alpha} \int \omega_3 \wedge *f. \tag{5.8}$$

We can show that (5.8) has the gauge symmetry as follows. Using (5.3), (5.4), and (5.7), and the Poincare duality, we find that S transforms to

$$S' = S - \frac{1}{3\alpha} \int_{C_3} \mathrm{Tr}(dUU^{-1} \wedge dUU^{-1} \wedge dUU^{-1}), \tag{5.9}$$

where C_3 is the 3-cycle dual to $*f$. As in the case of three-dimensional CS theory, if we normalize α appropriately, we obtain

$$S' = S + 2\pi n \tag{5.10}$$

for an integer n, so that e^{iS} is indeed invariant.

Equation (5.8) is rewritten as

$$\begin{aligned} S &= \frac{1}{6\alpha} \int d^D x \sqrt{G} E_A^M E_B^N E_C^L f^{ABC} \mathrm{Tr}\Big(A_M \partial_N A_L \\ &\quad + \frac{2i}{3} A_M A_N A_L\Big) \\ &= \frac{1}{6\alpha} \int dg f^{ABC} \mathrm{Tr}\Big(iX_A \mathcal{L}_B X_C + \frac{1}{2} f_{BCD} X_A X_D \\ &\quad + \frac{2i}{3} X_A X_B X_C\Big). \end{aligned} \tag{5.11}$$

The reduced model of (5.11) is

$$\begin{aligned} S_r = \frac{v}{6\alpha} f^{ABC} \mathrm{Tr}\Big(&i\hat{X}_A [\hat{L}_B, \hat{X}_C] + \frac{1}{2} f_{BCD} \hat{X}_A \hat{X}_D \\ &+ \frac{2i}{3} \hat{X}_A \hat{X}_B \hat{X}_C\Big), \end{aligned} \tag{5.12}$$

which retrieves the planar limit of (5.8) in the limit (3.27). By making the redefinition $\hat{L}_A + \hat{X}_A \to \hat{X}_A$, we obtain from (5.12) up to an irrelevant constant term

$$S_r' = \frac{v}{6\alpha} f^{ABC} \mathrm{Tr}\Big(\frac{1}{2} f_{BCD} \hat{X}_A \hat{X}_D + \frac{2i}{3} \hat{X}_A \hat{X}_B \hat{X}_C\Big). \tag{5.13}$$

$\hat{X}_A = \hat{L}_A$ is a classical solution of (5.13). We expand (5.13) around $\hat{X}_A = \hat{L}_A$ and take the limit (3.27). Then, (5.13) retrieves the planar limit of the original CS-like theory. For $G \simeq SU(2)$, (5.8) is nothing but pure CS theory on the 3-sphere.[5]

Next, we study the CS-like theory on G/H. It is easy to see from (2.21) that

$$\delta_A (f^{BCD} E_B^M E_C^N E_D^L) = 0. \tag{5.14}$$

This implies that (5.11) is invariant under the transformation (4.10). Hence, by imposing the constraints (4.32) and (4.28) on (5.11), we can truncate (5.11) to a theory on G/H as in the case of YM theory on G. The resulting theory is a CS-like theory on G/H which takes the form

$$\begin{aligned} S^{G/H} &= \frac{w}{6\alpha} \int d^{D-d} \sigma \sqrt{K} f^{\alpha\beta\gamma} \mathrm{Tr}\Big(iX_\alpha \mathcal{L}_\beta X_\gamma \\ &\quad + \frac{1}{2} f_{\beta\gamma\delta} X_\alpha X_\delta + \frac{2i}{3} X_\alpha X_\beta X_\gamma\Big) \\ &= \frac{w}{\alpha} \int \tilde{w}_3 \wedge *\tilde{f}. \end{aligned} \tag{5.15}$$

Here $\tilde{\omega}_3$ is the CS 3-form on G/H:

$$\tilde{\omega}_3 = \mathrm{Tr}\Big(a \wedge \tilde{d}a + \frac{2i}{3} a \wedge a \wedge a\Big), \tag{5.16}$$

with

$$a = a_\mu d\sigma^\mu, \qquad \tilde{d} = d\sigma^\mu \frac{\partial}{\partial \sigma^\mu}. \tag{5.17}$$

$\tilde{f}$ is a 3-form on G/H, which is analogous to f on G:

$$\tilde{f} = f_{\alpha\beta\gamma} e^\alpha \wedge e^\beta \wedge e^\gamma. \tag{5.18}$$

$*$ stands for the Hodge dual on G/H. (4.16) has been used to obtain the second line of (5.15). By construction, (5.15) has the symmetry under the gauge transformation (5.2) with $\delta_a U = 0 (\partial U / \partial y^m = 0)$. Indeed, we can easily show that

$$\tilde{d} * \tilde{f} = 0 \tag{5.19}$$

which means that $*\tilde{f} \in H^{D-d-3}(G/H)$. Hence, under the gauge transformation

$$a' = i\tilde{d}uu^{-1} + uau^{-1} \tag{5.20}$$

with u an arbitrary σ-dependent $N \times N$ unitary matrix, (5.15) is transformed as $S' = S + 2\pi n$, as in the case of a CS-like theory on G.

The reduced model of (5.15) is

$$\begin{aligned} S_r^{G/H} = \frac{v}{6\alpha} f^{\alpha\beta\gamma} \mathrm{Tr}\Big(&i\hat{X}_\alpha [\hat{L}_\beta, \hat{X}_\gamma] + \frac{1}{2} f_{\beta\gamma\delta} \hat{X}_\alpha \hat{X}_\delta \\ &+ \frac{2i}{3} \hat{X}_\alpha \hat{X}_\beta \hat{X}_\gamma\Big), \end{aligned} \tag{5.21}$$

with the constraint (4.30). The redefinition $\hat{L}_\alpha + \hat{X}_\alpha \to \hat{X}_\alpha$ in (5.21) leads to

$$\begin{aligned} S_r^{G/H\prime} = \frac{v}{6\alpha} f^{\alpha\beta\gamma} \mathrm{Tr}\Big(&\frac{1}{2} f_{\beta\gamma\delta} \hat{X}_\alpha \hat{X}_\delta + \frac{2i}{3} \hat{X}_\alpha \hat{X}_\beta \hat{X}_\gamma \\ &+ f_{\beta\gamma a} \hat{L}_a \hat{X}_\alpha\Big), \end{aligned} \tag{5.22}$$

up to an irrelevant constant term.

[5] In [26,27], the different type of the large N reduction on S^3 developed in [20] was explicitly demonstrated for this theory.

VI. CONCLUSION AND DISCUSSION

In this paper, we showed that the large N reduction holds on coset spaces. The reduced models of large N field theories on coset spaces are obtained by imposing the constraints on the reduced models of the corresponding theories on group manifolds. We also constructed CS-like theories on group manifolds and coset spaces, and gave their reduced models.

As an application of our findings in this paper, we can define large N field theories on $S^4 \simeq SO(5)/SO(4)$ nonperturbatively in terms of their reduced models. In particular, it is interesting to construct the reduced models of supersymmetric gauge theories on S^4. While the reduced models of those on $R \times S^3$ constructed in [19,20] still has the continuous time direction, the reduced models of those on S^4 are indeed defined in zero dimension so that they would be more tractable. The large N reduction for CS-like theories can be applied to the study of the theory proposed by Aharony *et al.* theory [40].

We hope to find reduced models of large N field theories on a wider class of curved spaces and eventually to make progress in the description of curved space-times in the matrix models conjectured to give a nonperturbative formulation of superstring theory.

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