Report on a local in time solvability of free surface problems for the Navier–Stokes equations with surface tension

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Dedicated to Professor Vsevolod Alekseevich Solonnikov
on the occasion of his 75th birthday.

Abstract. We consider the free boundary problem of the Navier-Stokes equation with surface tension. Our initial domain Ω is one of a bounded domain, an exterior domain, a perturbed half-space or a perturbed layer in $\mathbb{R}^n$ $(n \ge 2)$. We report a local in time unique existence theorem in the space $W^{2,1}_{q,p} = L_p((0,T), W^2_q(\Omega)) \cap W^1_q((0,T), L_q(\Omega))$ with some $T > 0$, $2 < p < \infty$ and $n < q < \infty$ for any initial data which satisfy compatibility condition. Our theorem can be proved by the standard fixed point argument based on the L_p-L_q maximal regularity theorem for the corresponding linearized equations. Our results cover the cases of a drop problem and an ocean problem that were studied by Solonnikov [15, 16, 18, 19], Beale [3] and Tani [21].

1 Introduction and Results

In this paper we would like to report a result concerning the local in time existence theorem of the free boundary problems of the motion of a viscous, incompressible fluid for the Navier-Stokes equations. In the models the effect of surface tension on free surface is included.

Let Ω_0 be an initial domain in $\mathbb{R}^n$ $(n \ge 2)$ and v_0 be an initial velocity. Both are given. Throughout the paper, we assume that Ω_0 is one of the following domains:

- a bounded domain;
- an exterior domain, that is the complement of Ω_0 is a bounded domain;
- a perturbed half space, that is there exist a constant $R > 0$ and a function $\eta(x')$, $x' = (x_1, \ldots, x_{n-1})$ such that
$$\Omega_0 \cap B^R = \{x = (x', x_n) \in \mathbb{R}^n \mid x_n < \eta(x')\} \cap B^R, \tag{1.1}$$
where $B^R = \{x \in \mathbb{R}^n \mid |x| > R\}$;
- a perturbed layer, that is there exist a constant $R > 0$ and two functions $\eta_1(x')$ and $\eta_2(x')$ such that
$$\Omega_0 \cap B^R = \{x = (x', x_n) \in \mathbb{R}^n \mid \eta_1(x') < x_n < \eta_2(x')\} \cap B^R. \tag{1.2}$$

*Partially supported by JSPS Grant-in-aid for Scientific Research (C) #20540164.
MOS Subject Classification: 35Q30, 35R35, 76D05.
Keywords: Navier-Stokes equation, free boundary problem, surface tension, local in time solvability.

Concerning the boundary of Ω_0, we consider the following cases:

- When Ω_0 is a bounded domain, we consider the two cases:
  - the boundary of Ω_0 consists of two hypersurfaces Γ_0 and Γ_b such that $\Gamma_0 \cap \Gamma_b = \emptyset$,
  - the boundary of Ω consists of only one hypersurface Γ_0. In this case, $\Gamma_b = \emptyset$.
- When Ω_0 is an exterior domain or a perturbed half-space, the boundary of Ω_0 consists of only one hypersurface Γ_0. In this case, $\Gamma_b = \emptyset$.
- When Ω_0 is a perturbed hypersurface, the boundary of Ω_0 consists of two hypersurfaces Γ_0 and Γ_b such that $\Gamma_0 \cap \Gamma_b = \emptyset$. Moreover, we assume that there exists an h such that $\Gamma_0 \subset \{x \in \mathbb{R}^n \mid x_n > 3h\}$ and $\Gamma_b \subset \{x \in \mathbb{R}^n \mid x_n < h\}$.

Our problem is to find the domain Ω_t for $t > 0$ occupied by the fluid, the velocity vector field $v(x, t)$ and the scalar pressure $\theta(x, t)$, $x \in \Omega_t$, satisfying the Navier-Stokes equations:

$$
\begin{aligned}
&\partial_t v + (v \cdot \nabla)v - \operatorname{Div} S(v, \theta) = f(x, t) && \text{in } \Omega_t,\ t > 0,\\
&\operatorname{div} v = 0 && \text{in } \Omega_t,\ t > 0,\\
&S(v, \theta)\nu_t = \sigma \mathcal{H} \nu_t - g_a x_n \nu_t && \text{on } \Gamma_t,\ t > 0,\\
&V_n = v \cdot \nu_t && \text{on } \Gamma_t,\ t > 0,\\
&v = 0 && \text{on } \Gamma_b,\ t > 0,\\
&v|_{t=0} = v_0 && \text{in } \Omega_0,
\end{aligned}
\tag{1.3}
$$

where the boundary of Ω_t is denoted by $\partial\Omega_t = \Gamma_t \cup \Gamma_b$ with Γ_t being the free (deformable) part. In (1.3), ν_t is the unit outward normal to Γ_t, $S(v, \theta) = \mu D(v) - \theta I$ is the stress tensor, $D(v) = (D(v))_{ij} = \partial v_i/\partial x_j + \partial v_j/\partial x_i$ is a deformation tensor, $\mathcal{H}$ is the mean curvature which is given by $\mathcal{H}\nu_t = \Delta_{\Gamma(t)} x$, $\Delta_{\Gamma(t)}$ is the Laplace-Beltrami operator on Γ_t, μ and σ denote the coefficient of viscosity and the coefficient of surface tension, that are positive constants, respectively, and $g_a > 0$ is the acceleration of gravity. V_n is the velocity of the evolution of Γ_t in the direction of outward normal ν_t. For the differentiation, we use the symbols: $\operatorname{div} v = \sum_{j=1}^n D_j v_j$,

$$
(v \cdot \nabla)v = (\sum_{j=1}^n v_j D_j v_1, \ldots, \sum_{j=1}^n v_j D_j v_n)^*, \quad \operatorname{Div} S = (\sum_{j=1}^n D_j S_{1,j}, \ldots, \sum_{j=1}^n D_j S_{n,j})^*
$$

where $D_j = \partial/\partial x_j$, M^* denotes the transposed M, $v = (v_1, \ldots, v_n)^*$ and $S = (S_{ij})$ ($n \times n$ matrix).

The problem (1.3) contains the following special cases:

- If Ω_0 is a bounded domain and $\Gamma_b = \emptyset$ and $g_a = 0$, then (1.3) is a drop problem.
- If Ω_0 is a perturbed layer then (1.3) is an ocean problem.
- If Ω_0 is a perturbed half-space with x_n being the vertical component and $\Gamma_b = \emptyset$, then (1.3) is an ocean problem without bottom.

