On presentations and a derived equivalence classification of orbit categories

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学 位 論 文 要 旨

Abstract of Doctoral Thesis

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論文要旨：

Abstract：

In this paper we investigated problems on orbit categories.

The first result gives presentations of Grothendieck constructions. Let I be a small category and $\Bbbk$ a commutative ring, and denote by $\Bbbk$-**Cat** the 2-category of all $\Bbbk$-categories.

The Grothendieck construction is a way to form a single category $\mathrm{Gr}(X)$ from a diagram X of small categories indexed by I. This can be also regarded as a generalization of orbit category construction from a category with a group action.

In [4] Asashiba has shown that if (colax) functors $X, X' \colon I \to \Bbbk\text{-}\mathbf{Cat}$ are derived equivalent, then so are their Grothendieck constructions $\mathrm{Gr}(X)$ and $\mathrm{Gr}(X')$. If categories $\mathcal{C}$ and $\mathcal{C}'$ are derived equivalent, then so are their diagonal functors $\Delta(\mathcal{C})$ and $\Delta(\mathcal{C}')$, and hence so are $\mathrm{Gr}(\Delta(\mathcal{C}))$ and $\mathrm{Gr}(\Delta(\mathcal{C}'))$. Therefore it will be important to investigate structures of Grothendieck constructions of functors.

Let A be a $\Bbbk$-algebra, which we regard as a $\Bbbk$-category with a single object, and let $\Delta(A)\colon I \to \Bbbk\text{-}\mathbf{Cat}$ be the diagonal functor (i.e., a functor sending all objects of I to A and all morphisms in I to the identity functor $\mathbb{1}_A$). In [4] it is noted that if I is a semigroup G, a poset S, or the free category $\mathbb{P}Q$ of a quiver Q, then the Grothendieck construction $\mathrm{Gr}(\Delta(A))$ of the diagonal functor $\Delta(A)$ is isomorphic to the semigroup algebra AG, the incidence algebra AS, or the path-algebra AQ, respectively. Further in [3] a quiver presentation of the orbit category $\mathcal{C}/G$ for each $\Bbbk$-category $\mathcal{C}$ with a G-action is given in the case that $\Bbbk$ is a field (cf. [5] for the finite group case). This result can be seen as a computation of a quiver presentation of the Grothendieck construction $\mathrm{Gr}(X)$ of each functor $X\colon G \to \Bbbk\text{-}\mathbf{Cat}$. In chapter 1 we generalize the two results above to obtain the following.

(1) We compute the Grothendieck construction $\mathrm{Gr}(\Delta(A))$ of the diagonal functor $\Delta(A)$ for each $\Bbbk$-algebra A and each small category I.
(2) We give a quiver presentation of the Grothendieck construction $\mathrm{Gr}(X)$ for each functor $X\colon I \to \Bbbk\text{-}\mathbf{Cat}$ and each small category I when $\Bbbk$ is a field.

The second result gives the derived equivalence classification of generalized multifold extensions of piecewise hereditary algebras of tree type. Let $\Bbbk$ be an algebraically closed field, and assume that all algebras are basic and finite-dimensional $\Bbbk$-algebras and that all categories are $\Bbbk$-categories.

In [1] Asashiba gave the classification of representation-finite self-injective algebras. The technique used there (a covering technique for derived equivalences) is applicable also for representation-infinite algebras; it requires that the algebras in consideration have the form of orbit categories (usually of repetitive categories of some algebras having no oriented cycles in their ordinary quivers). In fact, it was applied in [2] to give the classification of twisted

multifold extensions of piecewise hereditary algebras of tree type by giving a complete invariant. Here an algebra is called a *twisted multifold extension* of an algebra A if it has the form

$$T^n_\psi(A) := \hat{A}/\langle\hat{\psi}\nu_A^n\rangle \tag{0.1}$$

for some positive integer n and some automorphism ψ of A, where $\hat{A}$ is the repetitive algebra of A, ν_A is the Nakayama automorphism of $\hat{A}$ and $\hat{\psi}$ is the automorphism of $\hat{A}$ naturally induced from ψ; and an algebra A is called a *piecewise hereditary algebra of tree type* if A is an algebra derived equivalent to a hereditary algebra whose ordinary quiver is an oriented tree. In chapter 2 we extend this classification to a wider class of algebras. To state this class of algebras we introduce the following terminologies. For an integer n we say that an automorphism ϕ of $\hat{A}$ has a *jump* n if $\phi(A^{[0]}) = A^{[n]}$. An algebra of the form

$$\hat{A}/\langle\phi\rangle$$

for some automorphism ϕ of $\hat{A}$ with jump n for some positive integer n is called a *generalized multifold extension* of A. Since obviously $\hat{\psi}\nu_A^n$ is an automorphism of $\hat{A}$ with jump n in the formula (0.1), twisted multifold extensions are generalized multifold extensions. In chapter 2 we will give the derived equivalence classification of generalized multifold extensions of piecewise hereditary algebras of tree type by giving a complete invariant.

References

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