On local Lp−Lq well-posedness of incompressible two phase flows with phase transitions

（課程博士・様式７）（Doctoral qualification by coursework,Form 7）

学 位 論 文 要 旨

Abstract of Doctoral Thesis

専　　攻：情報科学　　　　　　　　　　氏　名：八木真太郎

Course：Information Science and Technology　　　　　Name：Shintaro Yagi

論文題目：相転移を伴う非圧縮性２相流の局所 L_p-L_q 適切性について

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論文要旨：

Abstract： We consider incompressible two-phase flows with phase transitions where the interface nearly flat in the case of non-equal densities and equal densities. We prove local well-posedness of the problem L_p in time L_q in space setting based on maximal L_p-L_q regularity of a linearized problem. The problem in the case of non-equal densities is quite different from that in the case of equal densities. In the case of equal densities, the phase flux does not enter the Navier-Stokes problem, and the system is called temperature dominated. But in the case of different densities, the phase flux causes a jump in the velocity field on the interface, and the system is called velocity dominated.

First, we introduce two-phase flows with phase transitions and state the main result of this paper. Specifically, we interpret physical quantity, the difference between the case with phase transitions and the case without phase transitions, and each equations of continuity in fluid mechanics for compressible case and incompressible case. Additionally, we explain previous works and key embedding theorem, interpolation theory and so on that are useful for the nonlinear problem. The main result of ours holds for p, q that satisfies all of the followings, $2<p<\infty$, $n<q<\infty$, $2/p+n/q<1$. J. Pruess and S. Shimizu prove local well-posedness of the case of $p=q$ and $n+2<p<\infty$ for the same model under some smallness condition. It holds that $2q/(q-n)<n+2$ if $n<q<\infty$, therefore our result is an extension of the previous work by J. Pruess and S. Shimizu. We reduce the nonlinear problem that describes incompressible two-phase flows with phase transitions to a quasilinear problem in a fixed domain with changing of variables. We gather nonlinear terms into the right hand side and replace these nonlinear terms with given functions. We state results for the linearized problem and explain the reason why we prove maximal L_p-L_q regularity of the linearized problem in detail in view of estimates in semi-group theory, where we set $1<p,q<\infty$. We do not argue for the problem abstractly in this paper but contents are the background of argument and calculus. We state the definition of R-boundedness,

operator valued Fourier multiplier theorem by L. Weis and an useful lemma that gives an sufficient condition for R-boundedness. The lemma changes argument for boundedness of operators into that for boundedness of functions. We prove maximal L_p-L_q regularity of the linearized problem with methods by Y. Shibata and S. Shimizu. Applying Laplace transform for time and Fourier transform for tangential space variables, the linearized problem is converted into an ordinary differential equation with respect to normal space variable and whose characteristic roots play important roles for estimates and extension. We really solve the ordinary differential equation. We suitably decompose the multiplier of the height function when we estimate higher order derivatives of the extended height function in time. It is important that we analyze whether or not the solution satisfies the condition of the lemma of which that we make use to prove R-boundedness. We describe the quasilinear problem to which we refer, and prove our main result: local L_p-L_q well-posedness. Moreover, we explain interpolation theory mentioned with argument in Bessel potential space, Triebel-Lizorkin space and Besov space. First, we decompose the solution into the sum of the term whose initial value is 0 and the term that is near the initial value, and then we could reduce the original problem to a quasilinear problem whose initial values are 0. We need to estimate nonlinear terms in the right hand side by lemmas introduced in Section 1 but it is difficult point that we derive the power of time, T. For terms from which we could not derive the power of T, we have to use the smallness condition for the initial data. Incidentally, we could hardly solve this nonlinear problem in a class whose regularity is lower. We treat the case of equal densities. This case is Stefan problem with Gibbs-Thomson correction, which is only weakly coupled to the standard Navier-Stokes equation. We could solve the problem essentially in the same way as the case non-equal densities, therefore first we derive maximal L_p-L_q regularity of the linearized problem which consists of linearized Stefan problem and standard Stokes equation. However, the regularity of the solution for the case of equal densities is different from that for the case of non-equal densities. Moreover, descriptions of the solutions for the linearized problems are clearly different from each other. If we compare the case of non-equal densities, the expression of the height function for the linearized problem in the case of equal-densities is simple.

The local well-posedness of the model in L_p in time L_q in space setting was proved by Shimizu and Yagi under smallness assumption for initial data. In Section 6-10, we remove the smallness assumption for initial data in latter sections of the first part. By appropriate technique, we reduce variable coefficients problem into constant coefficients problem.