Now, we shall discuss some known results of the unique existence theorem of problem (1.3). Let $W^{2,1}_{q,p}(\Omega \times (0, T)) = L_p((0, T), W^2_q(\Omega)) \cap W^1_p((0, T), L_q(\Omega))$ and for simplicity, we write $W^{2,1}_{q,p} = W^{2,1}_{q,p}(\Omega \times (0, T))$ for some $T > 0$ and $W^{2,1}_p = W^{2,1}_{p,p}$. We shall state mainly the case that the surface tension is taken into account.

First we mention about local in time solvability. Solonnikov formulated the drop problem and proved the local in time solvability of (1.3) for arbitrary initial data in the Sobolev-Slobodetskii space $W_2^{2+\alpha, 1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ in [15, 18, 19]. Moglilevskiĭ and Solonnikov [5] proved the local in time solvability in Hölder spaces. Schweizer [10] proved the local in time unique existence for small initial data by using the semigroup approach. Concerning the ocean problems, Allain [2] proved local in time unique solvability when $n = 2$. Tani [21] proved the local in time unique solvability in $W_2^{2+\alpha, 1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$. Prüss and Simonett [9] proved local in time unique solvability in $W_p^{2,1}$ ($p > n+2$) for two phase free boundary problem under the assumption of the smallness of the first derivative of a height function.

Concerning the global in time solvability, Solonnikov [16] proved the global in time solvability of the drop problem in $W_2^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$ for $f = 0$ provided that initial data are sufficiently small and the initial domain Ω_0 is sufficiently close to a ball. Padula and Solonnikov [8] proved the global in time unique solvability of the drop problem in Hölder spaces by using the mapping of Ω_t on a ball instead of Lagrangean coordinates. In [3], Beale proved the global in time unique solvability of the ocean problem in $H_2^{\ell,\frac{\ell}{2}}$ with $3 < \ell < \frac{7}{2}$ for $\sigma > 0$, $n = 3$ and $f = 0$ provided that the initial data are sufficiently small. Beale and Nishida [4] obtained the asymptotic power-like in time decay of the global solutions of the ocean problem. Tani and Tanaka [22] proved the global in time solvability of the ocean problem in $W_2^{2+\alpha}$ with $\alpha \in (\frac{1}{2}, 1)$ for $\sigma \geqq 0$ and $n = 3$ provided that initial data are sufficiently small by using Solonnikov's method.

Concerning the problem without surface tension, the local in time unique solvability for any initial data and the global in time unique solvability for small initial data of the drop problem were proved by Solonnikov [17] in $W_p^{2,1}$ ($n < p < \infty$), and by Shibata and Shimizu [11], [12] in $W_{q,p}^{2,1}$ ($2 < p < \infty$ and $n < q < \infty$). Also the local in time solvability in $W_p^{2,1}$ ($n < p < \infty$) was proved by Mucha and Zajączkowski [6, 7] for the drop problem, and Abels [1] for the ocean problem.

In this paper, we shall report a local in time unique existence theorem of (1.3) in the space $W_{q,p}^{2,1}$ ($2 < p < \infty$ and $n < q < \infty$) for any initial data which satisfy compatibility condition. In fact, all the results mentioned above, except for Moglilevskiĭ and Solonnikov [5] and Prüss and Simonett [9], are obtained in some Sobolev-Slobodetskii space $W_2^{2+\alpha,1+\frac{\alpha}{2}}$ with $\alpha \in (\frac{1}{2}, 1)$, that is in the L_2 framework. So far, there were no results in $W_{q,p}^{2,1}$ in the case that the surface tension is taken into account. But, to solve the problem (1.3) in the space $W_{q,p}^{2,1}$ is important from the viewpoint of lower regularity condition on the initial data.

Before stating our main results, first of all we shall discuss the formulation of the problem (1.3) by the Lagrange coordinate, instead of the Euler coordinate. Aside from the dynamical boundary condition, a further kinematic condition for Γ_t is satisfied which gives Γ_t as a set of points $x = x(\xi, t)$, $\xi \in \Gamma_0$, where $x(\xi, t)$ is the solution of the Cauchy problem:

$$\frac{dx}{dt} = v(x, t), \quad x|_{t=0} = \xi. \tag{1.4}$$

This expresses the fact that the free surface Γ_t consists for all $t > 0$ of the same fluid particles, which do not leave it and are not incident on it from Ω_t.

From now on, we write $\Omega = \Omega_0$ and $\Gamma = \Gamma_0$. The problem (1.3) can therefore be written as an initial boundary value problem in the given region Ω if we go over the Euler coordinates $x \in \Omega_t$ to the Lagrange coordinates $\xi \in \Omega$ connected with x by (1.4). If a velocity vector field $u(\xi, t) = (u_1, \dots, u_n)^*$ is known as a function of the Lagrange coordinates ξ, then this connection can be written in the form:

$$x = \xi + \int_0^t u(\xi, \tau)\, d\tau \equiv X_u(\xi, t). \tag{1.5}$$

Passing to the Lagrange coordinates in (1.3) and setting $\theta(X_u(\xi, t), t) = \pi(\xi, t)$, we obtain

$$\begin{aligned}
&\partial_t u - \operatorname{Div} S(u, \pi) = \operatorname{Div} Q(u) + R(u)\nabla\pi + f(X_u(\xi, t), t) && \text{in } \Omega,\ t > 0,\\
&\operatorname{div} u = E(u) = \operatorname{div}\tilde{E}(u) && \text{in } \Omega,\ t > 0,\\
&(S(u, \pi) + Q(u))\nu_{tu} - \sigma\mathcal{H}\nu_{tu} + g_a X_{u,n}\nu_{tu} = 0 && \text{on } \Gamma,\ t > 0,\\
&u = 0 && \text{on } \Gamma_b,\ t > 0,\\
&u|_{t=0} = u_0(\xi) && \text{in } \Omega,
\end{aligned} \tag{1.6}$$

where $u_0(\xi) = v_0(x)$ and $X_{u,n}$ stands for the n-th component of X_u. Moreover, ν_{tu} stands for the unit outer normal to Γ_t given by $\nu_{tu} = {}^tA^{-1}\nu_0/|{}^tA^{-1}\nu_0|$, where A is the matrix whose element $\{a_{jk}\}$ is the Jacobian of (1.5):

$$a_{jk} = \frac{\partial x_j}{\partial \xi_k} = \delta_{jk} + \int_0^t \frac{\partial u_j}{\partial \xi_k}\, d\tau,$$

and $Q(u)$, $R(\pi)$, $E(u)$ and $\tilde E(u)$ are nonlinear terms of the following forms:

$$Q(u) = \mu V_1(\int_0^t \nabla u\,d\tau)\nabla u, \quad R(u) = V_2(\int_0^t \nabla u\,d\tau)$$
$$E(u) = V_3(\int_0^t \nabla u\,d\tau)\nabla u, \quad \tilde E(u) = V_4(\int_0^t \nabla u\,d\tau)u \tag{1.7}$$

with some polynomials $V_j(\cdot)$ of $\int_0^t \nabla u\,d\tau$, $j = 1, 2, 3, 4$, such as $V_j(0) = 0$ (cf. Appendix in [11]).

We shall discuss a local in time unique existence theorem for the problem (1.6) instead of the problem (1.3). In order to state our main results precisely, at this point we introduce the function spaces. For any domain D in $\mathbb{R}^n$, integer m and $1 \le q \le \infty$, $L_q(D)$ and $W_q^m(D)$ denote the usual Lebesgue space and Sobolev space of functions defined on D with norms: $\|\cdot\|_{L_q(D)}$ and $\|\cdot\|_{W_q^m(D)}$, respectively. For any Banach space X with norm $\|\cdot\|_X$, X^n stands for the n product space defined by

$$X^n = \{f = (f_1, \ldots, f_n)^* \mid f_i \in X \ \ (i = 1, \ldots, n)\}.$$

The norm of X^n denotes also $\|\cdot\|_X$ which is defined by the formula: $\|f\|_X = \sum_{j=1}^n \|f_j\|_X$ for any $f = (f_1, \ldots, f_n)^* \in X^n$. For the space of the pressure term, we introduce $\hat W_q^1(D)$ which is defined by

$$\hat W_q^1(D) = \{u \in L_{q,loc}(D) \mid \nabla u \in L_q(D)^n\}.$$

For any interval $I \subset \mathbb{R}$, integer ℓ and $1 \le p \le \infty$, $L_p(I, X)$ and $W_p^\ell(I, X)$ denote the usual Lebesgue space and Sobolev space of the X-valued functions defined on I with norms: $\|\cdot\|_{L_p(I,X)}$ and $\|\cdot\|_{W_p^\ell(I,X)}$, respectively. Set

$$W_{q,p}^{m,\ell}(D \times I) = L_p(I, W_q^m(D)) \cap W_p^\ell(I, L_q(D)),$$
$$\|u\|_{W_{q,p}^{m,\ell}(D\times I)} = \|u\|_{L_p(I,W_q^m(D))} + \|u\|_{W_p^\ell(I,L_q(D))} \quad (u \in W_{q,p}^{2,1}(D \times I)). \tag{1.8}$$

Note that $W_q^0(D) = L_q(D)$ and $W_p^0(I, X) = L_p(I, X)$.

The following theorem will be proved in the forthcoming paper [14] based on Theorem 4.1 below.

Theorem 1.1. *Let $\Omega \subset \mathbb{R}^n$ $(n \ge 2)$ be one of a bounded domain, an exterior domain, a perturbed half-space or a perturbed layer. Let $\Gamma \in W_q^3$ and $\Gamma_b \in W_q^2$. Let $2 < p < \infty$ and $n < q < \infty$. Then, for any initial data $u_0 \in [L_q(\Omega), W_q^2(\Omega)]_{1-1/p,p}$ which satisfies the compatibility conditions:*

$$\operatorname{div} u_0 = 0 \ \ \text{in } \Omega, \quad D(u_0)\nu_0 - (D(u_0)\nu_0, \nu_0)\nu_0 = 0 \ \ \text{on } \Gamma, \quad u_0 = 0 \ \ \text{on } \Gamma_b, \tag{1.9}$$

and $f \in L_p(\mathbb{R}_+, L_q(\mathbb{R}^n))^n$ such that $D_j f \in L_\infty(\mathbb{R}^n \times \mathbb{R}_+)^n$ for $j = 1, \ldots, n$, there exists a $T > 0$ such that the problem (1.6) *admits a unique solution*

$$(u, \pi) \in W_{q,p}^{2,1}(\Omega \times (0, T)) \times L_p((0, T), \hat W_q^1(\Omega)).$$

Here, $[\cdot,\cdot]_{1-(1/p),p}$ denotes the real interpolation functor.

In the rest of the paper, we shall give a sketch of our idea of the proof of Theorem 1.1.

2 Reduction of the boundary condition to linearized problems

In this section, we shall discuss the reduction of the boundary condition:

$$(S(u, \pi) + Q(u))\nu_{tu} - \sigma\mathcal{H}\nu_{tu} + g_a X_{u,n}\nu_{tu} = 0, \tag{2.1}$$

which is the first key step in our proof of Theorem 1.1. Let $\mathbf{\Pi}_t$ and $\mathbf{\Pi}_0$ be projections to tangent hyperplanes of Γ_t and Γ_0, which are defined by

$$\mathbf{\Pi}_t d = d - (d, \nu_{tu})\nu_{tu}, \quad \mathbf{\Pi}_0 d = d - (d, \nu_0)\nu_0. \tag{2.2}$$

for an arbitrary vector field d defined on Γ_t and Γ_0, respectively. We know the following fact (cf. Solonnikov [20] and also the appendix below).

Lemma 2.1. *If $\nu_t \cdot \nu_0 \neq 0$, then for arbitrary vector d, $d = 0$ is equivalent to*

$$\mathbf{\Pi}_0\mathbf{\Pi}_t d = 0, \quad \nu_0 \cdot d = 0. \tag{2.3}$$

We apply Lemma 2.1 for (2.1). Since we obtain

$$\mathbf{\Pi}_t(\mu D(u) + Q(u))\nu_{tu} = 0 \tag{2.4}$$

by applying $\mathbf{\Pi}_t$ to the left hand side of (2.1), the first equation of (2.3) for (2.1) is given by

$$\mathbf{\Pi}_0\mu D(u)\nu_0 = -\mathbf{\Pi}_0(\mathbf{\Pi}_t - \mathbf{\Pi}_0)(\mu D(u)\nu_{tu}) - \mathbf{\Pi}_0\mu D(u)(\nu_{tu} - \nu_0) - \mathbf{\Pi}_0\mathbf{\Pi}_t(Q(u)\nu_{tu}), \tag{2.5}$$

where we have used $\mathbf{\Pi}_0\mathbf{\Pi}_0 = \mathbf{\Pi}_0$.

On the other hand, we shall consider the innerproduct of the boundary condition with ν_0. Using the fact that $\mathcal{H}\nu_{tu} = \Delta_{\Gamma(t)}X_u$ and substituting (1.5) for (2.1), we obtain

$$\begin{aligned}
&\nu_0 \cdot (S(u,\pi) + Q(u))\nu_{tu} - \sigma\nu_0 \cdot (\Delta_{\Gamma(t)} - \Delta_\Gamma)\Big(\xi + \int_0^t u(\xi,\tau)\,d\tau\Big) \\
&\quad - \sigma\nu_0 \cdot \Delta_\Gamma\Big(\xi + \int_0^t u(\xi,\tau)\,d\tau\Big) + g_a\nu_0 \cdot \Big(\xi_n + \int_0^t u_n(\xi,\tau)\,d\tau\Big)\nu_{tu} = 0.
\end{aligned} \tag{2.6}$$

Taking a commutator between Δ_Γ and $\nu_0\cdot$, we have

$$\nu_0 \cdot \Delta_\Gamma \int_0^t u\,d\tau = \Delta_\Gamma\Big(\int_0^t \nu_0 \cdot u\,d\tau\Big) - (\Delta_\Gamma\nu_0) \cdot \int_0^t u\,d\tau - 2\Big(\nabla_\Gamma \cdot \int_0^t u\,d\tau\Big)\nabla_\Gamma \cdot \nu_0. \tag{2.7}$$

By (2.6) and (2.7), we obtain

$$\begin{aligned}
&\nu_0 \cdot S(u,\pi)\nu_0 - \sigma\Delta_\Gamma \int_0^t \nu_0 \cdot u\,d\tau \\
&+ \sigma\Big[\Delta_\Gamma\nu_0 \cdot \int_0^t u\,d\tau + \nu_0 \cdot (\Delta_\Gamma - \Delta_{\Gamma(t)}) \int_0^t u\,d\tau + \nu_0 \cdot (\Delta_\Gamma - \Delta_{\Gamma(t)})\xi\Big] \\
&\quad = \nu_0 \cdot (S(u,\pi)(\nu_0 - \nu_{tu})) - \nu_0 \cdot (Q(u)\nu_{tu}) + \sigma\mathcal{H}_0(\Gamma) - g_a\xi_n \\
&\qquad - g_a \int_0^t u_n\,d\tau\,\nu_0 \cdot \nu_{tu} - g_a\xi_n\,\nu_0 \cdot (\nu_{tu} - \nu_0) - 2\sigma\Big(\nabla_\Gamma \cdot \int_0^t u\,d\tau\Big)\nabla_\Gamma \cdot \nu_0,
\end{aligned} \tag{2.8}$$

where we have used $\nu_0 \cdot \Delta_\Gamma\xi = \mathcal{H}_0(\Gamma)$. We denote the terms in the bracket of the left hand side of (2.8) by $F(u)$, that is

$$F(u) = \Delta_\Gamma\nu_0 \cdot \int_0^t u\,d\tau + \nu_0 \cdot (\Delta_\Gamma - \Delta_{\Gamma(t)}) \int_0^t u\,d\tau + \nu_0 \cdot (\Delta_\Gamma - \Delta_{\Gamma(t)})\xi.$$

In (2.8), since $\Delta_{\Gamma(t)}$ and Δ_Γ contain the second order tangential derivatives of X_u, in order to avoid the loss of regularity we apply the inverse operator $(m - \Delta_\Gamma)^{-1}$ with sufficiently large number m to $F(u)$. Namely, we proceed as follows:

$$\begin{aligned}
&\nu_0 \cdot S(u,\pi)\nu_0 + \sigma(m - \Delta_\Gamma)\left(\nu_0 \cdot \int_0^t u\,d\tau + (m - \Delta_\Gamma)^{-1}F(u)\right) - \sigma m\nu_0 \cdot \int_0^t u\,d\tau \\
&\quad = \nu_0 \cdot (S(u,\pi)(\nu_0 - \nu_{tu})) - \nu_0 \cdot (Q(u)\nu_{tu}) + \sigma\mathcal{H}_0(\Gamma) - g_a\xi_n \\
&\qquad - g_a \int_0^t u_n\,d\tau\,\nu_0 \cdot \nu_{tu} - g_a\xi_n\,\nu_0 \cdot (\nu_{tu} - \nu_0) - 2\sigma\Big(\nabla_\Gamma \cdot \int_0^t u\,d\tau\Big)\nabla_\Gamma \cdot \nu_0.
\end{aligned} \tag{2.9}$$

We introduce a function η by the formula:

$$\eta = \nu_0 \cdot \int_0^t u\,d\tau + (m - \Delta_\Gamma)^{-1}F(u) \quad \text{on } \Gamma. \tag{2.10}$$

Then, from (2.9) and (2.10), we obtain the two equations on the boundary Γ as follows:

$$\begin{aligned}
&\nu_0\cdot S(u,\pi)\nu_0+\sigma(m-\Delta_\Gamma)\eta \\
&\quad =\nu_0\cdot(S(u,\pi)(\nu_0-\nu_{tu}))-\nu_0\cdot(Q(u)\nu_{tu})+\sigma\mathcal{H}_0(\Gamma)-g_a\xi_n-g_a\int_0^t u_n\,d\tau \\
&\qquad -g_a\int_0^t u_n\,d\tau\,\nu_0\cdot\nu_{tu}-g_a\xi_n\,\nu_0\cdot(\nu_{tu}-\nu_0)-2\sigma\Big(\nabla_\Gamma\cdot\int_0^t u\,d\tau\Big)\nabla_\Gamma\cdot\nu_0 \\
&\qquad +\sigma m\nu_0\cdot\int_0^t u\,d\tau \qquad (2.11)\\
&\partial_t\eta-\nu_0\cdot u=(m-\Delta_\Gamma)^{-1}\dot F(u), \qquad (2.12)
\end{aligned}$$

where $\dot F(u)$ denotes the derivative of $F(u)$ with respect to t.

Finally we arrive at the equivalent equation to (1.6) as follows:

$$\begin{aligned}
&\partial_t u-\operatorname{Div}S(u,\pi)=\operatorname{Div}Q(u)+R(u)\nabla\pi+f(X_u(\xi,t),t) && \text{in } \Omega,\ t>0,\\
&\operatorname{div}u=E(u)=\operatorname{div}\tilde E(u) && \text{in } \Omega,\ t>0,\\
&\partial_t\eta-\nu_0\cdot u=G(u) && \text{on } \Gamma,\ t>0,\\
&\mathbf{\Pi}_0\mu D(u)\nu_0=H_t(u) && \text{on } \Gamma,\ t>0,\\
&\nu_0\cdot S(u,\pi)\nu_0+\sigma(m-\Delta_\Gamma)\eta=H_n(u,\pi)+\sigma\mathcal{H}_0(\Gamma)-g_a\xi_n && \text{on } \Gamma,\ t>0,\\
&u=0 && \text{on } \Gamma_b,\ t>0,\\
&u|_{t=0}=u_0(\xi)\ \text{in } \Omega,\quad \eta|_{t=0}=0\ \text{on } \Gamma, && (2.13)
\end{aligned}$$

where

$$\begin{aligned}
&G(u)=(m-\sigma\Delta_\Gamma)^{-1}\dot F(u),\\
&\dot F(u)=\Delta_\Gamma\nu_0\cdot u-\nu_0\cdot\dot\Delta_{\Gamma(t)}\int_0^t u\,d\tau+\nu_0\cdot(\Delta_\Gamma-\Delta_{\Gamma(t)})u-\nu_0\cdot\dot\Delta_{\Gamma(t)}\xi,\\
&H_t(u)=-\mathbf{\Pi}_0(\mathbf{\Pi}_t-\mathbf{\Pi}_0)(\mu D(u)\nu_{tu})-\mathbf{\Pi}_0(\mu D(u)(\nu_{tu}-\nu_0))-\mathbf{\Pi}_0\mathbf{\Pi}_t(Q(u)\nu_{tu}),\\
&H_n(u,\pi)=\nu_0\cdot(S(u,\pi)(\nu_0-\nu_{tu}))-\nu_0\cdot(Q(u)\nu_{tu})-g_a\int_0^t u_n\,d\tau\\
&\quad -g_a\int_0^t u_n\,d\tau\,\nu_0\cdot\nu_{tu}-g_a\xi_n\,\nu_0\cdot(\nu_{tu}-\nu_0)-2\sigma\Big(\nabla_\Gamma\cdot\int_0^t u\,d\tau\Big)\nabla_\Gamma\cdot\nu_0+\sigma m\nu_0\cdot\int_0^t u\,d\tau,
\end{aligned}$$

and $Q(u)$, $R(u)$, $E(u)$ and $\tilde E(u)$ are nonlinear terms defined by (1.7).

3 Initial flow

In this section, we shall discuss the initial flow to reduce the problem (2.13) to the case where $u_0(\xi)=0$ and $\sigma\mathcal{H}_0(\Gamma)-g_a\xi_n=0$. We study the problem in the two steps.

Step 1 Let (u_1,π_1) be a solution to the problem:

$$\begin{aligned}
&\lambda u_1-\operatorname{Div}S(u_1,\pi_1)=0 && \text{in } \Omega,\\
&\operatorname{div}u_1=0 && \text{in } \Omega,\\
&S(u_1,\pi_1)\nu_0=(\sigma\mathcal{H}_0(\Gamma)-g_a\xi_n)\nu_0 && \text{on } \Gamma,\\
&u_1=0 && \text{on } \Gamma_b. \qquad (3.1)
\end{aligned}$$

If positive number λ is large enough, then we know that (3.1) admits a unique solution

$$(u_1,\pi_1)\in W^2_q(\Omega)\times\hat W^1_q(\Omega)$$

which satisfies the estimate

$$|\lambda|\|u_1\|_{L_q(\Omega)} + \|\nabla^2 u_1\|_{L_q(\Omega)} + \|\nabla \pi_1\|_{L_q(\Omega)} \leq C_1(\sigma\|\mathcal{H}_0(\Gamma)\|_{W_q^{1-1/q}(\Gamma)} + g_a\|\xi_n\|_{W_q^{1-1/q}(\Gamma)}). \tag{3.2}$$

Step 2 We consider the linear time-dependent problem in the time interval $(0,2)$:

$$\begin{aligned}
&\partial_t u_2 - \mathrm{Div}\, S(u_2,\pi_2) = -\lambda u_1 && \text{in } \Omega \times (0,2)\\
&\mathrm{div}\, u_2 = 0 && \text{in } \Omega \times (0,2)\\
&S(u_2,\pi_2)\nu_0 = 0 && \text{on } \Gamma \times (0,2)\\
&u_2 = 0 && \text{on } \Gamma_b \times (0,2)\\
&u_2|_{t=0} = u_0(\xi) - u_1(\xi) \text{ in } \Omega.
\end{aligned} \tag{3.3}$$

If the initial data $u_0 \in [L_q(\Omega), W_q^2(\Omega)]_{1-(1/p),p}$ satisfies the compatibility condition (1.9), then problem (3.3) admits a unique solution

$$u_2 \in W_{q,p}^{2,1}(\Omega \times (0,2)), \quad \pi_2 \in L_p((0,2), \hat{W}_q^1(\Omega)).$$

Moreover, π_2 has an additional information about the regularity at boundary with respect to time variable t. To state this fact, we give a functional space. Given $\alpha \geq 0$, we set

$$\begin{aligned}
< D_t >^\alpha u(t) &= \mathcal{F}^{-1}[(1+s^2)^{\alpha/2}\mathcal{F}u(s)](t),\\
H_p^\alpha(\mathbb{R}, X) &= \{u \in L_p(\mathbb{R}, X) : < D_t >^\alpha u \in L_p(\mathbb{R}, X)\},\\
\|u\|_{H_p^\alpha(\mathbb{R},X)} &= \| < D_t >^\alpha u\|_{L_p(\mathbb{R},X)} + \|u\|_{L_p(\mathbb{R},X)}.
\end{aligned}$$

Here and hereafter, $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse formula, respectively. Set

$$\begin{aligned}
H_{q,p}^{1,1/2}(D \times \mathbb{R}) &= H_p^{1/2}(\mathbb{R}, L_q(D)) \cap L_p(\mathbb{R}, W_q^1(D)),\\
H_{q,p}^{1,1/2}(D \times (0,T)) &= \{u \mid v \in H_{q,p}^{1,1/2}(D \times \mathbb{R}),\ \ u = v \text{ on } D \times (0,T)\},\\
\|u\|_{H_{q,p,0}^{1,1/2}(D\times(0,T))} &= \inf\{\|v\|_{H_{q,p}^{1,1/2}(D\times\mathbb{R})} \mid v \in H_{q,p}^{1,1/2}(D \times \mathbb{R}),\ \ v = u \quad \text{on } D \times (0,T)\}.
\end{aligned}$$

By using these symbols, we can state the additional property of π_2 on the boundary Γ as follows: There exists $\bar{\pi}_2 \in H_{q,p}^{1,1/2}(\Omega \times \mathbb{R})$ such that $\bar{\pi}_2|_\Gamma = \pi_2|_\Gamma$. Moreover, u_2, π_2 and $\bar{\pi}_2$ satisfy the estimate:

$$\begin{aligned}
&\|u_2\|_{W_{q,p}^{2,1}(\Omega\times(0,2))} + \|\pi_2\|_{L_p((0,2),\hat{W}_q^1(\Omega))} + \|\bar{\pi}_2\|_{H_{q,p}^{1,1/2}(\Omega\times(0,\infty))}\\
&\qquad \leq C_2(\|u_0\|_{B_{q,p}^{2(1-1/p)}(\Omega))} + \sigma\|\mathcal{H}_0(\Gamma)\|_{W_q^{1-1/q}(\Gamma)} + g_a\|\xi_n\|_{W_q^{1-1/q}(\Gamma)}).
\end{aligned} \tag{3.4}$$

If we set $z = u_1 + u_2$ and $\tau = \pi_1 + \pi_2$, then (z,τ) satisfies the time-dependent linear equation in the time interval $(0,2)$:

$$\begin{aligned}
&\partial_t z - \mathrm{Div}\, S(z,\tau) = 0 && \text{in } \Omega \times (0,2)\\
&\mathrm{div}\, z = 0 && \text{in } \Omega \times (0,2)\\
&S(z,\tau)\nu_0 = (\sigma\mathcal{H}_0(\Gamma) - g_a\xi_n)\nu_0 && \text{on } \Gamma \times (0,2)\\
&z = 0 && \text{on } \Gamma_b \times (0,2)\\
&z|_{t=0} = u_0 && \text{in } \Omega.
\end{aligned} \tag{3.5}$$

z is our initial flow.

Now, we look for a solution (u,π) of the equation (2.13) of the form: $u = z + w$ and $\pi = \tau + \kappa$ in the time interval $(0,T)$ with $0 < T \leq 1$. Setting $\bar{\tau} = \pi_1 + \bar{\pi}_2$, we see that v, θ and η should satisfy the equations:

$$\begin{aligned}
&\partial_t w - \mathrm{Div}\, S(w,\kappa) = \mathrm{Div}\, Q(z+w) + R(z+w)\nabla(\tau+\kappa) + f(X_{z+w}(\xi,t),t) && \text{in } \Omega \times (0,T)\\
&\mathrm{div}\, w = E(z+w) = \mathrm{div}\tilde{E}(z+w) && \text{in } \Omega \times (0,T)
\end{aligned}$$

$$
\begin{aligned}
&\partial_t\eta - \nu_0\cdot w = G(z+w) + \nu_0\cdot z && \text{on } \Gamma\times(0,T)\\
&\mathbf{\Pi}_0\mu\, D(w)\nu_0 = H_t(z+w) && \text{on } \Gamma,\ t>0\\
&\nu_0\cdot S(w,\kappa)\nu_0 + \sigma(m-\Delta_\Gamma)\eta = H_n(z+w,\bar\tau+\kappa) && \text{on } \Gamma,\ t>0\\
&w=0 && \text{on } \Gamma_b,\ t>0\\
&w|_{t=0}=0 \text{ in } \Omega, \quad \eta|_{t=0}=0 \text{ on } \Gamma. && (3.6)
\end{aligned}
$$

4 L_p-L_q maximal regularity

In order to solve (3.6) locally in time, we consider the following time-dependent problem:

$$
\begin{aligned}
&\partial_t u - \operatorname{Div} S(u,\pi) = f, && \text{in } \Omega,\ t>0,\\
&\operatorname{div} u = f_d = \operatorname{div}\tilde f_d && \text{in } \Omega,\ t>0,\\
&\partial_t\eta - \nu_0\cdot u = d && \text{on } \Gamma,\ t>0,\\
&S(u,\pi)\nu_0 + \sigma(m-\Delta_\Gamma)\eta\,\nu_0 = h && \text{on } \Gamma,\ t>0,\\
&u=0 && \text{on } \Gamma_b,\ t>0,\\
&u|_{t=0}=0,\ \eta|_{t=0}=0. && (4.1)
\end{aligned}
$$

Our L_p-L_q maximal regularity result about the problem (4.1) is the following, which will be proved in a forthcoming paper based on [13].

Theorem 4.1. *Let $\Omega\subset\mathbb{R}^n$ $(n\ge 2)$ be one of a bounded domain, an exterior domain, a perturbed half-space or a perturbed layer. Let $1<p,q<\infty$ and r be a number such that $n<r<\infty$ and $q\le r$. Assume that $\Gamma\in W_r^{3-1/r}$ and $\Gamma_b\in W_r^{2-1/r}$, respectively. Let T_0 be any positive number and $0<T\le T_0$. Then, for any f, f_d, $\tilde f_d$, d and h in (4.1) satisfying the regularity conditions:*

$$
\begin{gathered}
f\in L_p((0,T),L_q(\Omega))^n,\ \ f_d\in L_p((0,T),W_q^1(\Omega)),\ \ \tilde f_d\in W_p^1((0,T),L_q(\Omega))^n,\\
d\in L_p((0,T),W_q^{2-1/q}(\Gamma)),\ \ h\in H_{q,p}^{1,1/2}(\Omega\times(0,T))^n
\end{gathered}
$$

and compatibility conditions:

$$
\tilde f_d|_{t=0}=0,\ \ h|_{t=0}=0,
$$

the problem (4.1) admits a unique solution (u,π,η) which satisfies the regularity condition:

$$
\begin{aligned}
&u\in W_{q,p}^{2,1}(\Omega\times(0,T)), \quad \pi\in L_p((0,T),\hat W_q^1(\Omega)),\\
&\partial_t\eta\in L_p((0,T),W_q^{2-1/q}(\Gamma)), \quad \eta\in L_p((0,T),W_q^{3-1/q}(\Gamma)).
\end{aligned}
$$

Moreover there exists a $\bar\pi|_\Gamma=\pi|_\Gamma$ such that $\bar\pi\in H_{q,p}^{1,1/2}(\Omega\times(0,T))$, and u, π, η and $\bar\pi$ satisfy the estimate:

$$
\begin{aligned}
&\|u\|_{W_{q,p}^{2,1}(\Omega\times(0,T))} + \|\nabla\pi\|_{L_p((0,T),L_q(\Omega))} + \|\bar\pi\|_{H_{q,p}^{1,1/2}(\Omega\times(0,T))}\\
&\quad + \|\partial_t\eta\|_{L_p((0,T),W_q^{2-(1/q)}(\Gamma))} + \|\eta\|_{L_p((0,T),W_q^{3-(1/q)}(\Gamma))}\\
&\le C\Big(\|f\|_{L_p((0,T),L_q(\Omega))} + \|d\|_{L_p((0,T),W_q^{2-1/q}(\Gamma))}\\
&\quad + \|f_d\|_{L_p((0,T),W_q^1(\Omega))} + \|\tilde f_d\|_{W_p^1((0,T),L_q(\Omega))} + \|h\|_{H_{q,p}^{1,1/2}(\Omega\times(0,T))}\Big).
\end{aligned}
$$

with some constant C independent of T whenever $0<T\le T_0$.

Using the standard fixed point argument based on Theorem 4.1 (cf. [11]), we can solve (3.6) locally in time. Concerning the estimates of the nonlinear terms appearing in (3.6), we use the following facts.

Lemma 4.2. *Let $n<q<\infty$. Then, for any f, $g\in W_q^1(\Omega)$ and $u\in W_{1-(1/q)}^1(\Gamma)$ and $v\in W_{1-(1/q)}^1(\Gamma)$, we have*

$$
\|fg\|_{W_q^1(\Omega)}\le C_{\Omega,q}\|f\|_{W_q^1(\Omega)}\|g\|_{W_q^1(\Omega)}, \quad \|uv\|_{W_q^{1-(1/q)}(\Gamma)}\le C_{\Gamma,q}\|u\|_{W_q^{1-(1/q)}(\Gamma)}\|v\|_{W_q^{1-(1/q)}(\Gamma)}
$$

for some constants $C_{\Omega,q}$ and $C_{\Gamma,q}$.

Lemma 4.3. *$BUC((0,2),[L_q(\Omega),W^2_q(\Omega)]_{1-(1/p),p})$ is continuously imbedded into $W^{2,1}_{q,p}(\Omega\times(0,2))$. Here, $BUC((0,2),X)$ denotes the set of all bounded uniformly continuous X-valued function on $(0,2)$.*

Lemma 4.4. *Let $1<p<\infty$, $n<q<\infty$ and $0<T\leqq 1$. Set*

$$\hat W^{1,1}_{q,p}(\Omega\times I)=\{f\in W^{1,1}_{q,\infty}(\Omega\times I):\partial_t f\in L_p(I,W^1_q(\Omega))\}.$$

If $f\in \hat W^{1,1}_{q,p}(\Omega\times\mathbb{R})$, $g\in H^{1,1/2}_{q,p}(\Omega\times\mathbb{R})$ and f vanishes when $t\not\in[0,2T]$, then we have

$$\|fg\|_{H^{1,1/2}_{q,p}(\Omega\times\mathbb{R})}\leqq C_{p,q}[\|f\|_{L_\infty(\mathbb{R},W^1_q(\Omega))}+T^{(q-n)/(pq)}\|f_t\|^{(1-n/(2q))}_{L_\infty(\mathbb{R},L_q(\Omega))}\|f_t\|^{n/(2q)}_{L_p(\mathbb{R},W^1_q(\Omega))}]\|g\|_{H^{1,1/2}_{q,p}(\Omega\times\mathbb{R})}.$$

Lemma 4.5. *Let $\Omega\subset\mathbb{R}^n$ $(n\ge 2)$ be one of a bounded domain, an exterior domain, a perturbed half-space or a perturbed layer. Assume that $\Gamma\in C^2$. Let $1<q<\infty$ and let $W^{-1/q}_q(\Gamma)$ be the dual space of $W^{1-(1/q')}_{q'}(\Gamma)$ with $(1/q)+(1/q')=1$. Then, we have the following two assertions:*
(1) There exists an $m\ge 1$ such that $(m-\Delta_\Gamma)$ is a bijection from $W^{2-(1/q)}_q(\Gamma)$ onto $W^{-1/q}_q(\Gamma)$. (2) For any $f\in W^{-1/q}_q(\Gamma)$ and $g\in W^{1-(1/q)}_q(\Gamma)$ we have

$$\|fg\|_{W^{-1/q}_q(\Gamma)}\le C_{\Gamma,q}\|f\|_{W^{-1/q}_q(\Gamma)}\|g\|_{W^{1-(1/q)}_q(\Gamma)}.$$

A A proof of Lemma 2.1

Solonnikov used the fact formulated in Lemma 2.1 without proof to formulate his linearization of the nonlinear problem, that is different from ours given in Section 2. We did not find any proof of Lemma 2.1 and it is important to our formulation, so that we will give its proof below.

To prove Lemma 2.1, we use the following fact about the determinant which can be proved by mathematical induction.

Lemma A.1. *Let $a_1,\dots,a_n$ be n real numbers such that $a_1^2+\cdots a_n^2<1$ and let δ_{ij} be the Kronecker's delta symbol, that is $\delta_{ii}=1$ and $\delta_{ij}=0$ for $i\neq j$. Let A be an $n\times n$ matrix whose (i,j) component is $\delta_{ij}-a_ia_j$, that is*

$$A=\begin{pmatrix}1-a_1^2 & -a_1a_2 & \cdots & -a_1a_n\\ -a_2a_1 & 1-a_2^2 & \cdots & -a_2a_n\\ \vdots & \vdots & \ddots & \vdots\\ -a_na_1 & -a_na_2 & \cdots & 1-a_n^2\end{pmatrix}$$

Then, $\det A=1-(\sum_{j=1}^n a_j^2)$.

Now we prove Lemma 2.1. If $d=0$, then obviously $\mathbf{\Pi}_0\mathbf{\Pi}_t d=0$ and $\nu_0\cdot d=0$. Therefore, we shall prove the opposite direction. Let $\mathbf{e}_1,\dots,\mathbf{e}_{n-1}$, ν_0 be an orthonormal basis of $\mathbb{R}^n$ and set $a_j=\mathbf{e}_j\cdot\nu_t$ $(j=1,\dots,n-1)$. Here, $\cdot$ stands for the standard inner-product of $\mathbb{R}^n$. Then, we have

$$\nu_t=\sum_{j=1}a_j\mathbf{e}_j+(\nu_t\cdot\nu_0)\nu_0.$$

Since $\nu_t\cdot\nu_0\neq 0$ and $\nu_t\cdot\nu_t=1$, we have

$$\sum_{j=1}^{n-1}a_j^2<1. \tag{A.1}$$

Let d be an n-vector such that $\mathbf{\Pi}_0\mathbf{\Pi}_t d=0$ and $\nu_0\cdot d=0$. Then, we have

$$d=\sum_{j=1}^{n-1}(\mathbf{e}_j\cdot d)\mathbf{e}_j \tag{A.2}$$

because $\nu_0 \cdot d = 0$. Since $d \cdot \nu_t = \sum_{j=1}^{n-1}(\mathbf{e}_j \cdot d)(\mathbf{e}_j \cdot \nu_t) = \sum_{j=1}^{n-1} a_j(\mathbf{e}_j \cdot d)$, we have $\mathbf{\Pi}_t d = d - (d \cdot \nu_t)\nu_t = d - (\sum_{j=1}^{n} a_j(\mathbf{e}_j \cdot d))\nu_t$. Therefore, we have

$$\begin{aligned}\mathbf{\Pi}_0\mathbf{\Pi}_t d &= d - (\sum_{j=1}^{n} a_j(\mathbf{e}_j \cdot d))\nu_t - (\nu_0 \cdot (d - (\sum_{j=1}^{n} a_j(\mathbf{e}_j \cdot d))\nu_t))\nu_0 \\ &= d - (\sum_{j=1}^{n-1} a_j(\mathbf{e}_j \cdot d))\nu_t + (\sum_{j=1}^{n-1} a_j(\mathbf{e}_j \cdot d))(\nu_0 \cdot \nu_t)\nu_0 \\ &= d - \sum_{k=1}^{n-1}(\sum_{j=1}^{n-1} a_j(\mathbf{e}_j \cdot \nu_t))(\mathbf{e}_k \cdot \nu_t)\mathbf{e}_k,\end{aligned}$$

where we have used $\nu_t = \sum_{k=1}^{n-1}(\mathbf{e}_k \cdot \nu_t)\mathbf{e}_k + (\nu_0 \cdot \nu_t)\nu_0$ in the final step. Now, substituting (A.2) into the last formula, we have

$$\mathbf{\Pi}_0\mathbf{\Pi}_t d = \sum_{k=1}^{n-1}(\mathbf{e}_k \cdot d - \sum_{j=1}^{n-1} a_j a_k(\mathbf{e}_j \cdot d))\mathbf{e}_k.$$

Since $\mathbf{\Pi}_0\mathbf{\Pi}_t d = 0$ and $\nu_0 \cdot d = 0$, we have

$$\mathbf{e}_k \cdot d - \sum_{j=1}^{n-1} a_j a_k(\mathbf{e}_j \cdot d) = 0 \quad (k = 1, \ldots, n-1). \tag{A.3}$$

If we define the $(n-1) \times (n-1)$ matrix A by the formula:

$$A = \begin{pmatrix} 1 - a_1^2 & -a_1 a_2 & \cdots & -a_1 a_{n-1} \\ -a_2 a_1 & 1 - a_2^2 & \cdots & -a_2 a_{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1}a_1 & -a_{n-1}a_2 & \cdots & 1 - a_{n-1}^2 \end{pmatrix},$$

then noting (A.1) and using Lemma A.1 we have $\det A \neq 0$. On the other hand, the equation (A.3) is written in the matrix form: $A(\mathbf{e}_1 \cdot d, \ldots, \mathbf{e}_{n-1} \cdot d)^* = 0$, which implies that $\mathbf{e}_1 \cdot d = \cdots = \mathbf{e}_{n-1} \cdot d = 0$. Since $\nu_0 \cdot d = 0$, we have $d = 0$, which completes the proof of Lemma 2.1. □

References

[1] H. Abels, *The initial-Value problem for the Navier-Stokes equations with a free surface in L_q Sobolev spaces*, Adv. Differential Equations, **10** (2005) 45–64.

[2] G. Allain, *Small-time existence for the Navier-Stokes equations with a free surface*, Appl. Math. Optim., **16** (1987) 37–50.

[3] J. T. Beale, *Large time regularity of viscous surface waves*, Arch. Rat. Mech. Anal., **84** (1984) 307–352.

[4] J. T. Beale and T. Nishida, *Large time behavior of viscous surface waves*, Lecture Notes in Numer. Appl. Anal., **8** (1985) 1–14.

[5] I. Sh. Mogilevskiĭ and V. A. Solonnikov, *On the solvability of a free boundary problem for the Navier-Stokes equations in the Hölder spaces of functions*, Nonlinear Analysis. A Tribute in Honour of Giovanni Prodi, Quaderni, Pisa (1991) 257–272.

[6] P. B. Mucha and W. Zajączkowski, *On the existence for the Cauchy-Neumann problem for the Stokes system in the L_p-framework*, Studia Mathematica, **143** (2000) 75–101.

[7] P. B. Mucha and W. Zajączkowski, *On local existence of solutions of the free boundary problem for an incompressible viscous self-gravitating fluid motion*, Applicationes Mathematicae, **27** (2000) 319–333.

[8] M. Padula, V. A. Solonnikov *On the global existence of nonsteady motions of a fluid drop and their exponential decay to a uniform rigid rotation*, Quad. Mat., **10** (2002) 185–218.

[9] J. Prüss and G. Simonett, *On the two-phase Navier-Stokes equations with surface tension*, to appear in Proc. Banach Center Publication.

[10] B. Schweizer, *Free boundary fluid systems in a semigroup approach and oscillatory behavior*, SIAM J. Math. Anal., **28** (1997) 1135–1157.

[11] Y. Shibata and S. Shimizu, *On some free boundary problem for the Navier-Stokes equations*, Differential Integral Equations, **20** (2007), 241–276.

[12] Y. Shibata and S. Shimizu, *On the L_p-L_q maximal regularity of the Neumann problem for the Stokes equations in a bounded domain*, J. reine angew. Math. (Crelles Journal), **615** (2008), 157–209.

[13] Y. Shibata and S. Shimizu, *On a resolvent estimate of the Stokes system in a half space arising from a free boundary problem for the Stokes equations*, Math. Nachr., **282** (2009), 482–499.

[14] Y. Shibata and S. Shimizu, *Local solvability of free surface problems for the Navier-Stokes equations with surface tension*, in preparation.

[15] V. A. Solonnikov, *Solvability of the evolution problem for an isolated mass of a viscous incompressible capillary liquid*, Zap. Nauchn. Sem. (LOMI), **140** (1984) 179–186 (in Russian); English transl.: J. Soviet Math., **32** (1986) 223–238.

[16] V. A. Solonnikov, *Unsteady motion of a finite mass of fluid, bounded by a free surface*, Zap. Nauchn. Sem. (LOMI), **152** (1986) 137–157 (in Russian); English transl.: J. Soviet Math., **40** (1988) 672–686.

[17] V. A. Solonnikov, *On the transient motion of an isolated volume of viscous incompressible fluid*, Izv. Akad. Nauk SSSR Ser. Mat., **51** (1987) 1065–1087 (in Russian); English transl.: Math. USSR Izv., **31** (1988) 381–405.

[18] V. A. Solonnikov, *On nonstationary motion of a finite isolated mass of self-gravitating fluid*, Algebra i Analiz, **1** (1989) 207–249 (in Russian); English transl.: Leningrad Math. J., **1** (1990) 227–276.

[19] V. A. Solonnikov, *Solvability of the problem of evolution of a viscous incompressible fluid bounded by a free surface on a finite time interval*, Algebra i Analiz, **3** (1991) 222–257 (in Russian); English transl.: St. Petersburg Math. J., **3** (1992) 189–220.

[20] V. A. Solonnikov, *Lectures on evolution free boundary problems: classical solutions*, L. Ambrosio et al.: LNM 1812, P.Colli and J.F.Rodrigues (Eds.), (2003) 123-175, Springer-Verlag, Berlin, Heidelberg.

[21] A. Tani, *Small-time existence for the three-dimensional incompressible Navier-Stokes equations with a free surface*, Arch. Rat. Mech. Anal., **133** (1996) 299–331.

[22] A. Tani and N. Tanaka, *Large time existence of surface waves in incompressible viscous fluids with or without surface tension*, Arch. Rat. Mech. Anal., **130** (1995) 303–314.